Online Appendix for: Optimal Contracts with Enforcement Risk

Nicola Gennaioli

CREI and Universitat Pompeu Fabra.

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1. State verification under a general contract $[x(\overline{r}), x(\underline{r})] \in [0,1]^2$

Consider contract $[x(\overline{r}), x(\underline{r})]$. After observing s, if the judge reports $\overline{r}$, he erroneously sets E-control with probability $x(\overline{r})\Pr(\underline{r}\,|s)$, if he reports $\underline{r}$, he erroneously sets I-control with probability $[1-x(\overline{r})]\Pr(\overline{r}\,|s)$. The judge's expected cost of reporting $\overline{r}$ is equal to $x(\overline{r})\beta_I(\lambda-\underline{r})\Pr(\underline{r}\,|s)+[1-x(\overline{r})]\,\beta_E(\overline{r}-\lambda)\Pr(\overline{r}\,|s)$, while his expected cost of reporting $\underline{r}$ is equal to $x(\underline{r})\beta_I(\lambda-\underline{r})\Pr(\underline{r}\,|s)+[1-x(\underline{r})]\,\beta_E(\overline{r}-\lambda)\Pr(\overline{r}\,|s)$. A judge having pro-enterpreneur bias $\beta \equiv \beta_E/\beta_I$ thus reports $\overline{r}$ if and only if:

$$[x(\overline{r})-x(\underline{r})]\,\beta(\overline{r}-\lambda)\Pr(\overline{r}\,|s) \geq [x(\overline{r})-x(\underline{r})]\,(\lambda-\underline{r})\Pr(\underline{r}\,|s). \tag{1}$$

If $x(\overline{r}) > x(\underline{r})$, the term $[x(\overline{r})-x(\underline{r})]$ drops from both sides and (1) is identical to Equation (9) in the text, which prevails under the fully contingent contract $[x(\overline{r})=1, x(\underline{r})=0]$. If instead $x(\overline{r}) < x(\underline{r})$, the adjudication rule is the opposite of (9) in the text, but this is just equivalent to considering contract $[x'(\overline{r}), x'(\underline{r})]$ where $x'(\overline{r}) = x(\underline{r})$, $x'(\underline{r}) = x(\overline{r})$.

Equation (1) can be derived from the judge's attempt to maximize his utility of control (rather than minimize his error costs). To see this, suppose that the judge views his payoff under E-control as being equal to $\widetilde{\beta}_E r$ and his payoff under I-control as being equal to $\widetilde{\beta}_I\lambda$, where $\widetilde{\beta}_E, \widetilde{\beta}_I \geq 0$ and $\widetilde{\beta}_E + \widetilde{\beta}_I = 1$. In this setting, a judge is pro-entrepreneur when $\widetilde{\beta}_E > 1/2$ and pro-investor when $\widetilde{\beta}_E < 1/2$. Upon observing s, the judge sets the probability

$\psi(s)$ of reporting $\overline{r}$ by solving:

$$\max_{\psi(s)} \; [x(\overline{r}) - x(\underline{r})] \left[\left(\widetilde{\beta}_E \overline{r} - \widetilde{\beta}_I \lambda \right) \Pr(\overline{r} \,|\, s) - \left(\widetilde{\beta}_I \lambda - \widetilde{\beta}_E \underline{r} \right) \Pr(\underline{r} \,|\, s) \right] \psi(s) + \\ + \left[\widetilde{\beta}_E E(r \,|\, s) - \widetilde{\beta}_I \lambda \right] x(\underline{r}) + \widetilde{\beta}_I \lambda. \tag{2}$$

The verification rule obtained in this problem is identical to the one of Equation (1) once judicial bias is redefined as $\beta_E \equiv \left(\widetilde{\beta}_E \overline{r} - \widetilde{\beta}_I \lambda \right) / (\overline{r} - \lambda)$ and $\beta_I \equiv \left(\widetilde{\beta}_I \lambda - \widetilde{\beta}_E \underline{r} \right) / (\lambda - \underline{r})$. As evident from these expressions, the moderate judges who try to avoid both error types are those having $\lambda / \overline{r} \leq \widetilde{\beta}_E / \widetilde{\beta}_I \leq \lambda / \underline{r}$.

2. Proofs

Proof of Proposition 2. E chooses $[x(r), d_E(r), d_I]$ so as to solve:

$$\max_{[x(r), d_E(r), d_I]} E \left\{ \omega(r) \left[r - d_E(r) \right] + \left[1 - \omega(r) \right] (\lambda - d_I) \right\} + \vartheta E \left\{ \omega(r) d_E(r) + \left[1 - \omega(r) \right] d_I - k \right\},$$

where $\omega(r)$ is defined in the text, ϑ is the multiplier of the break-even constraint. We can rewrite this problem as:

$$\max_{[x(r), d_E(r), d_I]} E \left\{ \omega(r) r + \left[1 - \omega(r) \right] \lambda \right\} + \nu E \left\{ \omega(r) d_E(r) + \left[1 - \omega(r) \right] d_I - k \right\},$$

where $\nu \equiv \vartheta - 1$ is the relevant Lagrange multiplier. The derivatives of the lagrangian with respect to $x(\overline{r}), x(\underline{r}), d_E(r)$ and d_I are:

$$x(\overline{r}) \;:\; \mu p_{\overline{r}} (\overline{r} - \lambda) + (1 - \mu)(1 - p_{\underline{r}})(\underline{r} - \lambda) + \nu \left\{ \begin{array}{c} \mu p_{\overline{r}} \left[d_E(\overline{r}) - d_I \right] + \\ +(1 - \mu)(1 - p_{\underline{r}}) \left[d_E(\underline{r}) - d_I \right] \end{array} \right\} \tag{3}$$

$$x(\underline{r}) \;:\; \mu (1 - p_{\overline{r}})(\overline{r} - \lambda) + (1 - \mu) p_{\underline{r}} (\underline{r} - \lambda) + \nu \left\{ \begin{array}{c} \mu (1 - p_{\overline{r}}) \left[d_E(\overline{r}) - d_I \right] + \\ +(1 - \mu) p_{\underline{r}} \left[d_E(\underline{r}) - d_I \right] \end{array} \right\} \tag{4}$$

$$d_E(r) \;:\; \nu \frac{\partial}{\partial d_E(r)} E \left\{ \omega(r) d_E(r) + \left[1 - \omega(r) \right] d_I \right\} \qquad \forall r \tag{5}$$

$$d_I \;:\; \nu \frac{\partial}{\partial d_I} E \left\{ \omega(r) d_E(r) + \left[1 - \omega(r) \right] d_I \right\} \qquad \forall r \tag{6}$$

Expressions (5) and (6) imply that $\nu \geq 0$ (i.e. $\vartheta \geq 1$): intuitively, investor break even is always binding because E always sets the lowest payments ensuring break-event. When $\nu = 0$ (i.e. $\vartheta = 1$), there is social indifference as to the funds transferred to I (so that E always sets the minimal payments consistent with break even), when $\nu > 0$ (i.e. $\vartheta > 1$) transferring funds to I is socially valuable because it allows the project to be undertaken. This is the case of Proposition 2, so that the optimal contract is described by (14) in the text. The proof of Proposition 2 then works in three steps A, B and C.

Step A. Consider first condition $\eta \geq (1 - p_{\underline{r}})/p_{\overline{r}}$, which implies that $x(\overline{r}) = 1$. By exploiting Equations (11) and (12) in the text and by changing the integration variable in $p_{\overline{r}}$ and in $p_{\underline{r}}$ one can rewrite it as:

$$H_1(z) \equiv e^z \int_{-\infty}^{z} e^{-(a+bt)^2/2} dt - \int_{-\infty}^{z} e^{-(bt-a)^2/2} dt \geq 0 \tag{7}$$

where $z \equiv \ln \eta$, $a \equiv (\overline{r} - \underline{r})/2\theta g$, $b \equiv \theta/(\overline{r} - \underline{r})g$ and $g \equiv \sqrt{1 + \theta^2\sigma^2/(\overline{r} - \underline{r})^2}$. Note that $g = 1$ if $\sigma = 0$ and $g > 1$ otherwise. Inequality (7) is always met for $z > 0$. To see what happens if $z \leq 0$, note that $\lim_{z \to -\infty} H_1(z) = 0$. Consider what happens as z increases from $-\infty$. The first derivative of $H_1(z)$ is:

$$H_1'(z) \equiv e^{-(bz-a)^2/2} \left[e^{z+(bz-a)^2/2} \int_{-\infty}^{z} e^{-(a+bt)^2/2} dt + e^{z(1-2ab)} - 1 \right] \tag{8}$$

If $\sigma = 0$, then $g = 1$ and $2ab = 1$. As a result, $H_1'(z) \geq 0$ for all $z \leq 0$. Since $\lim_{z \to -\infty} H_1(z) = 0$, then $H_1(z) \geq 0\ \forall z$, which implies that if $\sigma = 0$ the optimal contract sets $x(\overline{r}) = 1$ for every z. As we shall see, this property implies that if $\sigma = 0$ a contingent contract is always chosen. When instead $\sigma > 0$ and $g > 1$. The square bracketed term in (8) shows that $H_1'(z) \geq 0$ whenever:

$$\int_{-\infty}^{z} e^{-(a+bt)^2/2} dt \geq e^{-z-(bz-a)^2/2} - e^{-(bz+a)^2/2} \tag{9}$$

At $z = 0$, the left hand side above is positive while the right hand side is zero, so (9) is satisfied and thus $H_1'(0) > 0$. In general, the left hand side of (9) increases in z, the right hand side decreases in z for $z \geq z^*$ and increases in z for $z < z^*$, where z^* is a negative threshold smaller than $(ab - 1)/b^2$. Thus, when $z < z^*$ both the left and the right hand side of (9) increase in z. As $z \to -\infty$, both sides tend to 0, but by inspecting the first derivatives of both sides one sees that there exists a $z^{**} < z^*$ such that the right hand side of (9) increases faster than the left hand side for $z < z^{**}$. Hence, in proximity of $-\infty$, (9) is violated and parties set $x(\overline{r}) = 0$ for all values $z < z^{**}$. For $z > z^{**}$, the left hand side of (9) starts growing faster than the left hand side (which eventually becomes even decreasing) and thus expression (9) may become positive. We already know that at some point it becomes positive because at $z = 0$ (9) is satisfied. But then, this implies that there exists a unique point $\widetilde{z}$ such that $H_1'(z) \geq 0$ for $z \geq \widetilde{z}$ and $H_1'(z) < 0$ otherwise.

Thus, if $\sigma > 0$ there is one and only one $z_1 < 0$ such that $H_1(z) \geq 0$ for $z \geq z_1$ and $H_1(z) < 0$ otherwise. Uniqueness of z_1 follows from the fact that $H_1'(z)$ changes sign only once. Crucially, since $(1-p_{\underline{r}})/p_{\overline{r}} \leq p_{\underline{r}}/(1-p_{\overline{r}})$, if $z < z_1$ parties use contract $x(\overline{r}) = x(\underline{r}) = 0$. Consider how z_1 varies with σ. By noting that $da/d\sigma = -ag'(\sigma)/g$, $db/d\sigma = -bg'(\sigma)/g$ and that $g'(\sigma) > 0$, from the implicit function theorem [and since $H_1'(z_1) > 0$], it follows that

$$sign(\frac{dz_1}{d\sigma}) = -sign\left[e^{z_1}\int_{-\infty}^{z_1} e^{-(a+bt)^2/2}\,(a+bt)^2\,dt - \int_{-\infty}^{z_1} e^{-(bt-a)^2/2}\,(bt-a)^2\,dt\right].$$

The formula inside the square brackets is nothing else than a transformation of $H_1(z_1)$ where the first integrand is multiplied by $(a+bt)^2$ while the second integrand is multiplied by $(bt-a)^2$. Since $z_1 < 0$, we know that $t < 0$. Thus, since we also know that $b \geq 0$ and $a \geq 0$, the expression in square brackets is smaller than what is obtained by setting $b = 0$ in the functions that multiply the exponentials inside the integrals. Since the latter expression is precisely $H_1(z_1) = 0$, then the expression in square brackets in negative, which implies that z_1 increases in σ. Also, since as $\sigma \to +\infty$, both a and b tend to 0, z_1 tends to 0 as well. Since $z_1 < 0$, we know that $t < 0$. As a result, since we also know that $b \geq 0$ and $a \geq 0$, the expression in square brackets is smaller than the one obtained by setting $b = 0$ in the functions that multiply the exponentials inside the integrals. But then, since the latter expression is precisely $H_1(z_1) = 0$, then the expression in square brackets in negative, which implies that z_1 increases in σ. Also, since as $\sigma \to +\infty$, both a and b tend to 0, z_1 tends to 0 as well.

Step B. Consider now condition $\eta \leq p_{\underline{r}}/(1-p_{\overline{r}})$. By exploiting Equations (11) and (12) in the text one can rewrite it as:

$$H_2(z) \equiv e^{-z} \int_{-\infty}^{-(bz-a)} e^{-u^2/2} du - \int_{-\infty}^{-(a+bz)} e^{-u^2/2} du \geq 0 \tag{10}$$

where z, a, b and g are defined as before. Notice that $H_2(z) = H_1(-z)$. Thus, H_2 is a symmetric transformation of H_1.This has three implications. First, (10) is surely satisfied for $z \leq 0$, just as (7) is surely met for $z \geq 0$. Second, when $\sigma = 0$, $H_2(z) \geq 0$ $\forall z$ and thus $x(\underline{r}) = 0$ for every z. Third, for $\sigma > 0$ there exists a $z_2 = -z_1 > 0$ such that (10) is satisfied if and only if $z \leq z_2$, just as (7) is only met for $z \geq z_1$. Also, z_2 decreases in σ and $\lim_{\sigma \to +\infty} z_2 = 0$. This implies that when $z \geq z_2$ the parties use contract $x(\overline{r}) = x(\underline{r}) = 1$.

Step C. The previous analysis shows that if $\sigma = 0$ contract $x(\overline{r}) = 1$, $x(\underline{r}) = 0$ is always chosen. To see what happens when $\sigma > 0$, define $\kappa(\sigma) \equiv e^{-z_1(\sigma)}$. Then, the above analysis implies that if $\eta < 1/\kappa(\sigma)$ parties use $x(\overline{r}) = x(\underline{r}) = 0$, for $1/\kappa(\sigma) \leq \eta \leq \kappa(\sigma)$ they use $x(\overline{r}) = 1$ and $x(\underline{r}) = 0$, for $\eta > \kappa(\sigma)$ they use $x(\overline{r}) = x(\underline{r}) = 1$. Finally, since $z_1(\sigma)$ increases in σ then $\kappa(\sigma)$ falls in σ over the same range. This proves Proposition 2.

Proof of Proposition 3. After imposing on the feasibility condition for the truthful revelation contract the restriction $(1-\mu)/\mu = (\overline{r} - \lambda)/(\lambda - \underline{r})$ implied by assumption A.3 (i.e.$\eta = 1$), one finds that the truthful revelation contract is feasible provided:

$$\frac{\lambda - \underline{r}}{\overline{r} - \underline{r}} \cdot \left\{ \alpha \overline{r} + \frac{\overline{r} - \lambda}{\lambda - \underline{r}} \left[\lambda - (1-\alpha) \underline{r} \right] \right\} \geq k, \tag{11}$$

which can be rewritten as:

$$-\lambda^2 + \lambda \left[\overline{r} + \underline{r} + \alpha \left(\overline{r} - \underline{r} \right) \right] - \left[\overline{r} \underline{r} + k \left(\overline{r} - \underline{r} \right) \right] \geq 0. \tag{12}$$

The left hand side is inversely-U shaped in λ, and we now study the behavior of the expression for $\lambda \in (\underline{r}, \overline{r})$. The left hand side reaches its maximum at $\lambda = \left[\overline{r} + \underline{r} + \alpha \left(\overline{r} - \underline{r} \right) \right]/2 < \overline{r}$, implying that the left hand side is increasing in the domain of interest $\lambda \in (\underline{r}, \overline{r})$. At $\lambda = \underline{r}$, Equation (12) boils down to $(\alpha \underline{r} - k)(\overline{r} - \underline{r}) \geq 0$, which is fulfilled if and only if $\alpha \underline{r} \geq k$. As a result, if $\alpha \underline{r} \geq k$, the truthful revelation contract is always feasible, namely for $\lambda \geq \underline{r}$.

If instead $\alpha \underline{r} < k$, Equation (12) is not met at $\lambda = \underline{r}$. Can it be met at any other

$\lambda < \overline{r}$? A sufficient condition for the answer to be "yes" is that Equation (12) be satisfied at $\lambda = \overline{r}$. By substituting this value in the left hand side expression, I find that Equation (12) becomes $(\alpha\overline{r} - k)(\overline{r} - \underline{r}) \geq 0$. One can see that, given assumption A.3, this condition is always verified due to A.2 (evaluated at $\mu = 1$). As a result, there exists a threshold $\lambda_i^* \in (\underline{r}, \overline{r})$ such that the truthful revelation contract is feasible for $\lambda \geq \lambda_i^*$.

Overall, there exists a threshold $\lambda^* \in [\underline{r}, \overline{r})$, where $\lambda^* = \underline{r} \cdot \mathbf{I}(\alpha\underline{r} \geq k) + \lambda_i^* \cdot [1 - \mathbf{I}(\alpha\underline{r} \geq k)]$ and $\mathbf{I}(\cdot)$ is the indicator function, whereby the truthful revelation contract is feasible if and only if $\lambda > \lambda^*$. It is immediate to see that threshold λ^* decreases in α, as better pledgeability of cash flows improves the feasibility of truthful revelation.

When $\lambda < \lambda^*$ the truthful revelation contract is infeasible and a state verification contract is used. With respect to the latter contract, the proof Proposition 2 showed that $\eta = 1$, if $\nu = 0$ the optimal contract is $x(\overline{r}) = 1$, $x(\overline{r}) = 0$. Clearly, $\nu = 0$ as long as under such contract I's break-even is not binding namely if:

$$p[\mu(\alpha\overline{r} - \lambda) + (1 - \mu)(\lambda - \alpha\underline{r})] \equiv pA \geq B \equiv k - [\mu\lambda + (1 - \mu)\alpha\underline{r}] \tag{13}$$

By A2 we know that $A > B$. Since $p \in [1/2, 1]$, then (13) is always met provided $B < 0$. Consider now the behavior of B as a function of λ. It is easy to see that given assumption A.2 the function B always decreases in λ. Moreover, when $\alpha\underline{r} < k$, which is necessary for $\lambda < \lambda^*$, we also have $B > 0$ that at $\lambda = \underline{r}$. At $\lambda = \overline{r}$, instead, due to the fact that $\overline{r} > k$, we have that $B < 0$. As a result, there is a threshold $\lambda_1 > 0$ such that $B > 0$ for $\lambda < \lambda_1$ and $B \leq 0$ otherwise. This establishes that for $\lambda \geq \lambda_1$ (13) is met and thus $x(\overline{r}) = 1$, $x(\overline{r}) = 0$ is used for every σ. Note that threshold λ_1 decreases in α, implying that higher pledgeability of cash flows fosters the use of the fully contingent contract for $\lambda < \lambda^*$.

Suppose now that $\lambda < \lambda_1$. Then, given $A > B > 0$, condition (13) is met provided $p \geq B/A$. Since $p \geq 1/2$, condition (13) is always satisfied if $B/A < 1/2$. This is the case provided $2k < \lambda + \alpha E(r)$ which by using A3 becomes $\lambda > 2k/(1 + \alpha)$. For $\lambda > 2k/(1 + \alpha)$,the contingent contract is always feasible even if $\lambda < \lambda_1$. As a result, defined $\widetilde{\lambda} \equiv \min[2k/(1 + \alpha), \lambda_1]$ the contingent contract can only be infeasible provided $\lambda < \widetilde{\lambda}$; for $\lambda \geq \widetilde{\lambda}$ such contract is certainly feasible regardless of σ. Note that the threshold $\widetilde{\lambda}$ falls in

α. It is also immediate to find that for $\alpha = 1$ then, provided $k > \underline{r}$ (otherwise there is no financing problem), we have $\widetilde{\lambda} = k$.

Summarizing, so far I found that for $\lambda \geq \lambda^*$ parties attain the first best by choosing truthful revelation, while for $\lambda \geq \widetilde{\lambda}$ the fully contingent contract is feasible (and optimal in the class of state verification contracts) regardless of σ. Defining a new threshold $\widehat{\lambda} = \min\left[\widetilde{\lambda}, \lambda^*\right]$, we know that for $\lambda \geq \lambda^*$ truthful revelation is used, while for $\lambda \in \left[\widehat{\lambda}, \lambda^*\right)$ the fully contingent contract is used for any σ, where the latter region is non empty only when $\widehat{\lambda} = \widetilde{\lambda}$. Because thresholds λ^* and $\widetilde{\lambda}$ fall in α, we have that $\widehat{\lambda}$ also falls in α, implying that pledgeability of cash flows fosters the use of the fully contingent contract. Since for $\alpha = 1$ we have seen that $\widetilde{\lambda} = k$, then – given assumption A.1 – we know that only the fully contingent contract is used for $\lambda < \lambda^*$.

Suppose now that $\lambda < \widehat{\lambda}$. In this range, the truthful revelation contract is infeasible and whether the fully contingent is used or not depends on judicial dispersion σ. To see how this works, consider the expression for p obtained by plugging $\eta = 1$ into Equation (11) in the text, and substitute it in condition (13). Then, the latter condition identifies a threshold $\widehat{\sigma} \geq 0$, with $\widehat{\sigma}$ increasing in θ and $\lim_{\theta \to 0} \widehat{\sigma}(\theta) = +\infty$, such that parties set $x(\overline{r}) = 1$, $x(\overline{r}) = 0$ but only provided $\sigma < \widehat{\sigma}(\theta)$. Note that if θ is sufficiently large, there may exist no value of σ for which the fully contingent contract is feasible, in which case we set $\widehat{\sigma} = 0$. When this is the case, a condition on σ alone cannot ensure the use of the fully contingent contract for $\lambda < \widehat{\lambda}$. Once again, it is easy to see that $\widehat{\sigma}(\theta)$ increases in α, implying that better pledgeability improves the use of contingent contracts.

Finally, suppose that $\lambda < \widehat{\lambda}$ and $\sigma > \widehat{\sigma}$. Now, the investor break even constraint is binding, namely $\nu > 0$. Thus, (5) and (6) imply that payments are set at their maximum possible level $d_E(r) = \alpha r$ and $d_I = \lambda$. Because of $p > 1/2$, derivative (3) is strictly larger than (4), implying $x(\overline{r}) \geq x(\underline{r})$. Thus, (4) is negative and $x(\underline{r}) = 0$. If $x(\underline{r})$ were equal to 1, $x(\overline{r})$ would also be equal to 1, implying an average repayment to I of $\alpha E(r)$, which does not yield ex-ante break even. Given $x(\underline{r}) = 0$, to ensure break even it must be that:

$$x(\overline{r}) = \frac{\lambda - k}{(1-\mu)(1-p)(\lambda - \alpha\underline{r}) + \mu p(\lambda - \alpha\overline{r})} \tag{14}$$

When $\lambda \geq k$ the numerator of the right hand side is positive and for $\alpha < 1$ the denominator is positive as well. Expression (14) defines a function $x(\overline{r}\,|\sigma\,)$ decreasing in σ (and in θ). For $\lambda = k$ the optimal contract immediately jumps at $x(\overline{r}) = 0$ for $\sigma > \widehat{\sigma}$. For $\lambda > k$, if $\sigma \to +\infty$ then $p \to 1/2$ and the informational benefit of a contingent contract is zero. In particular, as $p \to 1/2$ the fully non-contingent contract $x(\underline{r}) = x(\overline{r}) = 0$ and $d_I = k$ is just as good for parties as the limit contract in (14). Thus, as polarization becomes extreme there is no benefit of writing state dependent contracts.

3. Ex-Post Renegotiation

Parties may try to reduce the costs of enforcement risk by renegotiating away judicial mistakes ex-post. To study this possibility suppose that E, the informed party, has all the bargaining power in renegotiation and makes a take-it-or-leave-it offer to I before going to court.[1] E's offer consists of a new allocation of control and repayment replacing those in the original contract. If I accepts the offer, parties settle out of court. If I declines the offer, a judge enforces their ex-ante contract. For ex-post renegotiation to remove all inefficiencies in the allocation of control, it must be that the bargaining equilibrium is separating or, put differently, that E has the incentive to truthfully report r.

Consider the possibility for parties to write an ex-ante feasible contract such that in renegotiation E has the incentive to truthfully report r and the first best is implemented. First, note that – foreseeing renegotiation – it may be optimal for I to lend an amount $D > k$ at $t = 0$. By doing so, I can improve the ability of E to renegotiate ex-post, allowing a more efficient outcome. The extra lending $D - k$ is akin to a $t = 1$ cash flow, so E can grab a fraction $(1 - \alpha)$ of it ex-post. As a result, repayments can also include the amount $\alpha(D - k)$ and in renegotiation E can always make an upfront payment of $(D - k)$ to the investor if he wants to.

If the initial contract promises $d_E(r)$ and d_I, by going to court I obtains on average $\omega(r)d_E(r) + [1 - \omega(r)]\, d_I$ in state r, where $\omega(r)$ is the probability that judges set E-control in r. Suppose that E reports r truthfully and pays I the latter's reservation value $\omega(r)d_E(r) + [1 - \omega(r)]\, d_I$, implementing the first best. Then, the repayment $\omega(\underline{r})d_E(\underline{r}) + [1 - \omega(\underline{r})]\, d_I$

[1] It is harder for renegotiation to avoid the ex-post inefficiencies created by judicial errors if I had all the bargaining power and tried to extract rents by screening E's information.

obtained by I in $\underline{r}$ must be strictly larger than $\alpha\underline{r} + (D - k)$. Suppose that this is not the case. Then, given that when E-control is efficiently set in $\overline{r}$ the investor cannot expect to get more than $\alpha\overline{r} + (D - k)$, in an ex-post efficient separating equilibrium ex-ante break even is always fulfilled provided:

$$\alpha E(r) + D - k \geq D \tag{15}$$

which is harder to fulfill that the break even condition under the truthful revelation contract. Renegotiation cannot obviously help to attain the first best because if (15) is met, the truthful revelation contract is feasible and so state verification contracts are not used.

Suppose then that $\omega(\underline{r})d_E(\underline{r}) + [1 - \omega(\underline{r})]\, d_I > \alpha\underline{r} + (D - k)$ and consider renegotiation in $\underline{r}$. By reporting $\underline{r}$, setting I-control and repaying $\omega(\underline{r})d_E(\underline{r}) + [1 - \omega(\underline{r})]\, d_I$ to I, E obtains $\lambda + (D - k) - \omega(\underline{r})d_E(\underline{r}) - [1 - \omega(\underline{r})]\, d_I$. Since E obtains all renegotiation surplus, this payoff is certainly larger than what E obtains by going to court. The problem is that E may want to misreport, claiming that the state is $\overline{r}$. Clearly, E is always able to misreport, if he wants to. Indeed, in every state r the entrepreneur has the same $D - k$ resources to make an upfront payment to I before the cash flow is generated at $t = 1$. Crucially, if E falsely claims that the true state is $\overline{r}$, I adjusts his reservation value to $\omega(\overline{r})d_E(\overline{r}) + [1 - \omega(\overline{r})]\, d_I$. But then, since upon implementing E-control in $\underline{r}$ the entrepreneur is always able to keep at least the private benefits of control $(1 - \alpha)\underline{r}$, a sufficient condition for the entrepreneur to truthfully reports $\underline{r}$ and forsake this private benefit is thus:

$$\lambda - \omega(\underline{r})d_E(\underline{r}) - [1 - \omega(\underline{r})]\, d_I + (D - k) \geq (1 - \alpha)\underline{r}$$

which can be rewritten as:

$$\lambda - (1 - \alpha)\underline{r} + (D - k) \geq \omega(\underline{r})d_E(\underline{r}) + [1 - \omega(\underline{r})]\, d_I$$

The above condition implies that, in a separating equilibrium, the average repayment to the investor in state $\underline{r}$ (net of the pledgeable portion of the ex-ante cash infusion $D - k$) must be smaller than $\lambda - (1 - \alpha)\underline{r} + (D - k)$. Given that in state r the investor can obtain at most $\alpha\overline{r}+(D-k)$, renegotiation implements the first best and I breaks even only if:

$$\mu\left(\alpha\overline{r}+D-k\right)+(1-\mu)\left[\lambda-(1-\alpha)\underline{r}+(D-k)\right]\geq D. \tag{16}$$

Which is the exact same condition as the one under which truthful revelation is feasible, implying that renegotiation can only avoid the cost of judicial bias if the truthful revelation contract is feasible. This implies that if α is so low that it is too costly for the parties to give E the incentive to report r in an ex-ante contract, it is also too costly to provide E with the same incentives through ex-post renegotiation. Hence, parties cannot renegotiate away all the ex-post misallocations of control created by judicial bias under state verification contracts.

4. Robustness to alternative contracts

I consider three contract types that in the interest of exposition I did not consider in the main body of the paper: i) contracts mixing truthful revelation and state verification, and ii) contracts inducing judges to truthfully report s (also by stipulating side payments to them), and iii) very open ended contracts allowing judges to "do what they want."

i) Mixture of truthful revelation and state verification. We consider three sub-cases.

i.a) Contracts where the judge enforces a truthful revelation contract with probability $t\in(0,1)$ and sets I-control with probability $1-t$ (so far we only considered the extremes where $t\in\{0,1\}$). Under this contract, the investor obtains:

$$t\left[\mu(\alpha\overline{r}-\lambda)-(1-\mu)(1-\alpha)\underline{r}\right]+\lambda,$$

which yields break even provided

$$t\leq t^{*}\equiv(\lambda-k)\,/\left[(1-\mu)(1-\alpha)\underline{r}-\mu(\alpha\overline{r}-\lambda)\right]. \tag{17}$$

With probability t^{*} this contract beneficially exploits E's information, but to ensures break-even it must set I-control with probability $1-t^{*}$, even if it is ex-post inefficient to do so. By using the Equation for social welfare in Section 4 in the text, one can see that the standard state verification contract studied in Section 4 is preferred to this mixture provided

$x(\overline{r})(2p-1) \geq 1-t^*$, namely when σ is sufficiently small. This condition is satisfied for many parameter values, in particular when $\sigma \leq \sigma^*$ where σ^* is the value of polarization at which $x(\overline{r})(2p-1) = 1-t^*$, where t^* is the threshold identified by Equation (17). For $\sigma \leq \sigma^*$, the comparative statics of Proposition 3 continue to hold; by contrast, as $\sigma > \sigma^*$ the state verification contract is no longer used because it is dominated by the above mixture.

i.b) A contract setting E-control in state $\overline{r}$ with probability less than 1. It is easy to see that this contract is dominated by the one considered in 1.a).

i.c) A contract exploiting both judicial reports $\widehat{r}_j$ and the entrepreneur's report $\widehat{r}_E$. According to this contract, the report by the court is issues to enforce a state contingent repayment $d_I(\widehat{r}_j)$ while the report issued by the entrepreneur is used to enforce the control allocation $x(\widehat{r}_E)$. Even if judges perfectly verify, namely $\widehat{r}_j = r$, in order for E to set I-control in state $\underline{r}$ is must be that $d_I(\underline{r}) = l - (1-\alpha)\underline{r}$. As a result, this contract cannot improve upon the truthful revelation contract considered in Section 2.

In sum, the result that the use of the contingent contract falls in σ is robust to including exotic mixtures among state verification and truthful revelation contracts.

ii) Contracts that try to induce judges to truthfully reveal signal s. By rendering s directly contractible, these contracts might avoid the costs of bias. Consider the formulation of judicial preferences introduced at the end of Appendix 1, where judicial preferences are encoded in the social welfare weights $\widetilde{\beta}_E, \widetilde{\beta}_I = 1 - \widetilde{\beta}_E$. The question is whether parties can write a signal and bias contingent contract $x(s, \widetilde{\beta}_E)$ in such a way that judges are induced to truthfully report s and $\widetilde{\beta}_E$, where judicial bias is naturally assumed to be unobservable to E and I. The contract may also provide the judge with monetary incentives for truthful reporting. I first consider the case where parties do not pay judges a bribe. This is probably the most realistic case, for contracts bribing judges for finding specific facts are illegal in most countries [see Bond (2004) for a study of court bribery]. I later consider also the role of such bribes.

ii.a) The case without bribes. Given $x(s, \widetilde{\beta}_E)$, a judge reporting $\left(\widehat{s}, \widehat{\widetilde{\beta}}_E\right)$ when the truth is $\left(s, \widetilde{\beta}_E\right)$ obtains utility:

$$U(\widehat{s}, \widehat{\widetilde{\beta}}_E \left| s, \widetilde{\beta}_E \right.) = x\left(\widehat{s}, \widehat{\widetilde{\beta}}_E\right) \cdot \widetilde{\beta}_E E(r\,|s) + \left[1 - x\left(\widehat{s}, \widehat{\widetilde{\beta}}_E\right)\right]\left(1 - \widetilde{\beta}_E\right) \cdot \lambda =$$
$$x\left(\widehat{s}, \widehat{\widetilde{\beta}}_E\right) \cdot \left\{\widetilde{\beta}_E \left[E(r\,|s) + \lambda\right] - \lambda\right\} + \left(1 - \widetilde{\beta}_E\right) \cdot \lambda.$$

Thus, judges with $\widetilde{\beta}_E > \lambda/\left[E(r\,|s) + \lambda\right]$ report $\left(s^*, \widetilde{\beta}_E^*\right) = \arg\max_{\widehat{s}, \widehat{\widetilde{\beta}}_E} x\left(\widehat{s}, \widehat{\widetilde{\beta}}_E\right)$ while judges with $\widetilde{\beta}_E < \lambda/\left[E(r\,|s) + \lambda\right]$ report $\left(s_*, \widetilde{\beta}_{E,*}\right) = \arg\min_{\widehat{s}, \widehat{\widetilde{\beta}}_E} x\left(\widehat{s}, \widehat{\widetilde{\beta}}_E\right)$. This boils down to reducing the contract space to two numbers $x(\overline{r})$ and $x(\underline{r})$. Hence, judges cannot be induced to reveal $\left(\widehat{s}, \widehat{\widetilde{\beta}}_E\right)$ when bribes are not used.

ii.a) The case with bribes. Now, besides specifying allocation $x\left(\widehat{s}, \widehat{\widetilde{\beta}}_E\right)$, the contract also pays the judge $b(\widehat{s}, \widehat{\widetilde{\beta}}_E) \geq 0$ (due to limited liability, judges cannot be forced to pay money). My aim here is not to derive the optimal contract, but to show that the effectiveness of bribes is limited (which perhaps helps explain why they are not used in reality). In particular, I show that bribes cannot induce judges to implement the adjudication policy of an unbiased judge who optimally uses s. To see this, note that the unbiased judge's optimal policy is equal to:

$$x^{optimal}\left(s, \widetilde{\beta}_E\right) = \begin{cases} 1 & for \quad \widetilde{\beta}_E \in [0,1] \text{ and } s > s_t \\ 0 & for \quad \widetilde{\beta}_E \in [0,1] \text{ and } s < s_t \end{cases}. \tag{18}$$

Here s_t is implicitly defined by $E(r\,|s_t) = \lambda$, which implies that when $s > s_t$ we have $E(r\,|s_t) > \lambda$, while when $s < s_t$ we have $E(r\,|s_t) < \lambda$. Suppose that a judge chooses to report $\widetilde{\beta}_E$ and $s > s_t$. Then, conditional on setting allocation $x^{optimal}\left(s, \widetilde{\beta}_E\right) = 1$, he reports the vector $\left(\widehat{s}, \widehat{\widetilde{\beta}}_E\right) \in (s_t, +\infty) \times [0,1]$ that maximizes his bribe $b(\widehat{s}, \widehat{\widetilde{\beta}}_E)$. Truthful reporting then requires that $b(\widehat{s}, \widehat{\widetilde{\beta}}_E) = b_1 =$ constant for all $\left(\widehat{s}, \widehat{\widetilde{\beta}}_E\right) \in (s_t, +\infty) \times [0,1]$. By the same token, if a judge reports $\widetilde{\beta}_E$ and $s < s_t$ then, conditional on setting $x^{optimal}\left(s, \widetilde{\beta}_E\right) = 0$, he reports the vector $\left(\widehat{s}, \widehat{\widetilde{\beta}}_E\right) \in (-\infty, s_t) \times [0,1]$ that maximizes the bribe $b(\widehat{s}, \widehat{\widetilde{\beta}}_E)$. Truthful reporting then requires that $b(\widehat{s}, \widehat{\widetilde{\beta}}_E) = b_0 =$ constant for all $\left(\widehat{s}, \widehat{\widetilde{\beta}}_E\right) \in (-\infty, s_t) \times [0,1]$.

As a result, to implement the optimal policy of Equation (18) under truthful reporting, the contract can only specify two bribes b_1 and b_0 which are paid to the judge if $x^{optimal}\left(s, \widetilde{\beta}_E\right) = 1$ and $x^{optimal}\left(s, \widetilde{\beta}_E\right) = 0$ are set, respectively. A judge with pro-entrepreneur bias $\widetilde{\beta}_E$ observing s, reports $s > s_t$ instead of $s < s_t$ if and only if:

$$\widetilde{\beta}_E \geq \frac{\lambda + b_0 - b_1}{\lambda + E(r\,|s)}. \tag{19}$$

Equation (19) shows that it is impossible to set a fixed $b_0 - b_1$ such that the allocation is $x^{optimal}\left(s, \widetilde{\beta}_E\right)$ for every $\left(s, \widetilde{\beta}_E\right)$. If for instance $b_0 - b_1 \in (\lambda, -\lambda)$, in a neighborhood of s_t judges with relatively high $\widetilde{\beta}_E$ set $x = 1$ even if parties prefer $x = 0$. By contrast, judges with relatively low $\widetilde{\beta}_E$ set $x = 0$ even if parties prefer $x = 1$. That is, pro-entrepreneur judges set E-control too often, pro-investor judges set I-control too often, just as when bribes are not used at all. Figure 3 below graphically shows this point by plotting Equation (19) for a value $b_0 - b_1 \in (0, \lambda)$ as a function of $\widetilde{\beta}_E$ and s:

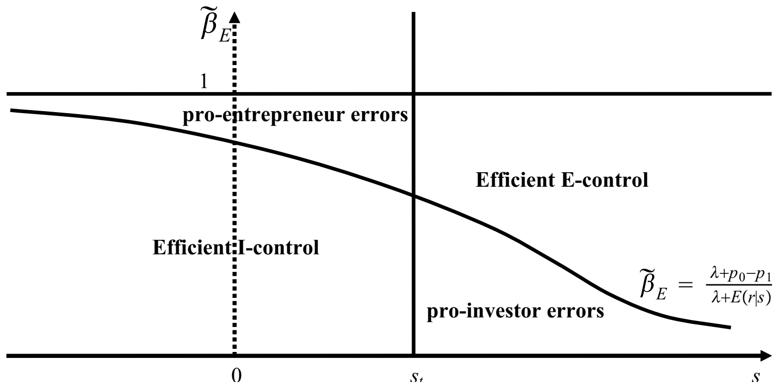

Figure 3

Thus, this cursory analysis shows that the very presence of judicial bias hinders the possibility of using judicial incentives to implement the optimal allocation of control. Even if bribes are used, judicial biases will introduce randomness in the allocation of control. Additionally, bribes are by their nature costly to the parties, and might even undermine investor break even. In this respect, non-contingent contracts have the advantage of: v) avoiding randomness (and thus costly errors) altogether, and vv) avoiding monetary costs for the parties, fostering break even.[2]

iii) Consider finally a drastically open ended contract allowing the judge to "do what he wants." This contract boils down to allowing the judge to maximize objective (2) not only with respect to $\psi(s)$ but also with respect to the control allocation $x(\overline{r}), x(\underline{r})$. It is then evident that this contract makes it impossible for the parties to set any state contingent allocation where $|x(\overline{r}) - x(\underline{r})| < 1$, namely even an allocation that is even slightly less contingent than the fully contingent one. This is because under a "let the judge do what he wants" contract the judge optimally sets $x(\overline{r}) = 1$ when his optimal policy is to find $\overline{r}$ [i.e. $\psi(s) = 1$] and $x(\underline{r}) = 0$ otherwise, thereby replicating the fully contingent contract. Crucially, then, the open ended contract does not allow parties to attain break even when contract $[x(\overline{r}) < 1, x(\underline{r}) = 0]$ is needed, such as in the cases highlighted by Proposition 3.

[2] This cost of bribes is best seen if judicial bias is observable. In this case, payments b_1 and b_0 can vary continuously as a function of $\widetilde{\beta}_E$.As a result, Equation (19) implies that by setting $b_0 - b_1 = \left(2\widetilde{\beta}_E - 1\right)\lambda$ parties can induce *all* judges to enforce the optimal policy of Equation (**??**). In the case of pro-entrepreneur judges (i.e. $\widetilde{\beta}_E > 1/2$), this requires to set $b_0 = \left(2\widetilde{\beta}_E - 1\right)\lambda$, $b_1 = 0$. In the case of pro-investor judges (i.e. $\widetilde{\beta}_E < 1/2$) this requires to set $b_0 = 0$, $b_1 = \left(1 - 2\widetilde{\beta}_E\right)\lambda$. The resulting ex-ante total cost for the parties of incentivizing judges is equal to:

$$\int_0^1 \lambda \cdot \left|2\widetilde{\beta}_E - 1\right| dF(\widetilde{\beta}_E) = 2\lambda \cdot \widetilde{\sigma},$$

where $F(\widetilde{\beta}_E)$ is the c.d.f. of bias and $\widetilde{\sigma} = \int_0^1 \left|\widetilde{\beta}_E - 1/2\right| dF(\widetilde{\beta}_E)$ is a measure of dispersion of judicial biases around the unbiased judge having $\widetilde{\beta}_E = 1/2$. Evidently, as judicial biases become more severe (i.e. as $\widetilde{\sigma}$ goes up), the cost of implementing the optimal policy increase as well, potentially undermining break even. If for instance judicial biases are symmetrically polarized at $\widetilde{\beta}_E = 0, 1$, then $\widetilde{\sigma} = 1/2$ and the cost of incentives becomes equal to λ, so that an amount of resoruces equal to the return under I-control must be pledged to judges. It is evident that, from the parties' standpoint, a non-contingent contract can be superior to this arrangment. The intuition here is that even if judicial bribes are allowed, state contingent allocations are very costly when dispersion of biases is high because the required bribes are very large in this case, in line with the conclusions reached by analyses of court corruption. As a result, a non-contingent contract may be optimal.