A Classical Monetary Model

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Assumptions

- Perfect competition in goods and labor markets
- Flexible prices and wages
- No capital accumulation
- No fiscal sector
- Closed economy

Outline

- The problem of households and firms
- Equilibrium: money neutrality and the determination of nominal variables
- A model with money in the utility function
- Optimal policy

Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, N_t\right) \tag{1}$$

subject to

$$P_t C_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t \tag{2}$$

for $t = 0, 1, 2, ...$and the solvency constraint

$$\lim_{T\to\infty} E_t \left\{\Lambda_{t,T}(B_T/P_T)\right\} \geq 0 \tag{3}$$

where $\Lambda_{t,T} \equiv \beta^{T-t} U_{c,T}/U_{c,t}$ is the stochastic discount factor.

Optimality conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \tag{4}$$

$$Q_t = \beta \; E_t \left\{\frac{U_{c,t+1}}{U_{c,t}} \; \frac{P_t}{P_{t+1}}\right\} \tag{5}$$

Specification of utility:

$$U(C_t, N_t) = \begin{cases} \frac{C_t^{1-\sigma}-1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} & \text{for } \sigma \neq 1 \\ \log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} & \text{for } \sigma = 1 \end{cases}$$

implied optimality conditions:

$$\frac{W_t}{P_t} = C_t^{\sigma} N_t^{\varphi} \tag{6}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\} \tag{7}$$

Log-linear versions

$$w_t - p_t = \sigma c_t + \varphi n_t \tag{8}$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \tag{9}$$

where $\pi_t \equiv p_t - p_{t-1}$, $i_t \equiv -\log Q_t$ and $\rho \equiv -\log \beta$

Steady state (zero growth):

$$i = \pi + \rho$$

implied real rate

$$r \equiv i - \pi = \rho$$

Ad-hoc money demand

$$m_t - p_t = c_t - \eta i_t$$

Firms

Representative firm with technology

$$Y_t = A_t N_t^{1-\alpha} \tag{10}$$

where $a_t \equiv \log A_t$ follows an exogenous process

$$a_t = \rho_a a_{t-1} + \varepsilon_t^a$$

Profit maximization:

$$\max \ P_t Y_t - W_t N_t$$

subject to (10), taking the price and wage as given (perfect competition)

Optimality condition:

$$\frac{W_t}{P_t} = (1-\alpha) A_t N_t^{-\alpha}$$

In log-linear terms

$$w_t - p_t = a_t - \alpha n_t + \log(1-\alpha)$$

Equilibrium

Goods market clearing

$$y_t = c_t$$

Labor market clearing

$$\sigma c_t + \varphi n_t = a_t - \alpha n_t + \log(1-\alpha)$$

Asset market clearing:

$$B_t = 0$$

$$\begin{aligned} r_t &\equiv i_t - E_t\{\pi_{t+1}\} \\ &= \rho + \sigma E_t\{\Delta c_{t+1}\} \end{aligned}$$

Aggregate output:

$$y_t = a_t + (1-\alpha) n_t$$

Implied equilibrium values for real variables:

$$n_t = \psi_{na} a_t + \psi_n$$

$$y_t = \psi_{ya} a_t + \psi_y$$

$$r_t = \rho - \sigma \psi_{ya} (1 - \rho_a) a_t$$

$$\begin{aligned} \omega_t &\equiv w_t - p_t \\ &= a_t - \alpha n_t + \log(1 - \alpha) \\ &= \psi_{\omega a} a_t + \psi_\omega \end{aligned}$$

where $\psi_{na} \equiv \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_n \equiv \frac{\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$;
$\psi_y \equiv (1-\alpha)\psi_n$; $\psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$; $\psi_\omega \equiv \frac{(\sigma(1-\alpha)+\varphi)\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$

$\Longrightarrow$ *neutrality*: real variables determined *independently of monetary policy*
$\Longrightarrow$ *optimal policy*: undetermined.
$\Longrightarrow$ specification of monetary policy needed to determine *nominal* variables

Monetary Policy and Price Level Determination

Example I: An Exogenous Path for the Nominal Interest Rate

$$i_t = i + v_t$$

where

$$v_t = \rho_v v_{t-1} + \varepsilon_t^v$$

Implied steady state inflation: $\pi = i - \rho$

Particular case: $i_t = i$ for all t.

Using definition of real rate:

$$\begin{aligned} E_t\{\pi_{t+1}\} &= i_t - r_t \\ &= \pi + v_t - \widehat{r}_t \end{aligned}$$

Equilibrium inflation:

$$\widehat{\pi}_t = v_{t-1} - \widehat{r}_{t-1} + \xi_t$$

for any $\{\xi_t\}$ sequence with $E_t\{\xi_{t+1}\} = 0$ for all t.

$$\Rightarrow \textit{nominal indeterminacy}$$

Example II: A Simple Interest Rate Rule

$$i_t = \rho + \pi + \phi_\pi(\pi_t - \pi) + v_t$$

where $\phi_\pi \geq 0$. Combined with definition of real rate:

$$\phi_\pi \widehat{\pi}_t = E_t\{\widehat{\pi}_{t+1}\} + \widehat{r}_t - v_t$$

If $\phi_\pi > 1$,

$$\begin{aligned} \widehat{\pi}_t &= \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t\{\widehat{r}_{t+k} - v_{t+k}\} \\ &= -\frac{\sigma(1-\rho_a)\psi_{ya}}{\phi_\pi - \rho_a}\, a_t - \frac{1}{\phi_\pi - \rho_v}\, v_t \end{aligned}$$

If $\phi_\pi < 1$,

$$\widehat{\pi}_t = \phi_\pi \widehat{\pi}_{t-1} - \widehat{r}_{t-1} + v_{t-1} + \xi_t$$

for any $\{\xi_t\}$ sequence with $E_t\{\xi_{t+1}\} = 0$ for all t

$\Longrightarrow$ *nominal indeterminacy*

$\Longrightarrow$ illustration of the "Taylor principle" requirement

Responses to a monetary policy shock ($\phi_\pi > 1$ case):

$$\frac{\partial \pi_t}{\partial \varepsilon_t^v} = -\frac{1}{\phi_\pi - \rho_v} < 0$$

$$\frac{\partial i_t}{\partial \varepsilon_t^v} = -\frac{\rho_v}{\phi_\pi - \rho_v} < 0$$

$$\frac{\partial m_t}{\partial \varepsilon_t^v} = \frac{\eta \rho_v - 1}{\phi_\pi - \rho_v} \lesseqgtr 0$$

$$\frac{\partial y_t}{\partial \varepsilon_t^v} = 0$$

Discussion: liquidity effect and price response.

Example III: An Exogenous Path for the Money Supply $\{m_t\}$

Combining money demand and the definition of the real rate:

$$p_t = \left(\frac{\eta}{1+\eta}\right) E_t\{p_{t+1}\} + \left(\frac{1}{1+\eta}\right) m_t + u_t$$

where $u_t \equiv (1+\eta)^{-1}(\eta r_t - y_t)$. Solving forward:

$$p_t = \frac{1}{1+\eta} \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\{m_{t+k}\} + \overline{u}_t$$

where $\overline{u}_t \equiv \sum_{k=0}^{\infty} \left(\frac{\eta}{1+\eta}\right)^k E_t\{u_{t+k}\}$

$\Rightarrow$ *price level determinacy*

In terms of money growth rates:

$$p_t = m_t + \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \{ \Delta m_{t+k} \} + \overline{u}_t$$

Nominal interest rate:

$$\begin{aligned} i_t &= \eta^{-1} \left(y_t - (m_t - p_t) \right) \\ &= \eta^{-1} \sum_{k=1}^{\infty} \left(\frac{\eta}{1+\eta} \right)^k E_t \{ \Delta m_{t+k} \} + \underline{u}_t \end{aligned}$$

where $\underline{u}_t \equiv \eta^{-1}(\overline{u}_t + y_t)$ is independent of monetary policy.

Example

$$\Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m$$

Assume no real shocks ($r_t = y_t = 0$).

Price response:

$$p_t = m_t + \frac{\eta \rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

$$\Rightarrow \textit{large price response}$$

Nominal interest rate response:

$$i_t = \frac{\rho_m}{1 + \eta(1 - \rho_m)} \Delta m_t$$

$$\Rightarrow \textit{no liquidity effect}$$

A Model with Money in the Utility Function

Preferences

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

Budget constraint

$$P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + D_t$$

with solvency constraint:

$$\lim_{T\to\infty} E_t \left\{\Lambda_{t,T}(\mathcal{A}_T/P_T)\right\} \geq 0$$

where $\mathcal{A}_t \equiv B_t + M_t$.

Equivalently:

$$P_t C_t + Q_t \mathcal{A}_t + (1 - Q_t) M_t \leq \mathcal{A}_{t-1} + W_t N_t + D_t$$

Interpretation: $1 - Q_t = 1 - \exp\{-i_t\} \simeq i_t$

$\Rightarrow$ opportunity cost of holding money

Optimality Conditions

$$-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}$$

$$Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}$$

$$\frac{U_{m,t}}{U_{c,t}} = 1 - \exp\{-i_t\}$$

Two cases:

- utility separable in real balances $\Rightarrow$ neutrality
- utility non-separable in real balances $\Rightarrow$ non-neutrality

Utility specification:

$$U\left(X_t, N_t\right) = \frac{X_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi}$$

where

$$X_t \equiv \left[(1-\vartheta)C_t^{1-\nu} + \vartheta\left(\frac{M_t}{P_t}\right)^{1-\nu}\right]^{\frac{1}{1-v}} \qquad for\ \nu \neq 1$$

$$\equiv C_t^{1-\vartheta}\left(\frac{M_t}{P_t}\right)^{\vartheta} \qquad for\ \nu = 1$$

Note that

$$U_{c,t} = (1-\vartheta)X_t^{\nu-\sigma}C_t^{-\nu}$$

$$U_{m,t} = \vartheta X_t^{\nu-\sigma}\left(M_t/P_t\right)^{-\nu}$$

$$U_{n,t} = -N_t^{\varphi}$$

Implied optimality conditions:

$$\frac{W_t}{P_t} = N_t^{\varphi} X_t^{\sigma-\nu} C_t^{\nu} (1-\vartheta)^{-1}$$

$$Q_t = \beta E_t \left\{ \left(\frac{C_{t+1}}{C_t} \right)^{-\nu} \left(\frac{X_{t+1}}{X_t} \right)^{\nu-\sigma} \left(\frac{P_t}{P_{t+1}} \right) \right\}$$

$$\frac{M_t}{P_t} = C_t \left(1 - \exp\{-i_t\}\right)^{-\frac{1}{\nu}} \left(\frac{\vartheta}{1-\vartheta} \right)^{\frac{1}{\nu}}$$

Log-linearized money demand equation:

$$m_t - p_t = c_t - \eta i_t$$

where $\eta \equiv 1/[\nu(\exp\{i\} - 1)]$.

Log-linearized labor supply equation:

$$\begin{aligned} w_t - p_t &= \sigma c_t + \varphi n_t + (\nu - \sigma)(c_t - x_t) \\ &= \sigma c_t + \varphi n_t + \chi(\nu - \sigma)\left(c_t - (m_t - p_t)\right) \\ &= \sigma c_t + \varphi n_t + \eta\chi(\nu - \sigma) i_t \end{aligned}$$

where $\chi \equiv \frac{\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}}{(1-\vartheta)^{\frac{1}{\nu}}+\vartheta^{\frac{1}{\nu}}(1-\beta)^{1-\frac{1}{\nu}}} = \frac{k_m(1-\beta)}{1+k_m(1-\beta)} \in [0,1)$ with $k_m \equiv \frac{M/P}{C} = \left(\frac{\vartheta}{(1-\beta)(1-\vartheta)}\right)^{\frac{1}{\nu}}$.

Equivalently,

$$w_t - p_t = \sigma c_t + \varphi n_t + \varpi i_t$$

where $\varpi \equiv \frac{k_m \beta(1-\frac{\sigma}{\nu})}{1+k_m(1-\beta)}$

Discussion

Equilibrium

Labor market clearing:

$$\sigma c_t + \varphi n_t + \varpi i_t = a_t - \alpha n_t + \log(1-\alpha)$$

which combined with aggregate production function:

$$y_t = \psi_{ya} a_t + \psi_{yi} i_t$$

where $\psi_{yi} \equiv -\frac{\varpi(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha}$ and $\psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha}$

Assessment of size of non-neutralities

Calibration: $\beta = 0.99$; $\sigma = 1$; $\varphi = 5$; $\alpha = 1/4$; $\nu = 1/\eta i$ "large"

$$\Rightarrow \varpi \simeq \frac{k_m \beta}{1 + k_m(1-\beta)} > 0 \quad ; \quad \psi_{yi} \simeq -\frac{k_m}{8} < 0$$

Monetary base inverse velocity: $k_m \simeq 0.3$ $\Rightarrow \psi_{yi} \simeq -0.04$

M2 inverse velocity: $k_m \simeq 3$ $\Rightarrow \psi_{yi} \simeq -0.4$

$\Rightarrow$ small output effects of monetary policy

Response to monetary policy shocks ($a_t = 0$)

$$y_t = \Theta(m_t - p_t)$$

$$i_t = -(1/\eta)(1 - \Theta)(m_t - p_t)$$

where $\Theta \equiv \frac{\varpi(1-\alpha)}{\eta[\sigma(1-\alpha)+\varphi+\alpha]+\varpi(1-\alpha)} \in [0, 1)$ (assuming $\varpi \geq 0$)

$$y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \varpi E_t\{\Delta i_{t+1}\} - \rho)$$

$$p_t = m_t + \frac{\eta}{\eta + \varpi\Lambda} \sum_{k=1}^{\infty} \left(\frac{\eta + \varpi\Lambda}{1 - \Theta + \eta + \varpi\Lambda} \right)^k E_t\{\Delta m_{t+k}\}$$

where $\Lambda \equiv \frac{\eta(\alpha+\varphi)}{\eta[\sigma(1-\alpha)+\varphi+\alpha]+\varpi(1-\alpha)} \in [0, 1)$.

Prediction (independent of rule):

persistent money growth $\Rightarrow cov(\Delta m, i) > 0$ and $cov(\Delta m, y) < 0$

Optimal Monetary Policy with Money in the Utility Function

Social Planner's problem

$$\max U\left(C_t, \frac{M_t}{P_t}, N_t\right)$$

subject to

$$C_t = A_t N_t^{1-\alpha}$$

Optimality conditions:

$$-\frac{U_{n,t}}{U_{c,t}} = (1-\alpha) A_t N_t^{-\alpha}$$

$$U_{m,t} = 0$$

Optimal policy (Friedman rule): $i_t = 0$ for all t .

Intuition

Implied average inflation: $\pi = -\rho < 0$

Implementation

$$i_t = \phi(r_{t-1} + \pi_t)$$

for some $\phi > 1$. Combined with the definition of the real rate:

$$E_t\{i_{t+1}\} = \phi i_t$$

whose only stationary solution is $i_t = 0$ for all t.

Implied equilibrium inflation:

$$\pi_t = -r_{t-1}$$