A Classical Monetary Model

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Assumptions

- Perfect competition in goods and labor markets
- Flexible prices and wages
- No capital accumulation
- No fiscal sector
- Closed economy

Outline

- The problem of households and firms
- Equilibrium: money neutrality and the determination of nominal variables
- A model with money in the utility function
- Optimal policy
Households

Representative household solves

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \tag{1}
\]

subject to

\[
P_tC_t + Q_t B_t \leq B_{t-1} + W_t N_t + D_t \tag{2}
\]

for \( t = 0, 1, 2, \ldots \) and the solvency constraint

\[
\lim_{T \to \infty} E_t \{ \Lambda_{t,T}(B_T/P_T) \} \geq 0 \tag{3}
\]

where \( \Lambda_{t,T} \equiv \beta^{T-t} U_{c,T}/U_{c,t} \) is the stochastic discount factor.

**Optimality conditions**

\[
\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \tag{4}
\]

\[
Q_t = \beta \ E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \tag{5}
\]
Specification of utility:

\[
U(C_t, N_t) = \begin{cases} 
\frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} & \text{for } \sigma \neq 1 \\
\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} & \text{for } \sigma = 1 
\end{cases}
\]

implied optimality conditions:

\[
\frac{W_t}{P_t} = C_t^\sigma N_t^\varphi
\]  \hspace{1cm} (6)

\[
Q_t = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \frac{P_t}{P_{t+1}} \right\}
\]  \hspace{1cm} (7)
Log-linear versions

\[ w_t - p_t = \sigma c_t + \varphi n_t \]  \hspace{1cm} (8)

\[ c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) \]  \hspace{1cm} (9)

where \( \pi_t \equiv p_t - p_{t-1} \), \( i_t \equiv -\log Q_t \) and \( \rho \equiv -\log \beta \)

Steady state (zero growth):

\[ i = \pi + \rho \]

implied real rate

\[ r \equiv i - \pi = \rho \]

Ad-hoc money demand

\[ m_t - p_t = c_t - \eta i_t \]
Firms

Representative firm with technology

\[ Y_t = A_t N_t^{1 - \alpha} \]  \hspace{1cm} (10)

where \( a_t \equiv \log A_t \) follows an exogenous process

\[ a_t = \rho_a a_{t-1} + \varepsilon_t^a \]

**Profit maximization:**

\[ \max P_t Y_t - W_t N_t \]

subject to (10), taking the price and wage as given (perfect competition)

**Optimality condition:**

\[ \frac{W_t}{P_t} = (1 - \alpha) A_t N_t^{-\alpha} \]

In log-linear terms

\[ w_t - p_t = a_t - \alpha n_t + \log (1 - \alpha) \]
Equilibrium

Goods market clearing

\[ y_t = c_t \]

Labor market clearing

\[ \sigma c_t + \varphi n_t = a_t - \alpha n_t + \log(1 - \alpha) \]

Asset market clearing:

\[ B_t = 0 \]

\[ r_t \equiv i_t - E_t\{\pi_{t+1}\} \]
\[ = \rho + \sigma E_t\{\Delta c_{t+1}\} \]

Aggregate output:

\[ y_t = a_t + (1 - \alpha)n_t \]
Implied equilibrium values for real variables:

\[ n_t = \psi_{na} a_t + \psi_n \]
\[ y_t = \psi_{ya} a_t + \psi_y \]
\[ r_t = \rho - \sigma \psi_{ya} (1 - \rho_a) a_t \]
\[ \omega_t \equiv w_t - p_t \]
\[ = a_t - \alpha n_t + \log(1 - \alpha) \]
\[ = \psi_{\omega a} a_t + \psi_\omega \]

where \( \psi_{na} \equiv \frac{1-\sigma}{\sigma(1-\alpha)+\varphi+\alpha} \); \( \psi_n \equiv \frac{\log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} \); \( \psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} \);

\( \psi_y \equiv (1 - \alpha) \psi_n \); \( \psi_{\omega a} \equiv \frac{\sigma+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} \); \( \psi_\omega \equiv \frac{(\sigma(1-\alpha)+\varphi) \log(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} \)

\( \rightarrow \) neutrality: real variables determined independently of monetary policy
\( \rightarrow \) optimal policy: undetermined.
\( \rightarrow \) specification of monetary policy needed to determine nominal variables
Monetary Policy and Price Level Determination

Example I: An Exogenous Path for the Nominal Interest Rate

\[ i_t = i + v_t \]

where

\[ v_t = \rho_v v_{t-1} + \varepsilon_t^v \]

Implied steady state inflation: \( \pi = i - \rho \)

Particular case: \( i_t = i \) for all \( t \).

Using definition of real rate:

\[ E_t\{\pi_{t+1}\} = i_t - r_t \]
\[ = \pi + v_t - \hat{r}_t \]

Equilibrium inflation:

\[ \hat{\pi}_t = v_{t-1} - \hat{r}_{t-1} + \xi_t \]

for any \( \{\xi_t\} \) sequence with \( E_t\{\xi_{t+1}\} = 0 \) for all \( t \).

\[ \Rightarrow \text{nominal indeterminacy} \]
Example II: A Simple Interest Rate Rule

\[ i_t = \rho + \pi + \phi_\pi (\pi_t - \pi) + v_t \]

where \( \phi_\pi \geq 0 \). Combined with definition of real rate:

\[ \phi_\pi \hat{\pi}_t = E_t\{\hat{\pi}_{t+1}\} + \hat{r}_t - v_t \]

If \( \phi_\pi > 1 \),

\[ \hat{\pi}_t = \sum_{k=0}^{\infty} \phi_\pi^{-(k+1)} E_t\{\hat{r}_{t+k} - v_{t+k}\} = -\frac{\sigma(1 - \rho_a) \psi_{ya}}{\phi_\pi - \rho_a} a_t - \frac{1}{\phi_\pi - \rho_v} v_t \]

If \( \phi_\pi < 1 \),

\[ \hat{\pi}_t = \phi_\pi \hat{\pi}_{t-1} - \hat{r}_{t-1} + v_{t-1} + \xi_t \]

for any \( \{\xi_t\} \) sequence with \( E_t\{\xi_{t+1}\} = 0 \) for all \( t \)

\( \implies \) nominal indeterminacy

\( \implies \) illustration of the "Taylor principle" requirement
Responses to a monetary policy shock ($\phi_\pi > 1$ case):

\[
\frac{\partial \pi_t}{\partial \varepsilon_t^v} = -\frac{1}{\phi_\pi - \rho_v} < 0
\]

\[
\frac{\partial i_t}{\partial \varepsilon_t^v} = -\frac{\rho_v}{\phi_\pi - \rho_v} < 0
\]

\[
\frac{\partial m_t}{\partial \varepsilon_t^v} = \frac{\eta \rho_v - 1}{\phi_\pi - \rho_v} \leq 0
\]

\[
\frac{\partial y_t}{\partial \varepsilon_t^v} = 0
\]

Discussion: liquidity effect and price response.
Example III: An Exogenous Path for the Money Supply \{m_t\}

Combining money demand and the definition of the real rate:

\[
p_t = \left( \frac{\eta}{1 + \eta} \right) E_t \{p_{t+1}\} + \left( \frac{1}{1 + \eta} \right) m_t + u_t
\]

where \(u_t \equiv (1 + \eta)^{-1}(\eta r_t - y_t)\). Solving forward:

\[
p_t = \frac{1}{1 + \eta} \sum_{k=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^k E_t \{m_{t+k}\} + \bar{u}_t
\]

where \(\bar{u}_t \equiv \sum_{k=0}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^k E_t \{u_{t+k}\}\)

\[
\Rightarrow \text{price level determinacy}
\]
In terms of money growth rates:

\[ p_t = m_t + \sum_{k=1}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^k E_t \{ \Delta m_{t+k} \} + \bar{u}_t \]

Nominal interest rate:

\[ i_t = \eta^{-1} (y_t - (m_t - p_t)) \]
\[ = \eta^{-1} \sum_{k=1}^{\infty} \left( \frac{\eta}{1 + \eta} \right)^k E_t \{ \Delta m_{t+k} \} + u_t \]

where \( u_t \equiv \eta^{-1} (\bar{u}_t + y_t) \) is independent of monetary policy.
Example

\[ \Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m \]

Assume no real shocks \((r_t = y_t = 0)\).

Price response:

\[ p_t = m_t + \frac{\eta \rho_m}{1 + \eta (1 - \rho_m)} \Delta m_t \]

\( \Rightarrow \) large price response

Nominal interest rate response:

\[ i_t = \frac{\rho_m}{1 + \eta (1 - \rho_m)} \Delta m_t \]

\( \Rightarrow \) no liquidity effect
A Model with Money in the Utility Function

Preferences

\[ E_0 \sum_{t=0}^{\infty} \beta^t U \left( C_t, \frac{M_t}{P_t}, N_t \right) \]

Budget constraint

\[ P_t C_t + Q_t B_t + M_t \leq B_{t-1} + M_{t-1} + W_t N_t + D_t \]

with solvency constraint:

\[ \lim_{T \to \infty} E_t \left\{ \Lambda_{t,T}(A_T/P_T) \right\} \geq 0 \]

where \( A_t \equiv B_t + M_t \).

Equivalently:

\[ P_t C_t + Q_t A_t + (1 - Q_t) M_t \leq A_{t-1} + W_t N_t + D_t \]

Interpretation:

\[ 1 - Q_t = 1 - \exp\{-i_t\} \approx i_t \]

\[ \Rightarrow \text{opportunity cost of holding money} \]
Optimality Conditions

\[-\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t}\]

\[Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\}\]

\[\frac{U_{m,t}}{U_{c,t}} = 1 - \exp\{-i_t\}\]

Two cases:

- utility separable in real balances \(\Rightarrow\) neutrality
- utility non-separable in real balances \(\Rightarrow\) non-neutrality
Utility specification:

\[ U(X_t, N_t) = \frac{X_t^{1-\sigma} - 1}{1 - \sigma} - \frac{N_t^{1+\varphi}}{1 + \varphi} \]

where

\[ X_t = \begin{cases} 
(1 - \vartheta)C_t^{1-\nu} + \vartheta \left( \frac{M_t}{P_t} \right)^{1-\nu} & \text{for } \nu \neq 1 \\
C_t^{1-\vartheta} \left( \frac{M_t}{P_t} \right)^{\vartheta} & \text{for } \nu = 1 
\end{cases} \]

Note that

\[ U_{c,t} = (1 - \vartheta)X_t^{\nu-\sigma}C_t^{-\nu} \]
\[ U_{m,t} = \vartheta X_t^{\nu-\sigma} (M_t/P_t)^{-\nu} \]
\[ U_{n,t} = -N_t^{\varphi} \]
Implied optimality conditions:

\[
\frac{W_t}{P_t} = N_t^{\varphi}X_t^{\sigma - \nu}C_t^{\nu}(1 - \vartheta)^{-1}
\]

\[
Q_t = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\nu} \left( \frac{X_{t+1}}{X_t} \right)^{\nu - \sigma} \left( \frac{P_t}{P_{t+1}} \right) \right\}
\]

\[
\frac{M_t}{P_t} = C_t (1 - \exp\{-i_t\})^{-\frac{1}{\nu}} \left( \frac{\vartheta}{1 - \vartheta} \right)^{\frac{1}{\nu}}
\]

Log-linearized money demand equation:

\[
m_t - p_t = c_t - \eta i_t
\]

where \(\eta \equiv 1/[\nu(\exp\{i\} - 1)]\) .
Log-linearized labor supply equation:

\[ w_t - p_t = \sigma c_t + \varphi n_t + (\nu - \sigma)(c_t - x_t) \]
\[ = \sigma c_t + \varphi n_t + \chi(\nu - \sigma)(c_t - (m_t - p_t)) \]
\[ = \sigma c_t + \varphi n_t + \eta \chi(\nu - \sigma)i_t \]

where \( \chi \equiv \frac{\vartheta^{\frac{1}{\beta}} (1-\beta)^{1-\frac{1}{\beta}}}{(1-\vartheta)^{\frac{1}{\beta}} + \vartheta^{\frac{1}{\beta}} (1-\beta)^{1-\frac{1}{\beta}}} = \frac{k_m (1-\beta)}{1 + k_m (1-\beta)} \in [0, 1) \) with \( k_m \equiv \frac{M}{P C} = \left( \frac{\vartheta}{(1-\beta)(1-\vartheta)} \right)^{\frac{1}{\nu}}. \)

Equivalently,

\[ w_t - p_t = \sigma c_t + \varphi n_t + \omega i_t \]

where \( \omega \equiv \frac{k_m \beta (1-\vartheta)}{1 + k_m (1-\beta)} \)

Discussion
Equilibrium

Labor market clearing:

\[ \sigma c_t + \varphi n_t + w_i_t = a_t - \alpha n_t + \log(1 - \alpha) \]

which combined with aggregate production function:

\[ y_t = \psi_{ya} a_t + \psi_{yi} i_t \]

where \( \psi_{yi} \equiv -\frac{\varpi(1-\alpha)}{\sigma(1-\alpha)+\varphi+\alpha} \) and \( \psi_{ya} \equiv \frac{1+\varphi}{\sigma(1-\alpha)+\varphi+\alpha} \)
Assessment of size of non-neutralities

Calibration: $\beta = 0.99 ; \sigma = 1 ; \varphi = 5 ; \alpha = 1/4 ; \nu = 1/\eta i \ "large"

$$\Rightarrow \omega \approx \frac{k_m\beta}{1 + k_m(1 - \beta)} > 0 \ ; \ \psi_{yi} \approx -\frac{k_m}{8} < 0$$

Monetary base inverse velocity: $k_m \approx 0.3 \quad \Rightarrow \psi_{yi} \approx -0.04$

M2 inverse velocity: $k_m \approx 3 \quad \Rightarrow \psi_{yi} \approx -0.4$

$\Rightarrow$ small output effects of monetary policy
Response to monetary policy shocks ($a_t = 0$)

\[ y_t = \Theta (m_t - p_t) \]
\[ i_t = -(1/\eta) (1 - \Theta) (m_t - p_t) \]

where \( \Theta \equiv \frac{\omega(1-\alpha)}{\eta[\sigma(1-\alpha)+\varphi+\alpha]+\omega(1-\alpha)} \in [0, 1) \) (assuming \( \omega \geq 0 \))

\[ y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}\} - \omega E_t\{\Delta i_{t+1}\} - \rho) \]
\[ p_t = m_t + \frac{\eta}{\eta + \omega \Lambda} \sum_{k=1}^{\infty} \left( \frac{\eta + \omega \Lambda}{1 - \Theta + \eta + \omega \Lambda} \right)^k E_t\{\Delta m_{t+k}\} \]

where \( \Lambda \equiv \frac{\eta(\alpha+\varphi)}{\eta[\sigma(1-\alpha)+\varphi+\alpha]+\omega(1-\alpha)} \in [0, 1). \)

Prediction (independent of rule):

persistent money growth \( \Rightarrow \text{cov}(\Delta m, i) > 0 \) and \( \text{cov}(\Delta m, y) < 0 \)
Optimal Monetary Policy with Money in the Utility Function

Social Planner’s problem

\[
\max U \left( C_t, \frac{M_t}{P_t}, N_t \right)
\]

subject to

\[ C_t = A_t N_t^{1-\alpha} \]

Optimality conditions:

\[
-\frac{U_{n,t}}{U_{c,t}} = (1 - \alpha) A_t N_t^{-\alpha}
\]

\[ U_{m,t} = 0 \]

Optimal policy (Friedman rule): \[ i_t = 0 \] for all \( t \).

Intuition

Implied average inflation: \( \pi = -\rho < 0 \)
Implementation

\[ i_t = \phi(r_{t-1} + \pi_t) \]

for some \( \phi > 1 \). Combined with the definition of the real rate:

\[ E_t\{i_{t+1}\} = \phi i_t \]

whose only stationary solution is \( i_t = 0 \) for all \( t \).

Implied equilibrium inflation:

\[ \pi_t = -r_{t-1} \]