Motivation and Outline

Evidence on Money, Output, and Prices:

- Macro evidence on the effects of monetary policy shocks
  (i) persistent effects on real variables
  (ii) slow adjustment of aggregate price level
  (iii) liquidity effect
- Micro evidence: significant price and wage rigidities
Figure 1. Estimated Dynamic Response to a Monetary Policy Shock

Source: Christiano, Eichenbaum and Evans (1999)
Figure 1 - Examples of individual price trajectories (French and Italian CPI data)

Note: Actual examples of trajectories, extracted from the French and Italian CPI databases. The databases are described in Baudry et al. (2004) and Veronese et al. (2005). Prices are in levels, denominated in French Francs and Italian Lira respectively. The dotted lines indicate events of price changes.

Source: Dhyne et al. (JEP, 2006)
Table 1. Measures of price stickiness in the euro area and the US (% per month unless otherwise stated).

<table>
<thead>
<tr>
<th>Statistics</th>
<th>Euro area</th>
<th>US</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPI(^*)</td>
<td>Frequency</td>
<td>15.1</td>
</tr>
<tr>
<td></td>
<td>Average duration (months)</td>
<td>13.0</td>
</tr>
<tr>
<td></td>
<td>Median duration (months)</td>
<td>10.6</td>
</tr>
<tr>
<td>PPI(^\d)</td>
<td>Frequency</td>
<td>20.0</td>
</tr>
<tr>
<td>Surveys(^\d)</td>
<td>Frequency</td>
<td>15.9</td>
</tr>
<tr>
<td></td>
<td>Average duration (months)</td>
<td>10.8</td>
</tr>
<tr>
<td>NKPC(^$)</td>
<td>Average durations (months)</td>
<td>13.5–19.2</td>
</tr>
<tr>
<td>Internet prices(^\d)</td>
<td>Frequency</td>
<td>79.2</td>
</tr>
</tbody>
</table>

Source: Álvarez et al. (JEEA, 2006)
Motivation and Outline

Evidence on Money, Output, and Prices:

• Macro evidence on the effects of monetary policy shocks
  
  (i) persistent effects on real variables
  (ii) slow adjustment of aggregate price level
  (iii) liquidity effect

• Micro evidence: significant price and wage rigidities
  
  ⇒ in conflict with the predictions of classical monetary models

A Baseline Model with Nominal Rigidities

• monopolistic competition

• sticky prices (staggered price setting)

• competitive labor markets, closed economy, no capital accumulation
Households

Representative household solves

$$\max E_0 \sum_{t=0}^{\infty} \beta^t U (C_t, N_t; Z_t)$$

where

$$C_t \equiv \left( \int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon-1}}$$

subject to

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + W_t N_t + D_t$$

for \( t = 0, 1, 2, \ldots \) plus solvency constraint.

Example:

$$\lim_{T \to \infty} E_t \left\{ \frac{B_T}{P_T} \right\} \geq 0$$
Optimality conditions

1. Optimal allocation of expenditures

\[ C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \]

implying

\[ \int_0^1 P_t(i) C_t(i) di = P_t C_t \]

where

\[ P_t = \left( \int_0^1 P_t(i)^{1-\epsilon} di \right)^{\frac{1}{1-\epsilon}} \]

2. Other optimality conditions

\[ \frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \]

\[ Q_t = \beta E_t \left\{ \frac{U_{c,t+1}}{U_{c,t}} \frac{P_t}{P_{t+1}} \right\} \]
**Specification of utility:**

\[ U(C_t, N_t; Z_t) = \begin{cases} 
\left( \frac{C_t^{1-\sigma} - N_t^{1+\varphi}}{1-\sigma} \right) Z_t & \text{for } \sigma \neq 1 \\
\left( \log C_t - N_t^{1+\varphi} \right) Z_t & \text{for } \sigma = 1 
\end{cases} \]

where

\[ z_t = \rho_z z_{t-1} + \varepsilon_t \]

**Optimality conditions:**

\[ w_t - p_t = \sigma c_t + \varphi n_t \]

\[ c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_z)z_t \]

where \( i_t \equiv -\log Q_t \) and \( \rho \equiv -\log \beta \)

**Ad-hoc money demand:**

\[ m_t - p_t = c_t - \eta i_t \]
Firms

- Continuum of firms, indexed by $i \in [0, 1]$
- Each firm produces a differentiated good
- Identical technology
  \[ Y_t(i) = A_t N_t(i)^{1-\alpha} \]
  where
  \[ a_t = \rho_a a_{t-1} + \varepsilon_t^a \]
- Probability of resetting price in any given period: $1 - \theta$, independent across firms (Calvo (1983)).
- $\theta \in [0, 1]$: index of price stickiness
- Implied average price duration $\frac{1}{1-\theta}$
Aggregate Price Dynamics

\[ P_t = \left[ \theta(P_{t-1})^{1-\epsilon} + (1 - \theta)(P^*_t)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \]

Dividing by \( P_{t-1} \):

\[ \Pi_t^{1-\epsilon} = \theta + (1 - \theta) \left( \frac{P^*_t}{P_{t-1}} \right)^{1-\epsilon} \]

Log-linearization around zero inflation steady state

\[ \pi_t = (1 - \theta)(p^*_t - p_{t-1}) \quad (1) \]

or, equivalently

\[ p_t = \theta p_{t-1} + (1 - \theta)p^*_t \]
Optimal Price Setting

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k}(1/P_{t+k}) \left( P_t^* Y_{t+k|t} - C_{t+k}(Y_{t+k|t}) \right) \right\}$$

subject to:

$$Y_{t+k|t} = \left( \frac{P_t}{P_{t+k}} \right)^{-\epsilon} C_{t+k}$$  \hspace{1cm} (2)

for $k = 0, 1, 2, \ldots$ where $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}$

Optimality condition:

$$\sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} Y_{t+k|t}(1/P_{t+k}) \left( P_t^* - \mathcal{M} \Psi_{t+k|t} \right) \right\} = 0$$

where $\Psi_{t+k|t} \equiv C'_{t+k}(Y_{t+k|t})$ and $\mathcal{M} \equiv \frac{\epsilon}{\epsilon-1}$.

Flexible price case ($\theta = 0$):

$$P_t^* = \mathcal{M} \Psi_{t|t}$$
Zero inflation steady state

\[ \Lambda_{t,t+k} = \beta^k ; \quad P_t^*/P_{t-1} = P_t/P_{t+k} = 1 \Rightarrow Y_{t+k|t} = Y ; \quad \Psi_{t+k|t} = \Psi_t ; \quad P_t = \mathcal{M}\Psi_t \]

Linearized optimal price setting condition:

\[ p_t^* = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\{\psi_{t+k|t}\} \]

where \( \psi_{t+k|t} \equiv \log \Psi_{t+k|t} \) and \( \mu \equiv \log \mathcal{M} \)
**Particular Case:** $\alpha = 0$ (constant returns)

$$\implies \psi_{t+k|t} = \psi_{t+k}$$

Recursive form:

$$p_t^* = \beta \theta E_t\{p_{t+1}^*\} + (1 - \beta \theta)p_t - (1 - \beta \theta)\hat{\mu}_t$$

where $\mu_t \equiv p_t - \psi_t$ and $\hat{\mu}_t \equiv \mu_t - \mu$

Combined with price dynamics equation yields:

$$\pi_t = \beta E_t\{\pi_{t+1}\} - \lambda \hat{\mu}_t$$

where

$$\lambda \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta}$$
General case: $\alpha \in [0, 1)$

$$
\psi_{t+k|t} = w_{t+k} - m_p n_{t+k|t} = w_{t+k} - (a_{t+k} - \alpha n_{t+k|t} + \log(1 - \alpha))
$$

$$
\psi_{t+k} \equiv w_{t+k} - (a_{t+k} - \alpha n_{t+k} + \log(1 - \alpha))
$$

$$
\psi_{t+k|t} = \psi_{t+k} + \alpha (n_{t+k|t} - n_{t+k})
$$

$$
= \psi_{t+k} + \frac{\alpha}{1 - \alpha} (y_{t+k|t} - y_{t+k})
$$

$$
= \psi_{t+k} - \frac{\alpha \epsilon}{1 - \alpha} (p^*_t - p_{t+k})
$$

Optimal price setting equation:

$$
p^*_t = (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \{p_{i+k} - \Theta \hat{\mu}_{t+k}\}
$$

where $\Theta \equiv \frac{1-\alpha}{1-\alpha + \alpha \epsilon} \in (0, 1]$. 
Recursive form:

\[ p_t^* = \beta \theta E_t\{p_{t+1}^*\} + (1 - \beta \theta)p_t - (1 - \beta \theta)\Theta \hat{\mu}_t \]

Combined with price dynamics equation yields:

\[ \pi_t = \beta E_t\{\pi_{t+1}\} - \lambda \hat{\mu}_t \]

where

\[ \lambda \equiv \frac{(1 - \theta)(1 - \beta \theta)}{\theta} \]
Equilibrium

Goods markets clearing

\[ Y_t(i) = C_t(i) \]

for all \( i \in [0, 1] \) and all \( t \).

Letting \( Y_t \equiv \left( \int_0^1 Y_t(i)^{1 - \frac{1}{\epsilon}} di \right)^{\frac{\epsilon}{\epsilon - 1}} \):

\[ Y_t = C_t \]

Combined with Euler equation:

\[ y_t = E_t\{y_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_z)z_t \]
Labor market clearing:

\begin{align*}
N_t &= \int_0^1 N_t(i) \, di \\
&= \int_0^1 \left( \frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}} \, di \\
&= \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\alpha}{1-\alpha}} \, di
\end{align*}

Up to a first order approximation:

\begin{align*}
n_t &= \frac{1}{1 - \alpha} (y_t - a_t)
\end{align*}
Average price markup and output

\[ \mu_t \equiv p_t - \psi_t \]
\[ = -(w_t - p_t) + (a_t - \alpha n_t + \log(1 - \alpha)) \]
\[ = -(\sigma y_t + \varphi n_t) + (a_t - \alpha n_t + \log(1 - \alpha)) \]
\[ = -\left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t + \left( \frac{1 + \varphi}{1 - \alpha} \right) a_t + \log(1 - \alpha) \]

Under flexible prices:

\[ \mu = -\left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) y_t^n + \left( \frac{1 + \varphi}{1 - \alpha} \right) a_t + \log(1 - \alpha) \]

implying

\[ y_t^n = \psi_y a_t + \psi_y \]

where \( \psi_y \equiv -\frac{(1-\alpha)(\mu - \log(1 - \alpha))}{\sigma(1 - \alpha) + \varphi + \alpha} > 0 \) and \( \psi_y a \equiv \frac{1 + \varphi}{\sigma(1 - \alpha) + \varphi + \alpha} \). Thus,

\[ \hat{\mu}_t = -\left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) (y_t - y_t^n) \]
New Keynesian Phillips Curve

\[ \pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \]

where \( \tilde{y}_t \equiv y_t - y_t^v \) and \( \kappa \equiv \lambda \left( \sigma + \frac{\sigma + \alpha}{1 - \alpha} \right) \).
The Non-Policy Block of the Basic New Keynesian Model

New Keynesian Phillips Curve

\[ \pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \]

Dynamic IS equation

\[ \tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) \]

where \( r_t^n \) is the natural rate of interest, given by

\[ r_t^n = \rho - \sigma(1 - \rho_a)\psi_y a_t + (1 - \rho_z)z_t \]

Missing block: description of monetary policy (determination of \( i_t \)).
Equilibrium under a Simple Interest Rate Rule

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t \]

where \( \tilde{y}_t \equiv y_t - y \) and

\[ v_t = \rho_v v_{t-1} + \varepsilon_t^v \]

Equivalently:

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \phi_y \tilde{y}_t^n + v_t \]

where \( \tilde{y}_t^n \equiv y_t^n - y \).
Equilibrium dynamics:

$$\begin{bmatrix} \tilde{\gamma}_t \\ \pi_t \end{bmatrix} = A_T \begin{bmatrix} E_t\{\tilde{\gamma}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + B_T \ u_t$$

where

$$u_t \equiv \tilde{r}_t^n - \phi_y \tilde{y}_t^n - v_t$$

$$= -\psi_y a_\pi (\phi_y + \sigma(1 - \rho_a)) a_t + (1 - \rho_z) z_t - v_t$$

and

$$A_T \equiv \Omega \begin{bmatrix} \sigma & 1 - \beta \phi_\pi \\ \sigma \kappa & \kappa + \beta (\sigma + \phi_\pi) \end{bmatrix} ; \quad B_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

with $$\Omega \equiv \frac{1}{\sigma + \phi_y + \kappa \phi_\pi}.$$ 

Uniqueness condition (Bullard and Mitra):

$$\kappa (\phi_\pi - 1) + (1 - \beta) \phi_y > 0$$

Exercise: analytical solution (method of undetermined coefficients).
Equilibrium under an Exogenous Money Growth Process

\[ \Delta m_t = \rho_m \Delta m_{t-1} + \varepsilon_t^m \]

Money demand

\[ l_t = m_t - p_t = \tilde{y}_t - \eta i_t + y_t^n \]

Substituting into dynamic IS equation

\[ (1 + \sigma \eta) \tilde{y}_t = \sigma \eta E_t\{\tilde{y}_{t+1}\} + \tilde{l}_t + \eta E_t\{\pi_{t+1}\} + \eta \hat{r}_t^n - \hat{y}_t^n \]

Identity:

\[ \hat{l}_{t-1} = \hat{l}_t + \pi_t - \Delta m_t \]
Equilibrium dynamics:

\[
A_{M,0} \begin{bmatrix} \tilde{y}_t \\ \pi_t \\ \hat{l}_{t-1} \end{bmatrix} = A_{M,1} \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \hat{l}_t \end{bmatrix} + B_M \begin{bmatrix} \hat{r}_t^n \\ \hat{y}_t^n \\ \Delta m_t \end{bmatrix}
\]

(3)

where

\[
A_{M,0} = \begin{bmatrix} 1 + \sigma \eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix}; \quad A_{M,1} = \begin{bmatrix} \sigma \eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix}; \quad B_M = \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{bmatrix}
\]

Uniqueness condition:

\[A_M \equiv A_{M,0}^{-1}A_{M,1}\] has two eigenvalues inside and one outside the unit circle.
Calibration

Households: $\sigma = 1 ; \varphi = 5 ; \beta = 0.99 ; \epsilon = 9 ; \eta = 4 ; \rho_z = 0.5$
Firms: $\alpha = 1/4 ; \theta = 3/4 ; \rho_a = 0.9$
Policy rules: $\phi_\pi = 1.5, \phi_y = 0.125 ; \rho_v = \rho_m = 0.5$

Dynamic Responses to Exogenous Shocks

- Monetary policy, discount rate, technology
- Interest rate rule vs. money growth rule
Dynamic responses to a monetary policy shock: Interest rate rule
Dynamic responses to a discount rate shock: Interest rate rule
Dynamic responses to a technology shock: Interest rate rule
Estimated Effects of Technology Shocks

Figure 4. Estimated Impulse Responses from a Five-Variable Model: U.S. Data, First-Differenced Hours (Point Estimates and ±2 Standard Error Confidence Intervals)

Source: Galí (1999)
Estimated Effects of Technology Shocks

Dynamic responses to a monetary policy shock: Money growth rule
Dynamic responses to a discount rate shock: Money growth rule
Dynamic responses to a technology shock: Money growth rule