Monetary Policy Design
in the Basic New Keynesian Model

by

Jordi Galí

October 2015
The Efficient Allocation

\[ \max U \left( C_t, N_t; Z_t \right) \]

where \( C_t \equiv \left( \int_0^1 C_t(i)^{1-\frac{1}{\alpha}} di \right)^{\frac{1}{\frac{1}{\alpha} - 1}} \) subject to:

\[ C_t(i) = A_t N_t(i)^{1-\alpha}, \text{ all } i \in [0, 1] \]

\[ N_t = \int_0^1 N_t(i) di \]

Optimality conditions:

\[ C_t(i) = C_t, \text{ all } i \in [0, 1] \]
\[ N_t(i) = N_t, \text{ all } i \in [0, 1] \]
\[ -\frac{U_{n,t}}{U_{c,t}} = MPN_t \]

where \( MPN_t \equiv (1 - \alpha) A_t N_t^{-\alpha} \)
Sources of Suboptimality of Equilibrium

1. Distortions unrelated to nominal rigidities: Market power

Optimal price setting: \( P_t = \mathcal{M} \frac{W_t}{MPN_t} \), where \( \mathcal{M} \equiv \frac{\varepsilon}{\varepsilon-1} > 1 \)

\[
\implies \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t
\]

Assume employment subsidy \( \tau \). Under flexible prices, \( P_t = \mathcal{M} \frac{(1-\tau)W_t}{MPN_t} \).

\[
\implies \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}(1-\tau)}
\]

Optimal subsidy: \( \mathcal{M}(1-\tau) = 1 \) or, equivalently, \( \tau = \frac{1}{\varepsilon} \).
2. Distortions associated with the presence of nominal rigidities:

- **Markup variations** resulting from sticky prices (assuming optimal subsidy):
  \[
  M_t = \frac{P_t}{(1 - \tau)(W_t/MPN_t)} = \frac{P_t M}{W_t/MPN_t}
  \]

  \[
  \implies -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t \frac{M}{M_t} \neq MPN_t
  \]

  Efficiency requirement: average markup = desired markup, all \( t \)

- **Relative price distortions** resulting from staggered price setting: \( C_t(i) \neq C_t(j) \) if \( P_t(i) \neq P_t(j) \). Optimal policy requires that prices and quantities (and hence marginal costs) are equalized across goods.
Optimal Monetary Policy in the Basic NK Model

Assumptions:

- optimal employment subsidy
  \[ \implies \text{flexible price equilibrium allocation is efficient} \]
- no inherited relative price distortions, i.e. \( P_{-1}(i) = P_{-1} \) for all \( i \in [0, 1] \)

Optimal policy: replicate flexible price equilibrium allocation.

Implementation: commit to stabilizing marginal costs at a level consistent with firms’ desired markup, given existing prices:

- no firm has an incentive to adjust its price, i.e. \( P_t^* = P_{t-1} \) and, hence, \( P_t = P_{t-1} \) for \( t = 0, 1, 2, \ldots \) (aggregate price stability)
- equilibrium output and employment match their natural counterparts.
Equilibrium under the Optimal Policy

\[ y_t = y_t^n \implies \tilde{y}_t = 0 \]
\[ \pi_t = 0 \]
\[ i_t = r^n_t \]
for all \( t \).

Implementation: Some Candidate Interest Rate Rules

Non-Policy Block:

\[ \tilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1} - r^n_t\}) + E_t\{\tilde{y}_{t+1}\} \]
\[ \pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \]

where \( r^n_t = \rho - \sigma(1 - \rho_a)\psi_y a_t + (1 - \rho_z)z_t \)
An Exogenous Interest Rate Rule

\[ i_t = r^n_t \]

Equilibrium dynamics:

\[
\begin{bmatrix}
\tilde{y}_t \\
\pi_t
\end{bmatrix}
= A_O
\begin{bmatrix}
E_t\{\tilde{y}_{t+1}\} \\
E_t\{\pi_{t+1}\}
\end{bmatrix}
\]

where

\[
A_O \equiv \begin{bmatrix}
1 & 1 \\
\kappa & \beta + \frac{\kappa}{\sigma}
\end{bmatrix}
\]

Shortcoming: the solution \( \tilde{y}_t = \pi_t = 0 \) for all \( t \) is not unique: one eigenvalue of \( A_O \) is strictly greater than one. \( \rightarrow \) indeterminacy (real and nominal).
An Interest Rate Rule with Feedback from Target Variables

\[ i_t = r^n_t + \phi_\pi \pi_t + \phi_y \tilde{y}_t \]

Equilibrium dynamics:

\[
\begin{bmatrix}
\tilde{y}_t \\
\pi_t
\end{bmatrix} = A_T \begin{bmatrix}
E_t\{\tilde{y}_{t+1}\} \\
E_t\{\pi_{t+1}\}
\end{bmatrix}
\]

where

\[
A_T \equiv \frac{1}{\sigma + \phi_y + \kappa \phi_\pi} \begin{bmatrix}
\sigma & 1 - \beta \phi_\pi \\
\sigma \kappa & \kappa + \beta (\sigma + \phi_y)
\end{bmatrix}
\]
Existence and uniqueness condition: (Bullard and Mitra (2002)): 

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$

Taylor-principle interpretation (Woodford (2000)): 

$$\lim_{k \to \infty} \frac{d_i t+k}{d\pi_t} = \phi_\pi + \phi_y \lim_{k \to \infty} \frac{d\tilde{y}_{t+k}}{d\pi_t}$$

$$= \phi_\pi + \frac{\phi_y(1 - \beta)}{\kappa}$$
Figure 4.1 Determinacy and Indeterminacy Regions: Standard Taylor Rule
A Forward-Looking Interest Rate Rule

\[ i_t = r^n_t + \phi_{\pi} E_t\{\pi_{t+1}\} + \phi_y E_t\{\tilde{y}_{t+1}\} \]

Equilibrium dynamics:

\[
\begin{bmatrix}
\tilde{y}_t \\
\pi_t
\end{bmatrix} = \mathbf{A}_F 
\begin{bmatrix}
E_t\{\tilde{y}_{t+1}\} \\
E_t\{\pi_{t+1}\}
\end{bmatrix}
\]

where

\[
\mathbf{A}_F \equiv \begin{bmatrix}
1 - \sigma^{-1} \phi_y & -\sigma^{-1}(\phi_{\pi} - 1) \\
\kappa (1 - \sigma^{-1} \phi_y) & \beta - \kappa \sigma^{-1}(\phi_{\pi} - 1)
\end{bmatrix}
\]

Existence and uniqueness conditions (Bullard and Mitra (2002):

\[\kappa (\phi_{\pi} - 1) + (1 - \beta)\phi_y > 0\]

\[\kappa (\phi_{\pi} - 1) + (1 + \beta)\phi_y < 2\sigma (1 + \beta)\]
Figure 4.2 Determinacy and Indeterminacy Regions: Forward Looking Taylor Rule
Shortcomings of Optimal Rules

- assumed observability of the natural rate of interest (in real time).
- this requires, in turn, knowledge of:
  
  (i) the true model  
  (ii) true parameter values  
  (iii) realized shocks

“Simple rules”:
- the policy instrument depends on observable variables only,
- do not require knowledge of the true parameter values
- ideally, they approximate optimal rule across different models
Simple Monetary Policy Rules

Welfare-based evaluation:

\[ \mathbb{W} \equiv -E_0 \sum_{t=0}^{\infty} \beta^t \left( \frac{U_t - U^*_t}{U_c C} \right) = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[ \left( \sigma + \varphi + \alpha \right) \tilde{y}_t^2 + \frac{\epsilon}{\lambda^2} \pi_t^2 \right] \]

\[ \implies \text{expected average welfare loss per period:} \]

\[ \mathbb{L} = \frac{1}{2} \left[ \left( \sigma + \varphi + \alpha \right) \text{var}(\tilde{y}_t) + \epsilon \text{var}(\pi_t) \right] \]
A Taylor Rule

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t \]

Equivalently:

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + v_t \]

where \( v_t \equiv \phi_y \hat{y}_t^n \)

Equilibrium dynamics:

\[
\begin{bmatrix}
\hat{y}_t \\
\pi_t
\end{bmatrix}
= A_T \begin{bmatrix}
E_t\{\hat{y}_{t+1}\} \\
E_t\{\pi_{t+1}\}
\end{bmatrix}
+ B_T(\hat{r}_t^n - v_t)
\]

where

\[
A_T \equiv \Omega \begin{bmatrix}
\sigma & 1 - \beta \phi_\pi \\
\sigma \kappa & \kappa + \beta (\sigma + \phi_y)
\end{bmatrix} \quad ; \quad B_T \equiv \Omega \begin{bmatrix}
1 \\
\kappa
\end{bmatrix}
\]

and \( \Omega \equiv \frac{1}{\sigma + \phi_y + \kappa \phi_\pi} \). Note that \( \hat{r}_t^n - v_t = -\psi_y a(\sigma(1 - \rho_a) + \phi_y) a_t + (1 - \rho_z) z_t \)

Exercise: \( \Delta a_t \sim AR(1) + \) modified Taylor rule \( i_t = \rho + \phi_\pi \pi_t + \phi_y \Delta y_t \)
<table>
<thead>
<tr>
<th></th>
<th>Technology</th>
<th>Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_\pi$</td>
<td>1.5 1.5 5 1.5</td>
<td>1.5 1.5 5 1.5</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>0.125 0 0 1</td>
<td>0.125 0 0 1</td>
</tr>
<tr>
<td>$\sigma(y)$</td>
<td>1.85 2.07 2.25 1.06</td>
<td>0.59 0.68 0.28 0.31</td>
</tr>
<tr>
<td>$\sigma(\tilde{y})$</td>
<td>0.44 0.21 0.03 1.23</td>
<td>0.59 0.68 0.28 0.31</td>
</tr>
<tr>
<td>$\sigma(\pi)$</td>
<td>0.69 0.34 0.05 1.94</td>
<td>0.20 0.23 0.09 0.10</td>
</tr>
<tr>
<td>$L$</td>
<td>1.02 0.25 0.006 7.98</td>
<td>0.10 0.13 0.02 0.02</td>
</tr>
</tbody>
</table>
Money Growth Peg

\[ \Delta m_t = 0 \]

Money demand:

\[ l_t = y_t - \eta i_t - \zeta_t \]

where \( l_t \equiv m_t - p_t \).

\[ \Delta \zeta_t = \rho \zeta \Delta \zeta_{t-1} + \varepsilon_t \]

where \( \rho \zeta \in [0, 1) \).

\[ \hat{l}_t = \tilde{y}_t + \tilde{y}_t^n - \eta \hat{i}_t - \zeta_t \]

Letting \( l^+_t \equiv l_t + \zeta_t \)

\[ \hat{i}_t = \frac{1}{\eta}(\tilde{y}_t + \tilde{y}_t^n - \hat{l}_t^+) \]

Imposing the assumed rule \( \Delta m_t = 0 \), and clearing of the money market:

\[ \hat{l}_{t-1}^+ = \hat{l}_t^+ + \pi_t - \Delta \zeta_t \]
Equilibrium dynamics:

\[
A_{M,0} \begin{bmatrix} \tilde{y}_t \\ \pi_t \\ \hat{l}^+_{t-1} \end{bmatrix} = A_{M,1} \begin{bmatrix} \bar{E}_t \{\tilde{y}_{t+1}\} \\ \bar{E}_t \{\pi_{t+1}\} \\ \hat{l}^+_t \end{bmatrix} + B_M \begin{bmatrix} \hat{r}^m_t \\ \hat{y}^m_t \\ \Delta \zeta_t \end{bmatrix}
\]

where

\[
A_{M,0} \equiv \begin{bmatrix} 1 + \sigma \eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad A_{M,1} \equiv \begin{bmatrix} \sigma \eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad B_M \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}
\]
### Table 4.2
Evaluation of Simple Rules: Constant Money Growth

<table>
<thead>
<tr>
<th></th>
<th>Technology</th>
<th>Demand</th>
<th>Money Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(y)$</td>
<td>1.72</td>
<td>0.59</td>
<td>1.07</td>
</tr>
<tr>
<td>$\sigma(\bar{y})$</td>
<td>0.92</td>
<td>0.59</td>
<td>1.07</td>
</tr>
<tr>
<td>$\sigma(\pi)$</td>
<td>0.35</td>
<td>0.12</td>
<td>0.55</td>
</tr>
<tr>
<td>$\mathbb{L}$</td>
<td>0.29</td>
<td>0.04</td>
<td>0.69</td>
</tr>
</tbody>
</table>
The Taylor Rule (Taylor 1993)
Clarida, Galí and Gertler (QJE 2000)

\[ i_t = \rho i_{t-1} + (1 - \rho) \left[ r + \pi^* + \beta E_t \{ \pi_{t+1} - \pi^* \} + \gamma E_t \{ y_{t+1} - y_{t+1}^* \} \right] \]

<table>
<thead>
<tr>
<th>TABLE II</th>
</tr>
</thead>
<tbody>
<tr>
<td>Baseline Estimates</td>
</tr>
<tr>
<td>-------------</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>( \pi^* )</th>
<th>( \beta )</th>
<th>( \gamma )</th>
<th>( \rho )</th>
<th>( p )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre-Volcker</td>
<td>4.24</td>
<td>0.83</td>
<td>0.27</td>
<td>0.68</td>
<td>0.834</td>
</tr>
<tr>
<td></td>
<td>(1.09)</td>
<td>(0.07)</td>
<td>(0.08)</td>
<td>(0.05)</td>
<td></td>
</tr>
<tr>
<td>Volcker-Greenspan</td>
<td>3.58</td>
<td>2.15</td>
<td>0.93</td>
<td>0.79</td>
<td>0.316</td>
</tr>
<tr>
<td></td>
<td>(0.50)</td>
<td>(0.40)</td>
<td>(0.42)</td>
<td>(0.04)</td>
<td></td>
</tr>
</tbody>
</table>

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation: output gap, the federal funds rate, the short-long spread, and commodity price inflation.
Orphanides (JME 2003)
Fig. 5. Then and now: Taylor rule with final and real-time data.