Monetary Policy Design in the Basic New Keynesian Model

by

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The Efficient Allocation

$$\max U\left(C_t, N_t; Z_t\right)$$

where $C_t \equiv \left(\int_0^1 C_t(i)^{1-\frac{1}{\epsilon}} di\right)^{\frac{\epsilon}{\epsilon-1}}$ subject to:

$$C_t(i) = A_t N_t(i)^{1-\alpha}, \ \ all \ \ i \in [0,1]$$

$$N_t = \int_0^1 N_t(i) di$$

Optimality conditions:

$$C_t(i) = C_t, \ \ all \ \ i \in [0,1]$$

$$N_t(i) = N_t, \ \ all \ \ i \in [0,1]$$

$$-\frac{U_{n,t}}{U_{c,t}} = MPN_t$$

where $MPN_t \equiv (1-\alpha) A_t N_t^{-\alpha}$

Sources of Suboptimality of Equilibrium

1. Distortions unrelated to nominal rigidities: Market power

Optimal price setting: $P_t = \mathcal{M}\frac{W_t}{MPN_t}$, where $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon-1} > 1$

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}} < MPN_t$$

Assume employment subsidy τ. Under flexible prices, $P_t = \mathcal{M}\frac{(1-\tau)W_t}{MPN_t}$.

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = \frac{MPN_t}{\mathcal{M}(1-\tau)}$$

Optimal subsidy: $\mathcal{M}(1-\tau) = 1$ or, equivalently, $\tau = \frac{1}{\varepsilon}$.

2. Distortions associated with the presence of nominal rigidities:

- *Markup variations* resulting from sticky prices (assuming optimal subsidy):

$$\mathcal{M}_t = \frac{P_t}{(1-\tau)(W_t/MPN_t)} = \frac{P_t \mathcal{M}}{W_t/MPN_t}$$

$$\Longrightarrow \quad -\frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} = MPN_t \frac{\mathcal{M}}{\mathcal{M}_t} \neq MPN_t$$

Efficiency requirement: average markup = desired markup, all t

- *Relative price distortions* resulting from staggered price setting: $C_t(i) \neq C_t(j)$ if $P_t(i) \neq P_t(j)$. Optimal policy requires that prices and quantities (and hence marginal costs) are equalized across goods.

Optimal Monetary Policy in the Basic NK Model

Assumptions:

- optimal employment subsidy

  $\Longrightarrow$ flexible price equilibrium allocation is efficient

- no inherited relative price distortions, i.e. $P_{-1}(i) = P_{-1}$ for all $i \in [0,1]$

Optimal policy: replicate flexible price equilibrium allocation.

Implementation: commit to stabilizing marginal costs at a level consistent with firms' desired markup, *given existing prices*:

- no firm has an incentive to adjust its price, i.e. $P_t^* = P_{t-1}$ and, hence, $P_t = P_{t-1}$ for $t = 0, 1, 2, \ldots$(aggregate price stability)

- equilibrium output and employment match their *natural* counterparts.

Equilibrium under the Optimal Policy

$$y_t = y_t^n \quad \Rightarrow \quad \widetilde{y}_t = 0$$
$$\pi_t = 0$$
$$i_t = r_t^n$$

for all t.

Implementation: Some Candidate Interest Rate Rules

Non-Policy Block:

$$\widetilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi_{t+1}\} - r_t^n) + E_t\{\widetilde{y}_{t+1}\}$$
$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \widetilde{y}_t$$

where $r_t^n = \rho - \sigma(1 - \rho_a)\psi_{ya} a_t + (1 - \rho_z) z_t$

An Exogenous Interest Rate Rule

$$i_t = r_t^n$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A_O} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A_O} \equiv \begin{bmatrix} 1 & \frac{1}{\sigma} \\ \kappa & \beta + \frac{\kappa}{\sigma} \end{bmatrix}$$

Shortcoming: the solution $\widetilde{y}_t = \pi_t = 0$ for all t is *not* unique: one eigenvalue of $\mathbf{A_O}$ is strictly greater than one. $\rightarrow$ indeterminacy (real and nominal).

An Interest Rate Rule with Feedback from Target Variables

$$i_t = r_t^n + \phi_\pi \pi_t + \phi_y \widetilde{y}_t$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

where

$$\mathbf{A}_T \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \begin{bmatrix} \sigma & 1 - \beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma + \phi_y) \end{bmatrix}$$

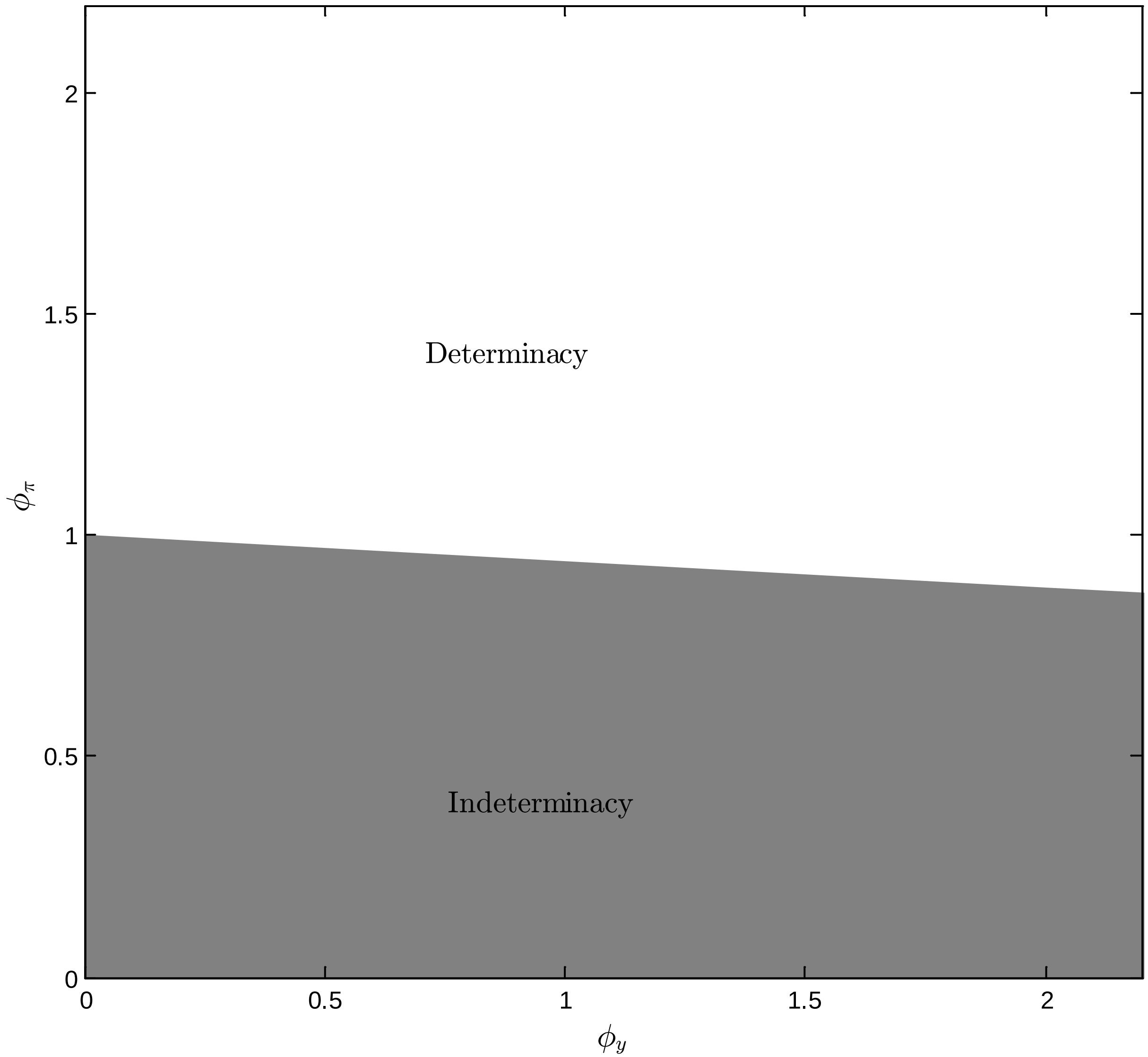
Existence and uniqueness condition: (Bullard and Mitra (2002)):

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$$

Taylor-principle interpretation (Woodford (2000)):

$$\begin{aligned}\lim_{k\to\infty}\frac{di_{t+k}}{d\pi_t} &= \phi_\pi + \phi_y \lim_{k\to\infty}\frac{d\widetilde{y}_{t+k}}{d\pi_t} \\ &= \phi_\pi + \frac{\phi_y(1-\beta)}{\kappa}\end{aligned}$$

Figure 4.1 Determinacy and Indeterminacy Regions: Standard Taylor Rule



A Forward-Looking Interest Rate Rule

$$i_t = r_t^n + \phi_\pi E_t\{\pi_{t+1}\} + \phi_y E_t\{\widetilde{y}_{t+1}\}$$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_F \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}$$

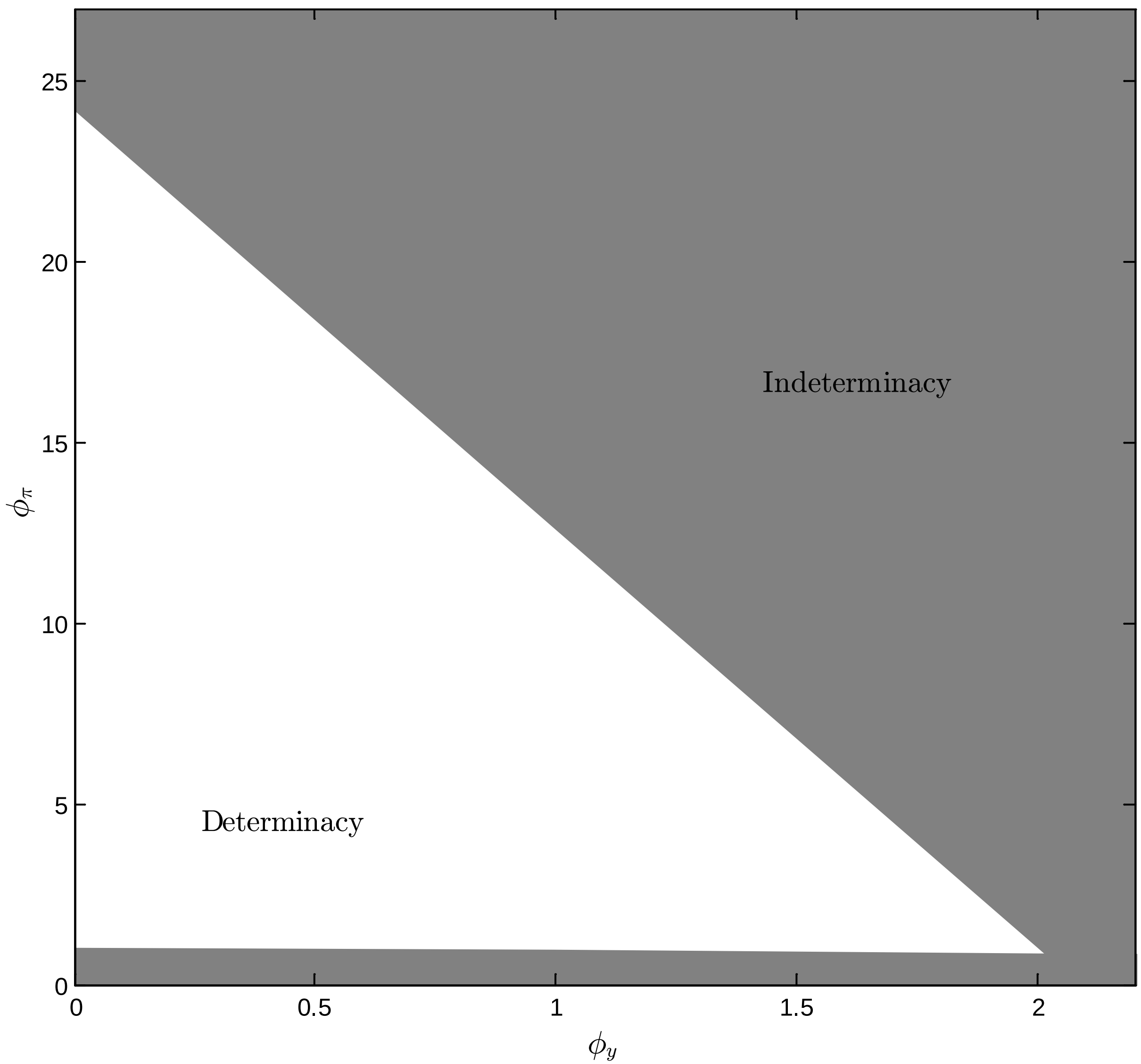
where

$$\mathbf{A}_F \equiv \begin{bmatrix} 1 - \sigma^{-1}\phi_y & -\sigma^{-1}(\phi_\pi - 1) \\ \kappa(1 - \sigma^{-1}\phi_y) & \beta - \kappa\sigma^{-1}(\phi_\pi - 1) \end{bmatrix}$$

Existence and uniqueness conditions (Bullard and Mitra (2002):

$$\kappa(\phi_\pi - 1) + (1 - \beta)\phi_y > 0$$
$$\kappa(\phi_\pi - 1) + (1 + \beta)\phi_y < 2\sigma(1 + \beta)$$

Figure 4.2 Determinacy and Indeterminacy Regions: Forward Looking Taylor Rule



Shortcomings of Optimal Rules

- assumed observability of the natural rate of interest (in real time).
- this requires, in turn, knowledge of:

  (i) the true model
  (ii) true parameter values
  (iii) realized shocks

"Simple rules" :

- the policy instrument depends on observable variables only,
- do not require knowledge of the true parameter values
- ideally, they approximate optimal rule across different models

Simple Monetary Policy Rules

Welfare-based evaluation:

$$\mathbb{W} \equiv -E_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{U_t - U_t^n}{U_c C} \right) = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \widetilde{y}_t^2 + \frac{\epsilon}{\lambda} \pi_t^2 \right]$$

$\Longrightarrow$ expected average welfare loss per period:

$$\mathbb{L} = \frac{1}{2} \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) var(\widetilde{y}_t) + \epsilon \ var(\pi_t) \right]$$

A Taylor Rule

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widehat{y}_t$$

Equivalently:

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \widetilde{y}_t + v_t$$

where $v_t \equiv \phi_y \widehat{y}_t^n$

Equilibrium dynamics:

$$\begin{bmatrix} \widetilde{y}_t \\ \pi_t \end{bmatrix} = \mathbf{A}_T \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \mathbf{B}_T(\widehat{r}_t^n - v_t)$$

where

$$\mathbf{A}_T \ \equiv \Omega \ \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma+\phi_y) \end{bmatrix} \qquad ; \qquad \mathbf{B}_T \equiv \Omega \begin{bmatrix} 1 \\ \kappa \end{bmatrix}$$

and $\Omega \equiv \frac{1}{\sigma+\phi_y+\kappa\phi_\pi}$. Note that $\widehat{r}_t^n - v_t = -\psi_{ya}(\sigma(1-\rho_a)+\phi_y)a_t + (1-\rho_z)z_t$

Exercise: $\Delta a_t \sim AR(1)$ + modified Taylor rule $i_t = \rho + \phi_\pi \pi_t + \phi_y \Delta y_t$

Table 4.1
Evaluation of Simple Rules: Taylor Rule

	Technology				*Demand*			
ϕ_π	1.5	1.5	5	1.5	1.5	1.5	5	1.5
ϕ_y	0.125	0	0	1	0.125	0	0	1
$\sigma(y)$	1.85	2.07	2.25	1.06	0.59	0.68	0.28	0.31
$\sigma(\widetilde{y})$	0.44	0.21	0.03	1.23	0.59	0.68	0.28	0.31
$\sigma(\pi)$	0.69	0.34	0.05	1.94	0.20	0.23	0.09	0.10
$\mathbb{L}$	1.02	0.25	0.006	7.98	0.10	0.13	0.02	0.02

Money Growth Peg

$$\Delta m_t = 0$$

Money demand:

$$l_t = y_t - \eta i_t - \zeta_t$$

where $l_t \equiv m_t - p_t$.

$$\Delta\zeta_t = \rho_\zeta \Delta\zeta_{t-1} + \varepsilon_t^\zeta$$

where $\rho_\zeta \in [0, 1)$.

$$\widehat{l}_t = \widetilde{y}_t + \widehat{y}_t^n - \eta \widehat{i}_t - \zeta_t$$

Letting $l_t^+ \equiv l_t + \zeta_t$

$$\widehat{i}_t = \frac{1}{\eta}(\widetilde{y}_t + \widehat{y}_t^n - \widehat{l}_t^+)$$

Imposig the assumed rule $\Delta m_t = 0$, and clearing of the money market:

$$\widehat{l}_{t-1}^+ = \widehat{l}_t^+ + \pi_t - \Delta\zeta_t$$

Equilibrium dynamics:

$$\mathbf{A}_{\mathbf{M},\mathbf{0}} \begin{bmatrix} \widetilde{y}_t \\ \pi_t \\ \widehat{l}^{+}_{t-1} \end{bmatrix} = \mathbf{A}_{\mathbf{M},\mathbf{1}} \begin{bmatrix} E_t\{\widetilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \\ \widehat{l}^{+}_{t} \end{bmatrix} + \mathbf{B}_{\mathbf{M}} \begin{bmatrix} \widehat{r}^{n}_t \\ \widehat{y}^{n}_t \\ \Delta\zeta_t \end{bmatrix}$$

where

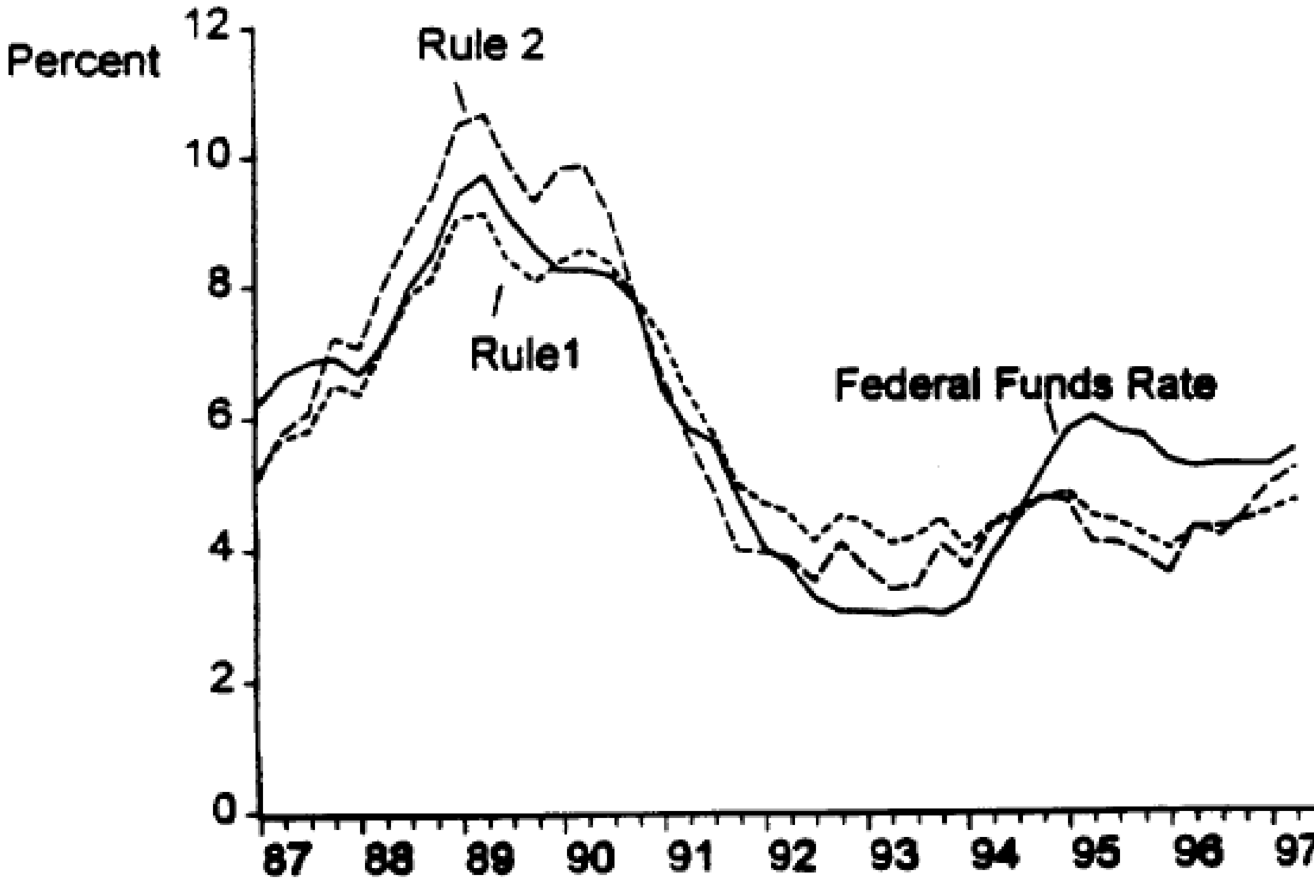
$$\mathbf{A}_{\mathbf{M},\mathbf{0}} \equiv \begin{bmatrix} 1+\sigma\eta & 0 & 0 \\ -\kappa & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} \quad ; \quad \mathbf{A}_{\mathbf{M},\mathbf{1}} \equiv \begin{bmatrix} \sigma\eta & \eta & 1 \\ 0 & \beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad ; \quad \mathbf{B}_{\mathbf{M}} \equiv \begin{bmatrix} \eta & -1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Table 4.2
Evaluation of Simple Rules:
Constant Money Growth

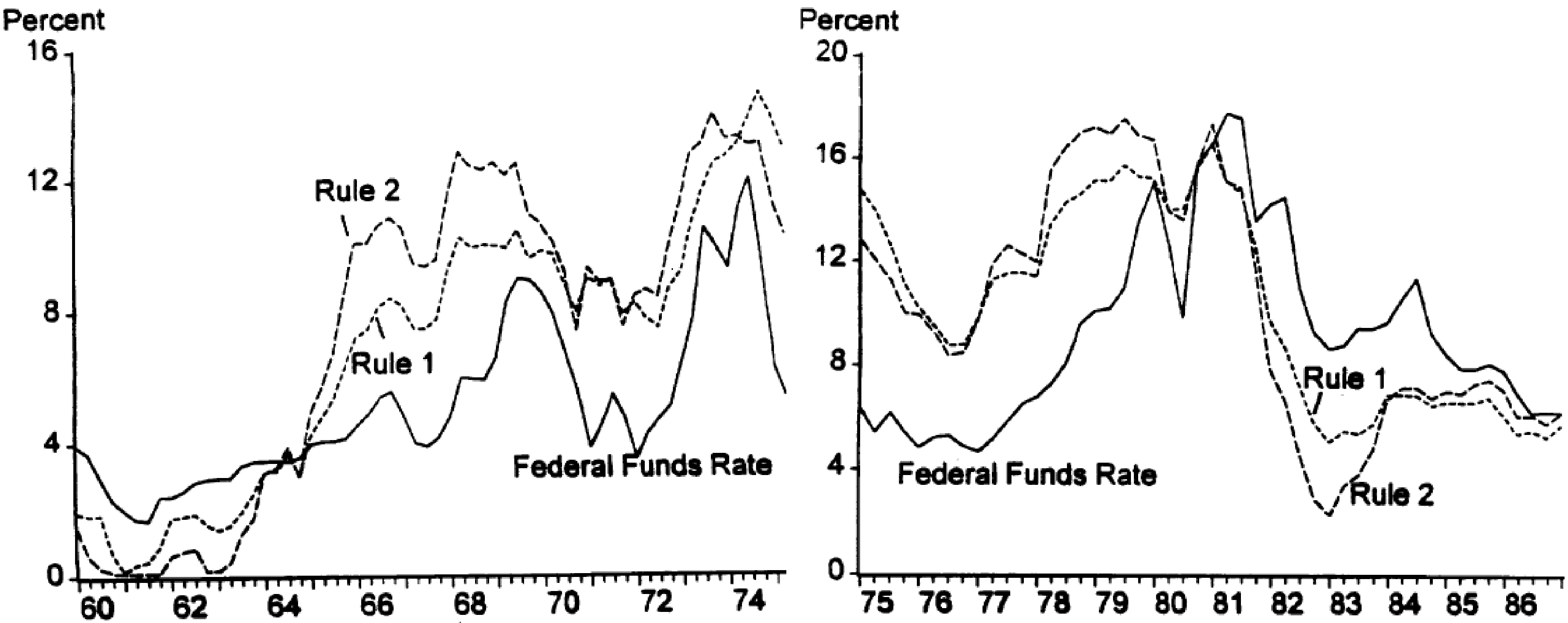
	Technology	*Demand*	*Money Demand*
$\sigma(y)$	1.72	0.59	1.07
$\sigma(\widetilde{y})$	0.92	0.59	1.07
$\sigma(\pi)$	0.35	0.12	0.55
$\mathbb{L}$	0.29	0.04	0.69

The Taylor Rule (Taylor 1993)

$$i_t = 4 + 1.5(\pi_t - 2) + 0.5\, y_t$$



Source: Taylor 1999



Source: Taylor 1999

Clarida, Galí and Gertler (QJE 2000)

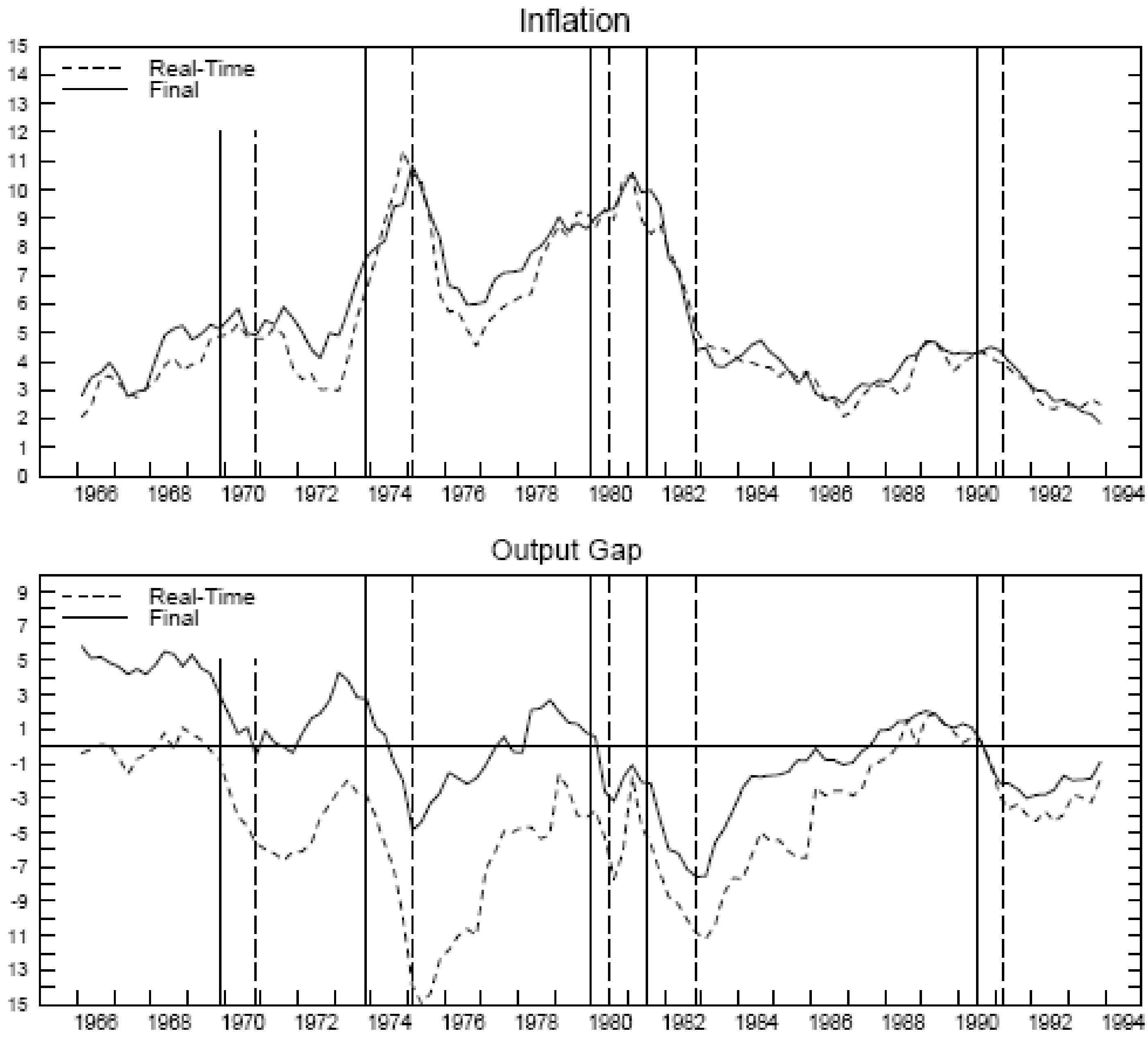
$$i_t = \rho i_{t-1} + (1-\rho)[r + \pi^* + \beta E_t\{\pi_{t+1} - \pi^*\} + \gamma E_t\{y_{t+1} - y^*_{t+1}\}]$$

TABLE II
BASELINE ESTIMATES

	π^*	β	γ	ρ	p
Pre-Volcker	4.24	0.83	0.27	0.68	0.834
	(1.09)	(0.07)	(0.08)	(0.05)	
Volcker-Greenspan	3.58	2.15	0.93	0.79	0.316
	(0.50)	(0.40)	(0.42)	(0.04)	

Standard errors are reported in parentheses. The set of instruments includes four lags of inflation: output gap, the federal funds rate, the short-long spread, and commodity price inflation.

Orphanides (JME 2003)



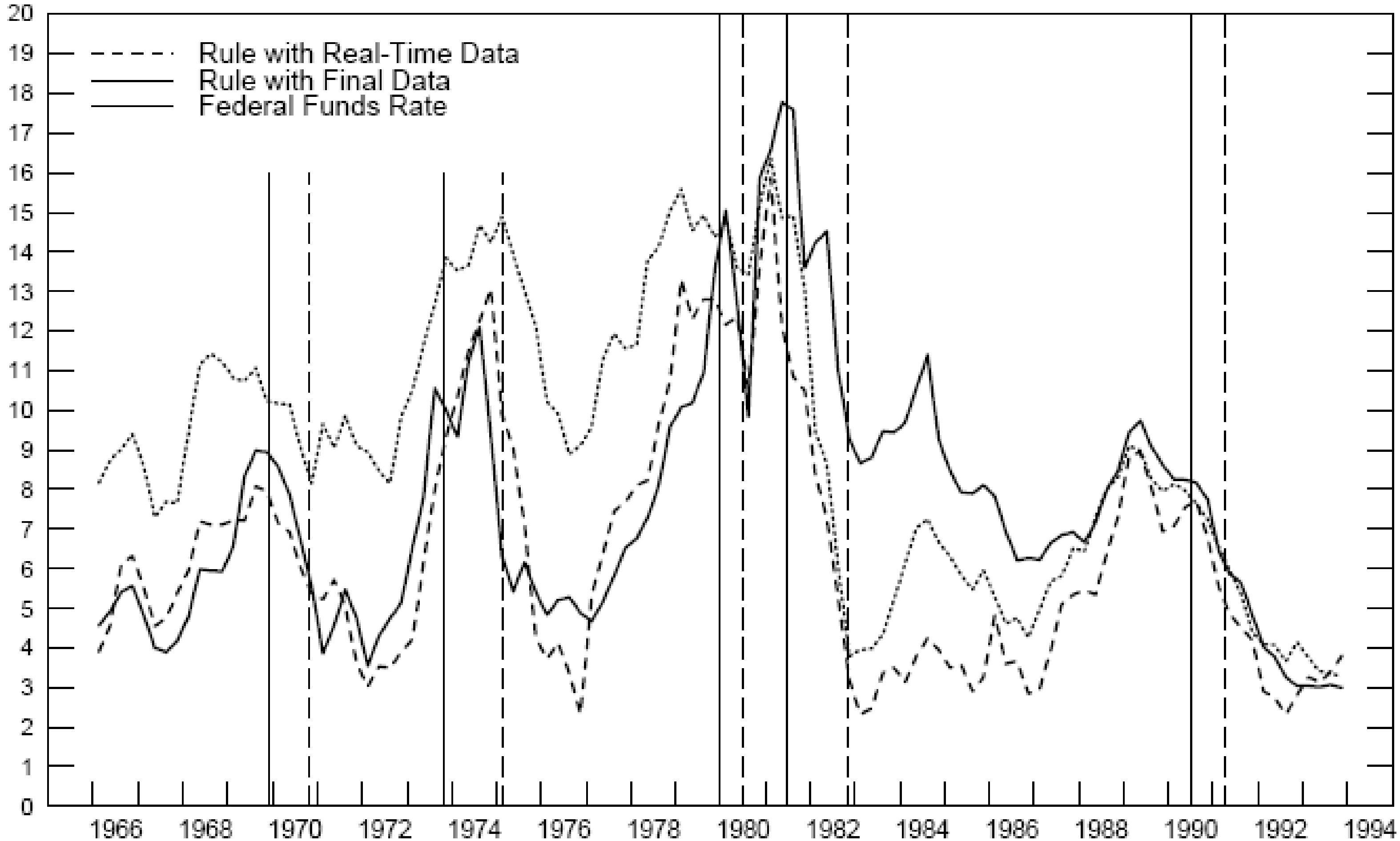


Fig. 5. Then and now: Taylor rule with final and real-time data.