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Expectations-driven spatial fluctuations

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Abstract

I develop a dynamic spatial model with monopolistic competition, increasing returns and labor mobility. Even when shocks to preferences or technology are absent, rational expectations equilibria characterized by stationary random fluctuations in the spatial allocation of resources are shown to exist. Such fluctuations result from the interaction between forward-looking location decisions and the agglomeration/congestion economies implied by the assumptions on market structure and preferences. Welfare losses result from the unnecessary randomness of equilibrium allocations along such equilibria.

Key words: Regional fluctuations; Sunspot equilibria; Location decisions

JEL classification: C32; C33; E32

1. Introduction

Characterizations of business cycles, both in econometric studies and in informal accounts, have traditionally relied on the observation and analysis of aggregate data at the country level. Such a practice has tended to conceal the important differences in the cyclical performance of the economy in different regions and cities within a given country. Despite that traditional neglect, and partly as a consequence of the diversity of regional experiences during the recent recession in the United States,[1] many econ-

[1] See, for example, the discussion in the Economic Report of the President (1992, p. 64).

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omists seem to be turning their attention towards regional fluctuations.[2]

Explanations for regional fluctuations found in the literature or in the popular press typically rely on some types of shocks to fundamentals that hit different regions in an asymmetric fashion. The interaction of sectoral shocks, and differences in the sectoral composition of employment across regions, is thus seen as a natural source of differences in regional economic performances (e.g. Dunn, 1980; Garcia-Milà and McGuire, 1993). Another explanation often given involves the existence of imperfectly correlated shocks to local fiscal policies (e.g. Bartik, 1991).

In a previous paper (Galí, 1994b) I explored an alternative explanation for regional fluctuations, one based on the possibility of variations in the spatial allocation of resources resulting from sunspot-driven revisions in expectations. That possibility was analyzed in the context of a two-period model with 'local' technological spillovers and convex moving costs, and shown to be the result of the interaction between forward-looking location decisions and the macroeconomic complementarities arising from the presence of local spillovers. In the present paper I extend that analysis in two respects. First, I use an infinite horizon framework, which allows me to focus on the possibility of persistent, stationary spatial fluctuations. Second, and following Krugman (1979), Stahl (1983), Rivera-Batiz (1983, 1988), Fujita (1988), and Matsuyama (1992) (among others) I introduce monopolistic competition, increasing returns and consumer's 'taste for diversity' as a source of agglomeration economies: the latter arise as a result of the mutual reinforcement between the number of goods available in a given location (equivalently, the number of active firms) and the size of the market (determined by the number of consumers/workers in the same location). The presence of congestion effects or agglomeration diseconomies (e.g. higher land rents, housing prices, and commuting costs) tends to offset the previous effect and, if sufficiently strong, helps rule out equilibrium allocations characterized by full concentration in one location.[3]

Unfortunately, most of the models found in the aforementioned literature have a static nature, and/or the dynamic stories that are often told to characterize spatial allocation changes over time are based on an ad-hoc

[2] See, for instance, work by Blanchard and Katz (1992), Quah (1993), Garcia-Milà and McGuire (1993), and Bartik (1991). Blanchard and Katz summarize the importance of regional fluctuations in the United States with a simple statistic: for the average state in the United States, as much as 34% of year-to-year movements in state employment over the postwar period are orthogonal to movements in U.S. aggregate employment.

[3] See, for example, Abdel-Rahman (1988).

process that does not take expectations explicitly into account.[4] The present paper, in contrast, embeds many of the elements found in that literature in an explicitly dynamic framework with rational expectations.

More specifically, I use a two-region version of an overlapping-generations (OLG) model, a key feature of which is the irreversible location decision that each agent faces at the beginning of his life, and which determines the labor and goods markets he will have access to during his lifetime. In each period and location a number of monopolistically competitive firms operate, each of which produces a differentiated non-tradable good using an increasing returns technology that requires a single input (labor services).[5] Our assumptions on technology, preferences and market structure imply that the number of goods available at each location – and, as a result, the level of utility derived by the local residents – is a function of the size of the labor force in that location. Consequently, and given the demographic structure of the model, the location decision made by a given agent will ultimately be based on the spatial allocation resulting from similar location decisions faced by the previous cohort and other agents in his cohort, as well as his expectations on the next cohort's spatial distribution. As I show below, the latter feature plays a key role in generating the possibility of stationary sunspot equilibria, i.e. stochastic fluctuations in the spatial allocation of employment and output, driven by self-fulfilling revisions in expectations. In fact, whether the possibility of such fluctuations arises or not depends on the extent to which the future "matters" in consumers' location decisions, a circumstance which depends, among other factors, on the endowment pattern over the life cycle and on the discount factor.

In my model, expectations-driven spatial fluctuations arise in an environment in which sectoral and/or fiscal shocks are absent. Needless to say, the paper's focus on such fluctuations should not be interpreted as a denial of the existence and/or importance of sectoral, policy or other fundamental shocks as sources of regional or urban fluctuations, but as a way of stressing the possibility of an independent, "non-fundamental" component in them.

[4] Notable exceptions include Krugman (1991) and Matsuyama (1991). For a yet different class of spatial models, involving explicit but ad-hoc dynamics (i.e. dynamics not as derived as the equilibrium of an explicit model) see the work of Dendrinos (1985) and some of the references therein.

[5] The introduction of imperfect competition as a source of endogenous fluctuations has some tradition in the business cycle literature. Example include Chaterjee et al. (1990), Woodford (1991), and Galí (1994a), among others.

2. The model

2.1. Consumers

I assume a demographic structure characterized by overlapping cohorts of non-altruistic, two-period-lived consumers. I normalize the size of each cohort to be equal to one, which implies a constant population size of two.

Each consumer is endowed with $(1+\delta)$ units of labor services when young and $(1-\delta)$ units when old, where $|\delta| \leq 1$. Before selling his first-period endowment, each consumer must choose a location among a number of possible alternatives. That location decision is irreversible, and constrains the consumer to sell his labor services to, and to purchase goods from, local firms. In other words, I make the (admittedly extreme) assumptions of no tradability of goods across locations, and of infinite moving costs. For simplicity I assume the existence of only two possible locations, a and b.

Having chosen location $i \in \{a, b\}$, a consumer belonging to the cohort born in period t – henceforth, cohort t – seeks to maximize

$$V_t^i \equiv \max U(c_{1t}^i) - Z(N_t^i) + \beta \mathrm{E}_t\{U(c_{2t+1}^i) - Z(N_{t+1}^i)\}\ , \tag{1}$$

subject to

$$c_{ht}^j = \sum_{j=1}^{M_t^i} (c_{ht}^{ij})^{(\sigma-1)/\sigma}\ , \quad \sigma > 1\ , \quad h = 1, 2\ , \tag{2}$$

$$\sum_{j=1}^{M_t^i} p_t^{ij} c_{1t}^{ij} = (1+\delta) - s_t^i \equiv a_{1t}^i\ , \tag{3}$$

$$\sum_{j=1}^{M_{t+1}^i} p_{t+1}^{ij} c_{2t+1}^{ij} = (1-\delta) + s_t^i R_t^i \equiv a_{2t+1}^i\ . \tag{4}$$

where U is a continuously differentiable utility function satisfying $U' > 0$, and $U'' \leq 0$. β is the discount factor. E_t is the usual expectations operator, conditional on an information set which includes all variables, both individual and aggregate, with a time subscript $t' \leq t$. c_{ht}^{ij} denotes the quantity of good j produced in location i and consumed in period t by an individual of age h ($h = 1$ if 'young', $h = 2$ if 'old'). According to (2), c_{ht}^i is a CES function of the quantities c_{ht}^{ij}, $j = 1, 2, \ldots, M_t^i$, where M_t^i denotes the number of different good types available at location i in period t. Parameter σ measures the elasticity of substitution across goods, and is assumed to be strictly greater than one.[6] p_t^{ij} is the price in period t of a unit of good j produced (and consumed) in location i. Period t consumption expenditures by a consumer of age h in location i are denoted by a_{ht}^i. s_t^i is the level of savings chosen by (young) consumers in location i, in period t. R_t^i is the corresponding return on those savings. Given the assumed endowment pattern, the stream of labor income accruing to a consumer is thus given by $(1+\delta, 1-\delta)$, as is reflected in the budget constraints (3) and (4). All prices and returns are expressed in terms of (local) labor service units (i.e. the wage is normalized to one in all periods and locations).

Finally, $Z(\cdot)$ measures the disutility resulting from the "congestion effects" experienced from having to share a limited amount of space with other agents. Thus, Z is meant to capture in an admittedly ad-hoc fashion the effects of population size and density on traffic congestion, the cost of housing rentals or purchases, lack of open spaces, crime, etc. I assume that those costs, as measured by Z, are increasing and convex in the level of *local* employment. The latter is denoted by N_t^i, and can be expressed as $N_t^i \equiv (1+\delta)n_t^i + (1-\delta)n_{t-1}^i$. Hence, I assume $Z'(N_t^i) > 0$, and $Z''(N_t^i) \geq 0$, for $0 \leq N_t^i \leq 2$.[7]

Throughout it is assumed that each individual perceives its location decision to have a negligible effect on the current value and the probability distribution of future values of M^i and N^i, and thus takes those variables (or, more precisely, their distribution) as given.

The solution to the problem faced by a consumer born in period t can be solved in three stages. In a first stage he solves for the optimal bundle of goods, conditional on being in location i and having chosen a pattern of expenditures (a_{1t}^i, a_{2t+1}^i). The solution to that first-stage problem is given by (see, for example, Dixit and Stiglitz, 1977):

$$c_{1t}^{ij} = [p_t^{ij}/P_t^i]^{-\sigma}[a_{1t}^i/(P_t^i M_t^i)] , \tag{5}$$

$$c_{2t+1}^{ij} = [p_{t+1}^{ij}/P_{t+1}^i]^{-\sigma}[a_{2t+1}^i/(P_{t+1}^i M_t^i)] , \tag{6}$$

where P_t^i is location i's price index in period t, defined as

$$P_t^i \equiv \left[(I/M_t^i) \sum_{j=1}^{M_t^i} (p_t^{ij})^{1-\sigma}\right]^{1/(1-\sigma)} . \tag{7}$$

The level of composite consumption is then given by

[6] The restriction $\sigma > 1$ is needed to guarantee the existence of a solution to the firm's problem.

[7] Though it may seem more natural to assume that Z depends on the size of the local population instead of local employment, the current specification simplifies the algebra substantially without affecting the basic results or intuition.

$$c^i_{1t} = (M^i_t)^{1/\sigma}[a^i_{1t}/P^i_t]^{(\sigma-1)/\sigma}\,, \tag{8}$$

$$c^i_{2t+1} = (M^i_{t+1})^{1/\sigma}[a^i_{2t+1}/P^i_{t+1}]^{(\sigma-1)/\sigma}\,, \tag{9}$$

implying that the level of composite consumption in any given period is increasing and concave in the number of goods available and in the level of real expenditures. Notice that the "taste for diversity" effect is stronger the smaller is σ, i.e. the more limited is the substitutability across goods. As σ approaches infinity (i.e. perfect substitutability) the consumption measure converges to the level of real expenditures.

Next I solve for the optimal expenditure pattern at each location, i.e. the sequence (a^i_{1t}, a^i_{2t+1}) that maximizes (1) subject to (8), (9), and

$$a^i_{2t+1} = (1-\delta) + ((1+\delta) - a^i_1)R^i_t\,. \tag{10}$$

A solution to that problem must satisfy, in addition to the above constraints, the first-order condition

$$U'(c^i_{1t}) = \beta \mathrm{E}_t U'(c^i_{2t+1})\rho^i_t\,, \tag{11}$$

where $\rho^i_t \equiv R^i_t(P^i_t/P^i_{t+1})^{(\sigma-1)/\sigma}(M^i_{t+1}/M^i_t)^{1/\sigma}$ can be interpreted as the interest rate in terms of composite consumption units. Finally, and given a level of expected utility V^i_t associated with the solution of the above problem for $i = a,b$, a consumer born at time t will choose the location that yields the highest expected utility. Formally,

$$V^*_t \equiv \max_{i\in\{a,b\}} V^i_t\,. \tag{12}$$

2.2. Firms

A firm located in region i and producing good j faces a cost function given by

$$l(y^{ij}_t) = \phi + \nu y^{ij}_t\,, \tag{13}$$

where $l(z)$ is the quantity of labor input required to produce z units of output, and where ϕ and ν can be respectively interpreted as fixed and marginal costs.

Given the demand schedule for good j, profit maximization requires that each firm set a constant markup $\mu \equiv \sigma/(\sigma-1)$ over marginal cost. Thus, all firms will set the same price (in terms of the local numéraire) $p^{ij}_t = \mu\nu$, regardless of the good they produce or their location. Given (7), that price will in turn be equal to the aggregate price level. Formally,

$$P^i_t = p^{ij}_t = \mu\nu \equiv P_t\,, \tag{14}$$

for all i, j, and t.

Accordingly, the profit of a typical firm at time t is given by

$$\pi_t^{ij} = (\nu/(\sigma - 1))y_t^{ij} - \phi . \tag{15}$$

Under the assumptions of free entry and zero profits, the levels of output and employment *per firm* are common to all active firms (regardless of their location) and constant over time,[8] being given by

$$y_t^{ij} = (\sigma - 1)\phi/\nu \equiv y , \tag{16}$$

$$l(y_t^{ij}) = \sigma\phi \equiv l . \tag{17}$$

Next I turn to a characterization of equilibrium in this economy.

3. Equilibrium

3.1. Definition

Equilibrium in the goods market requires

$$y_t^{ij} = n_t^i c_{1t}^{ij} + n_{t-1}^i c_{2t}^{ij} , \tag{18}$$

for $i = a,b$, $j = 1, 2, \ldots, M_t^i$, and $t = 1, 2, \ldots$.

Homogeneity of preferences and endowments across agents belonging to a given cohort, combined with the OLG-type demographic structure and the non-tradability of goods,[9] guarantees that savings equal zero for all agents and all periods, thus implying $a_{1t}^i = 1 + \delta$, and $a_{2t}^i = 1 - \delta$ for $i = a,b$, and $t = 1, 2, \ldots$. Let vectors $N_t = [N_t^a, N_t^b]'$ and $M_t = [M_t^a, M_t^b]'$ represent period t's spatial distribution of employment and firms, respectively. Using (5), (6), (14) and (16), equilibrium condition (18) can be shown to imply

$$M_t = (I/\sigma\phi)N_t , \tag{19}$$

for $t = 1, 2, \ldots$ which is, in turn, equivalent to the condition for equilibrium in the labor market, $N_t = lM_t$, given (17).

Equilibrium condition (19) determines the number of firms operating in

[8] Notice that in the absence of increasing returns to scale ($\phi = 0$) free entry would lead to an infinite number of firms and good types in both locations, with the scale of operation for each firm (and thus the output of each good) being infinitesimal. Thus, the increasing returns assumption is necessary to get a finite number of firms and good types at each location. The assumption of market power reconciles the presence of increasing returns and constant marginal costs with the possibility of non-negative profits.

[9] Notice that our assumption of non-tradability of goods effectively implies the absence of capital mobility across locations.

each location – and, thus, the number of different goods available to consumers in that location – as a function of the distribution of employment.

Similarly, one can use (16) and (19) to determine the vector $Y_t = [Y_t^a, Y_t^b]' = yM_t$, representing the spatial distribution of output:

$$Y_t = (I/\mu\nu)N_t \,. \tag{20}$$

Combining (8), (9), (14), and (19) one obtains a simple expression for the CES consumption index c_{ht}^i as a function of the level of expenditures and the number of goods available at location i, given by

$$c_{1t}^i = \omega_1 (N_t^i)^{1/\sigma} \equiv c_1^*(N_t^i)\,,$$

$$c_{2t}^i = \omega_2 (N_t^i)^{1/\sigma} \equiv c_2^*(N_t^i)\,,$$

where $\omega_1 \equiv ((1+\delta)/\mu\nu)^{1-1/\sigma}(1/\sigma\phi)^{1/\sigma}$, and $\omega_2 \equiv ((1-\delta)/\mu\nu)^{1-1/\sigma}(1/\sigma\phi)^{1/\sigma}$. Thus, in equilibrium, an agent's level of "composite consumption" is positively related to the size of the labor force in his location. The intuition underlying this linkage is straightforward: a larger labor force increases the demand for each local good and, as a result, the profits of local firms; the latter effect leads to the entry of new firms and a greater variety of products, with a consequent increase in composite consumption resulting from the "preference for diversity" effect and the constancy of real expenditures.[10] This phenomenon generates positive agglomeration economies on the consumption side. To the extent that each individual consumer does not account for the impact of his location decision on the range of goods available to other consumers (and, thus, on their utility) those agglomeration economies will have the nature of an externality, so I refer to them as "consumption externalities" in what follows. The strength of those externalities in any given period t and location i can be measured by $\mathrm{d}U(c_h^*)/\mathrm{d}N_t^i = (\omega_h/\sigma)(1/N_t^i)^{1-1/\sigma}U'(c_h^*)$, for $h = 1, 2$.

Letting

$$G(N_t^i) \equiv U(c_1^*(N_t^i)) - Z(N_t^i)\,,$$

$$H(N_{t+1}^i) \equiv U(c_2^*(N_{t+1}^i)) - Z(N_{t+1}^i)\,,$$

denote the first- and second-period utility levels conditional on local employment in each period, I can write the total expected utility achieved by an agent from cohort t choosing location i as

[10] Notice also that the consumption index is, in equilibrium, strictly concave in N_t^i, a property which follows from the concavity of optimal consumption with respect to the number of goods available [see (8) and (9)], combined with the fact that such a number is proportional to N_t^i in equilibrium.

$$V_t^i = G(N_t^i) + \beta \mathrm{E}_t H(N_{t+1}^i) \equiv V(N_t^i, \varphi_t(N_{t+1}^i)) ,$$

where $\varphi_t(N_{t+1}^i)$ denotes the probability distribution of N_{t+1}^i conditional on the information available in period t.

I define a *spatial allocation of cohorts* as a vector sequence $\{n_t\}_{t=0}^{\infty}$, where $n_t = [n_t^a, n_t^b]' > 0$, $n_t^a + n_t^b = 1$, $t = 1, 2, \ldots$, and where n_0 is an exogenously given initial condition. To a given sequence $\{n_t\}_{t=0}^{\infty}$ there corresponds a *spatial allocation of employment*, represented by a vector sequence $\{N_t\}_{t=1}^{\infty}$, where $N_t = (1-\delta)n_{t-1} + (1+\delta)n_t$.

Given an initial cohort distribution n_0, I define an *equilibrium* as a sequence $\{n_t, N_t, M_t, Y_t\}_{t=1}^{\infty}$ satisfying

$$\begin{aligned} & n_t^i \geq 0 , \quad n_t^a + n_t^b = 1 , \\ & N_t = (1-\delta)n_{t-1} + (1+\delta)n_t , \\ & V(N_t^i, \varphi_t(N_{t+1}^i)) - V_t^* \leq 0 \quad (< \text{only if } n_t^i = 0) , \\ & M_t = (1/\sigma\phi)N_t , \\ & Y_t = (1/\mu\nu)N_t , \end{aligned} \tag{21}$$

for $i = a,b$, and $t = 1, 2, \ldots$, and where (26) holds as a strict inequality only if $n_t^i = 0$.

3.2. Steady states

I define a steady state to be an equilibrium sequence satisfying (21) and such that $n_t = [n^a, n^b]' \equiv n$, and $N_t = 2n$, for $t = 1, 2, \ldots$. Let $W(x) \equiv G(x) + \beta H(x)$, $x \in [0, 2]$, measure the steady-state utility level in a location populated by a constant fraction x of the members of successive cohorts. Any interior steady state $N = (N^a, 2 - N^a)$ must satisfy $W(N^a) = W(2 - N^a)$. Furthermore, $N = (0, 2)$ and $N = (2, 0)$ will be non-interior steady states if and only if $W(0) \leq W(2)$.

Given the symmetry of the model there always exists at least one steady state, corresponding to the symmetric allocation $n = [1/2, 1/2]'$, and $N = [1, 1]'$. Whether other steady states exist or not depends on the strength of positive and negative agglomeration economies. Figs. 1–3 illustrate three possibilities, which I briefly discuss next.

In Fig. 1, congestion effects are assumed to be unimportant relative to consumption externalities, as reflected in the monotonicity of the W curve. As a result, $W(x)$ and $W(2-x)$ intersect only once at $x = 1$, so the symmetric steady state is the only interior steady state. In addition to this, there are two non-interior steady states, each of which is characterized by full concentration of the population in one of the locations. This is essentially the case found in Krugman (1979).

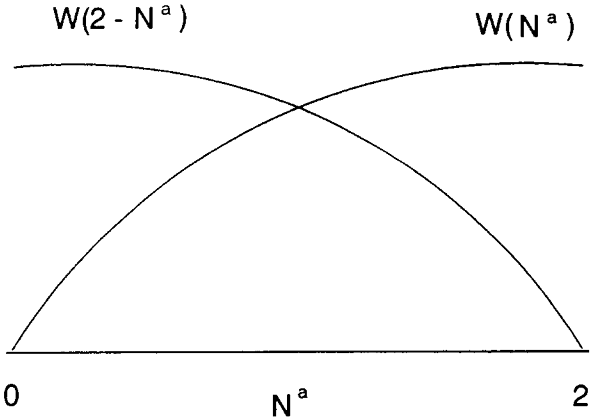

Fig. 1.

Fig. 2 corresponds to a case in which congestion costs initially increase very rapidly, more than offsetting consumption externalities at the symmetric allocation, i.e. $W'(1) < 0$. Yet, their subsequent increases are not large enough, so that $W(2) > W(0)$ holds. Three interior and two non-interior steady states exist in that case.

In Fig. 3 congestion costs are still dominated by positive consumption externalities at $N^a = 1$, but are assumed to increase very fast as higher employment levels. As a result $W(2) < W(0)$, and allocations characterized by a full concentration of the population in one location cannot be steady states. In those allocations, the congestion costs experienced by residents of the location that absorbed all the population would be so high that each of them would have an incentive to move to the uninhibited location (even

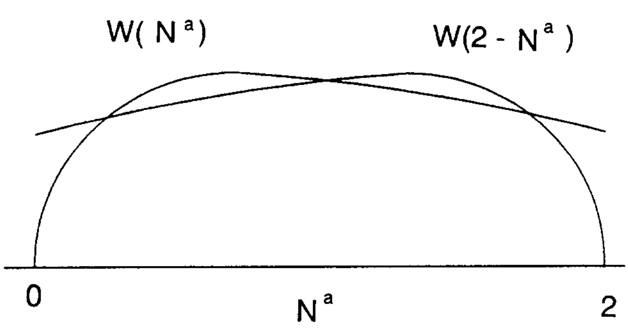

Fig. 2.

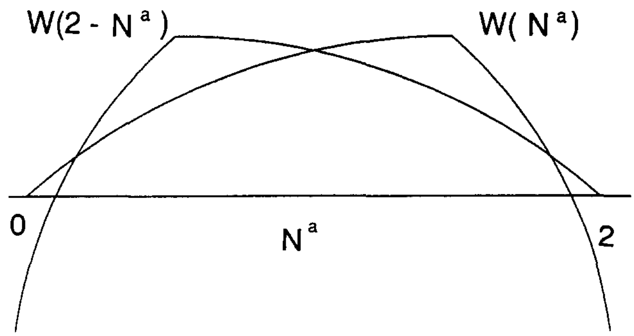

Fig. 3.

though no goods are available in it). Accordingly, there are only three steady states in that case, all of which are interior.

3.3. Equilibria with sunspot fluctuations

In this subsection I want to show the existence, for some parameter configurations, of equilibria characterized by persistent random fluctuations in the spatial allocation of resources. I restrict myself to the equilibrium dynamics in a neighborhood of an interior steady state.

Given the structure of equilibrium conditions in (21), the sequence of equilibrium values $\{N_t, M_t, Y_t\}_{t=1}^{\infty}$ is uniquely determined by the initial allocation n_0 together with the equilibrium sequence $\{n_t\}_{t=1}^{\infty}$, so I can concentrate on the latter for the purpose of characterizing our model's equilibrium dynamics.

On any equilibrium path in a small neighborhood of an interior steady state n, $V(N_t^i, \varphi_t(N_{t+1}^i)) - V_t^* \leq 0$ must hold as an equality. Accordingly,

$$V(N_t^a, \varphi_t(N_{t+1}^a)) = V(N_t^b, \varphi_t(N_{t+1}^b)),$$

must be satisfied for $t = 1, 2, \ldots$. Using the definition of V, and letting $\hat{n}_t \equiv (n_t^a - n^a)$, any interior equilibrium sequence $\{\hat{n}_t\}_{t=1}^{\infty}$ must thus satisfy the difference equation

$$\mathrm{E}_t F(\hat{n}_{t-1}, \hat{n}_t, \hat{n}_{t+1}; N^a) = 0, \quad t = 1, 2, \ldots, \tag{22}$$

where $F(w, x, z; N^a) \equiv G(N^a + D(w, x)) + \beta H(N^a + D(x, z)) - \{G(2 - N^a - D(w, x)) + \beta H(2 - N^a - D(x, z))\}$ and where $N^a = 2n^a$. Notice that $\mathrm{E}_t F$ measures the expected utility differential between the two locations for a given cohort t. Hence, condition (22) implies that along an interior equilibrium each agent facing a location decision should be ex ante indifferent between the two possible locations, given the spatial distribution of his cohort, and that of the two other cohorts with which he overlaps during his lifetime. Notice that from the viewpoint of a member of cohort t, indifference between locations requires that the actual values of $\hat{n}_{t-1}$ and $\hat{n}_t$ and the conditional probability distribution for $\hat{n}_{t+1}$ are such that (22) is satisfied.

Let $\{\varepsilon_t\}_{t=1}^{\infty}$ be a martingale difference sequence representing the innovations in a *sunspot variable* – using the terminology introduced by Cass and Shell (1983) – i.e. a variable unrelated to preferences, technology, and endowments. By definition, $\mathrm{E}_t\varepsilon_{t+1} = 0$, $t = 1, 2, \ldots$, and we can rewrite (22) in the equivalent fashion:

$$F(\hat{n}_{t-1}, \hat{n}_t, \hat{n}_{t+1}; N^a) = \varepsilon_{t+1}\,, \tag{23}$$

for $t = 1, 2, \ldots$. Given $\{\varepsilon_t\}_{t=1}^{\infty}$, any stationary stochastic process $\{\hat{n}_t\}_{t=1}^{\infty}$ satisfying (33) while remaining in an arbitrarily small neighborhood of 0 – the value taken by $\hat{n}$ in the nearby steady state – is a *stationary sunspot equilibrium* (SSE) of our model, using the terminology introduced in Woodford (1986).

The conditions for the existence of SSE involve the eigenvalues of the (2×2) matrix

$$A \equiv \begin{bmatrix} -(F_2/F_3) & -(F_1/F_3) \\ 1 & 0 \end{bmatrix},$$

where F_j denotes the partial derivative of F with respect to its jth argument, evaluated at $(0, 0, 0)$. Notice that A is just the matrix associated with the VAR representation of a linearized version of (23) about $(0, 0, 0)$, given by

$$\begin{bmatrix} \hat{n}_{t+1} \\ \hat{n}_t \end{bmatrix} = A \begin{bmatrix} \hat{n}_t \\ \hat{n}_{t-1} \end{bmatrix} + \begin{bmatrix} \varepsilon_{t+1} \\ 0 \end{bmatrix}. \tag{24}$$

Woodford (1986) establishes the necessary and sufficient conditions for the existence of SSE around a steady state in models whose equilibrium dynamics are represented by a general second-order stochastic difference equation including a predetermined variable. Applied to our case, Woodford's Theorem 1 implies that SSE will exist in an (arbitrarily) small neighborhood of the origin if and only if the two eigenvalues of A have modulus less than one.

Using the definition of F it is straightforward to compute the eigenvalues of A. Denoting the latter by λ_1 and λ_2, one can show

$$\lambda_1 = -\frac{1-\delta}{1+\delta}\,,$$

$$\lambda_2 = -\frac{G'(N^a) + G'(2 - N^a)}{\beta\{H'(N^a) + H'(2 - N^a)\}}\,.$$

As argued above, both conditions $|\lambda_1| < 1$ and $|\lambda_2| < 1$ must be satisfied if SSE are to exist around a steady state. Their economic interpretation is relatively simple. $|\lambda_1| < 1$ requires that $\delta > 0$, i.e. a declining labor supply over the consumer's life-cycle. Such a phenomenon has two key effects. First, it tends to increase the marginal utility of consumption in the second period, for any given level of local employment. Second, it enhances the effect of $\hat{n}_{t+1}$ (i.e. the spatial distribution of the next cohort) on the current cohort's utility, while diminishing the importance of the previous cohort's spatial distribution, $\hat{n}_{t-1}$.

In order to interpret the condition involving the second eigenvalue, λ_2, notice that the latter is a ratio of two terms. The numerator measures the sum (across locations) of the utility impact of a marginal change in *local* employment in the first period of an agent's lifetime, evaluated at the steady state. The denominator gives an analogous measure corresponding to the second period, appropriately discounted. Consequently, $|\lambda_2| < 1$ will be satisfied as long as the location decision faced by consumers "gives enough weight to the future". That will be the case if the impact of one-period-ahead changes in the spatial distribution of the population on the consumer's utility is sufficiently large relative to the impact of current changes, and/or the discount factor β is not too low.

Thus, we see that both eigenvalue conditions for the existence of SSE require that expectations about the future are sufficiently important in the consumer's location decision. Such strongly forward-looking behavior becomes crucial for generating the kind of expectations-driven fluctuations considered here.

3.4. Linear time series representation of spatial sunspot fluctuations

Given a parameter configuration consistent with the existence of SSE about a perfect foresight steady state it is straightforward to approximate the dynamic behavior of aggregate employment and output in a given location along such a SSE. We do so by linearizing the equilibrium conditions around the given steady state. Using (24), and letting $\hat{N}_t \equiv (N_t^a - N^a) = (1-\delta)\hat{n}_{t-1} + (1+\delta)\hat{n}_t$ denote the deviation of aggregate employment in location a from its steady-state value, the equilibrium behavior of $\hat{n}$ and $\hat{N}$ can be represented by the autoregressive processes

$$\hat{n}_t = (\lambda_1 + \lambda_2)\hat{n}_{t-1} - (\lambda_1\lambda_2)\hat{n}_{t-2} + \varepsilon_t \,, \tag{25}$$

$$\hat{N}_t = \lambda_2\hat{N}_{t-1} + (1+\delta)\varepsilon_t \,. \tag{26}$$

Finally, letting $\hat{M}_t \equiv \log[M_t^a/M^a]$, and $\hat{Y}_t \equiv \log[Y_t^a/Y^a]$ denote the percent deviations from their steady-state values of location a's output and number of firms, and using (19), (20), and the approximation $\log N_t \equiv \log(1 +$

$\hat{N}_t) \cong \hat{N}_t$ (for small values of the latter), we obtain $\hat{Y}_t = \hat{M}_t = \hat{N}_t$, a result which reflects the proportionality between local employment and (de-trended) output and number of firms implied by our preference and technology assumptions. Accordingly,

$$\hat{Y}_t = \lambda_2 \hat{Y}_{t-1} + (1+\delta)\varepsilon_t\,, \tag{27}$$

$$\hat{M}_t = \lambda_2 \hat{M}_{t-1} + (1+\delta)\varepsilon_t\,. \tag{28}$$

Notice that the sign of the serial correlation in (and output) at each location is given by the sign of λ_2. As long as δ is not too different from zero, G' and H' will be relatively close when evaluated at the same steady state, and λ_2 will be negative. The intuition behind the implied negative serial correlation in local employment goes as follows: any movement away from the steady state will imply that current utility is higher in one of the two regions; but agents must expect that ranking to be reversed in the following period if a fraction of the new cohort members is to be willing to locate today in the region that currently yields relatively low utility. The negative serial correlation implied by (26) has its origin in that "expected reversal".

3.5. An example

I end this section by simulating the employment dynamics along a SSE for a calibrated, linearized version of the model consistent with the existence of such equilibria. I specify the utility and congestion functions to be of the form $U(x) \equiv x^{1-\eta}/(1-\eta)$, and $Z(x) = \gamma x^{\rho}$, respectively, where $\eta, \gamma \geqslant 0$, and $\rho > 1$.

The parameter values underlying the reported simulations are $\eta = 0.2$, $\beta = 0.9$, $\sigma = 1.1$, $\gamma = 0.25$, $\delta = 0.5$, $\phi = 1$, and $\nu = 0.1$. under such parameter values our model economy has three steady states, $[0.328, 0.672]$, $[1, 1]$, and $[0.672, 0.328]$. In what follows I restrict myself to the dynamics about the first of these steady states. In that case, the eigenvalues of the linearized system are both less than one in absolute value ($\lambda_1 = -0.333$, $\lambda_2 = -0.6949$), so our parameter values are consistent with the existence of SSE.

I draw an i.i.d. sequence $\{\varepsilon_t\}$ from a uniform distribution on the interval $(-0.08,\ 0.08)$, and construct the corresponding equilibrium paths for $\{\hat{n}_t\}$ and $\{\hat{N}_t\}$ using (25) and (26). Figs. 4 and 5 plot the resulting sequences. As is clearly seen in these figures, sunspot fluctuations in our calibrated model generate a noticeable pattern of rise and decline in local economic activity.

4. Welfare

Even in the absence of "unnecessary" spatial fluctuations, the possibility of an inefficient equilibrium allocation arises in our model economy as a

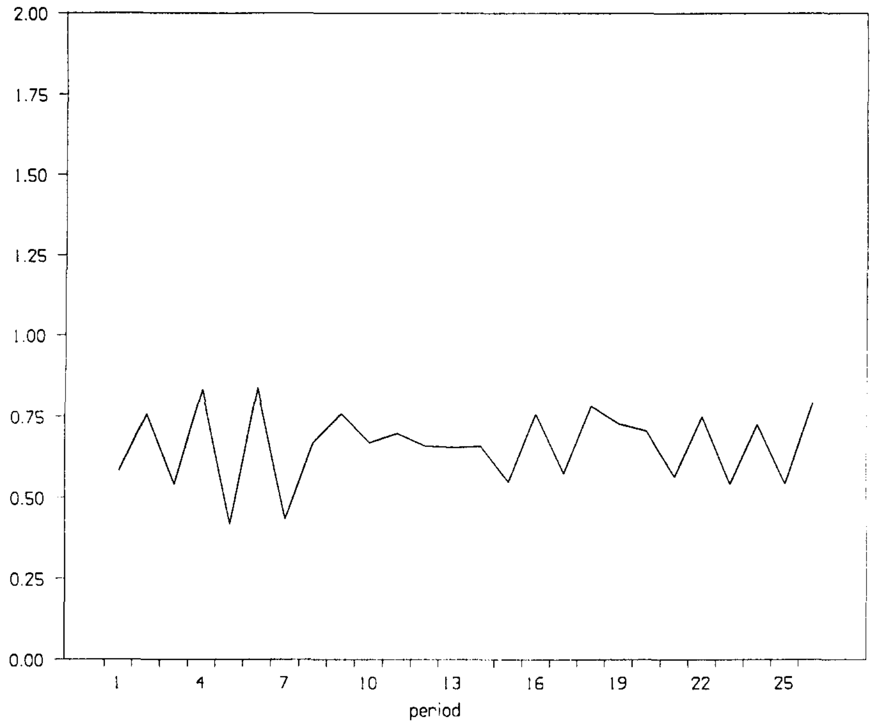

Fig. 4.

result of non-competitive behavior by firms and the presence of an infinite number of consumers, each of which may prevent the first welfare theorem from holding. Our interest here does not lie on those two sources of suboptimality, but on the potential role of sunspot spatial fluctuations as a source of *additional* inefficiency. In particular, I compare the welfare properties of a steady-state equilibrium with those of a sunspot equilibrium around it.

Let V_t denote the expected utility obtained by a member of cohort t (regardless of his location choice) along a sunspot equilibrium. The utility attained by a member of cohort t in an interior steady state (N^a, N^b) is given by $W \equiv W(N^a) = W(N^b)$. I can approximate the difference between V_t and W (the "welfare gap") using the second-order Taylor expansion

$$V_t - W \cong G'\hat{N}_t + \beta H' \mathrm{E}_t \hat{N}_{t+1} + (1/2)[G''\hat{N}_t^2 + \beta H'' \mathrm{E}_t \hat{N}_{t+1}^2],$$

where G', G'', H', and H'' are all evaluated at N^a. Using the fact that $\mathrm{E}_t \hat{N}_{t+1} = \lambda_2 \hat{N}_t$ along a sunspot equilibrium, and taking expectations, it is easy to derive the average welfare loss:

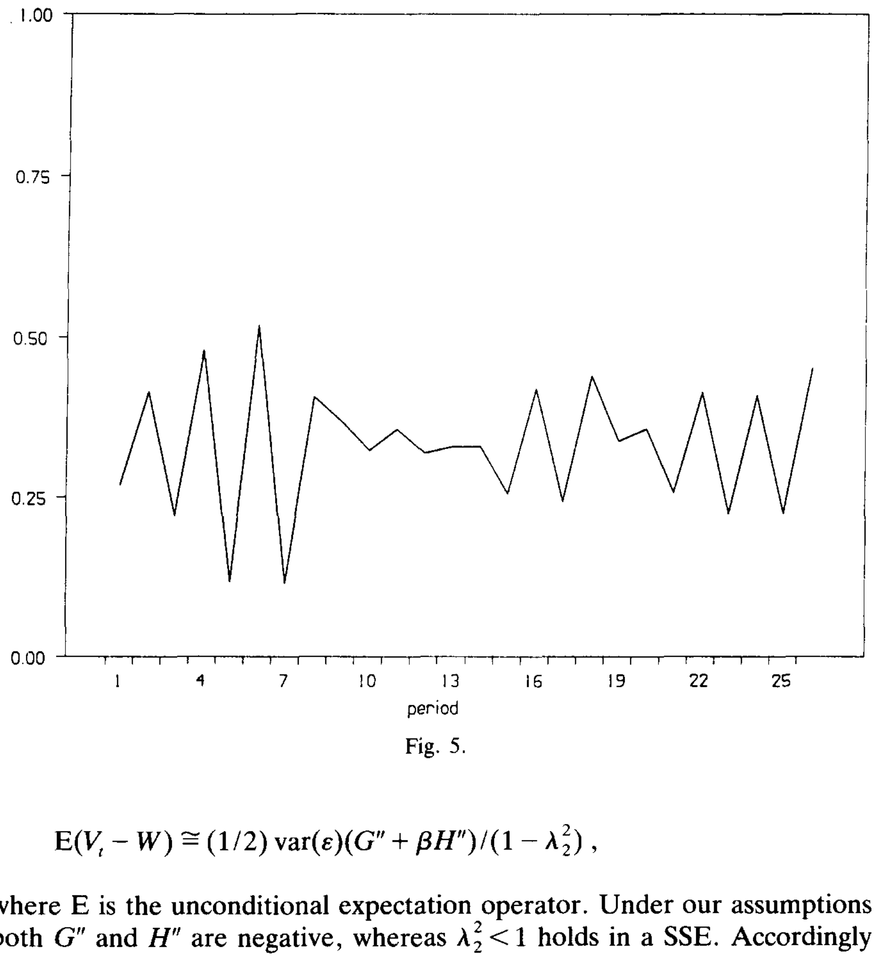

Fig. 5.

$$E(V_t - W) \cong (1/2)\,\mathrm{var}(\varepsilon)(G'' + \beta H'')/(1 - \lambda_2^2)\ ,$$

where E is the unconditional expectation operator. Under our assumptions, both G'' and H'' are negative, whereas $\lambda_2^2 < 1$ holds in a SSE. Accordingly, $E(V_t - W) < 0$, i.e. sunspot fluctuations of the kind considered here have, on average, a negative impact on consumers' expected utility. In other words, equilibria involving such fluctuations are not desirable (relative to the steady-state equilibria).[11] The source of the inefficiency brought about by spatial sunspot fluctuations lies in the "unnecessary" randomness in local employment generated by sunspots, combined with the concavity of the consumer's (reduced form) objective function V_t with respect to employment.

The previous analysis suggests a potential welfare-improving role for

[11] Our use of the unconditional expectations operator implies that our welfare measure must be interpreted in an ex ante (i.e. as of period 0) sense. Ex post, some cohorts will actually benefit from some sunspot realizations. Our result, however, implies that, on average, they will experience a utility loss.

policies that eliminate sunspot fluctuations. Thus, for instance, the government could establish a balanced-budget transfer program between overlapping cohorts that would make the *after tax* life-cycle income stream be given by $[(1+\delta'), (1-\delta')]$, where $\delta' \equiv \delta - \tau$, with τ being a net lump-sum tax on young consumers (or, equivalently, the net lump-sum transfer to old consumers).

Given a parameter configuration such that SSE exist in the absence of intervention, the policy maker could effectively eliminate sunspot fluctuations by setting a tax τ in the interval $(\delta, 1+\delta)$. Such a policy scheme is sufficient to violate at least one of the eigenvalue conditions that are necessary for the existence of SSE.

Unfortunately, there is no guarantee that such an intervention would be welfare-improving since the reallocation of consumption between the first and second period of a consumer's lifetime implied by the transfer program will also have a first-order effect on steady-state utility W. If negative, that effect could more than offset the gains resulting from the disappearance of fluctuations, leaving consumers worse off, on average.[12]

5. Summary and conclusions

In this paper I have explored a potential source of spatial fluctuations, based on the possibility of self-fulfilling revisions in expectations. In our model such fluctuations result from the interaction between forward-looking location decisions and the type of agglomeration/congestion economies found in the urban economics literature. Using an overlapping-generations model with two locations, I have shown that stationary sunspot equilibria around a deterministic steady state will exist whenever preference and endowments are such that expectations about the future are sufficiently important in the consumer's location decision.

I can think of several extensions of the analysis presented in this paper which may shed light upon the role of non-fundamental factors in the spatial allocation of resources. First, our model could be enriched by introducing several elements often found in spatial models but which have been left out here for the sake of analytic simplicity. The possibility of trade in some goods between locations seems a natural candidate. Among other things, departing from the extreme non-tradability assumption would make it possible for members of the same cohort choosing different locations to diversify risk by trading in sunspot-contingent assets. Second, one may want

[12] This is more likely if the size of sunspot fluctuations (which depends on var(ε), among other parameters) was small to begin with, for in that case the welfare losses associated with those fluctuations would be small.

to allow for the possibility of spatial mobility of workers subject to some adjustment costs as in Krugman (1991) and Galí (1994b). A third possible avenue of research that I am currently pursuing involves the development of a methodology (based on the current shift-share technique) to estimate and characterize the "non-fundamental" component of spatial fluctuations using regional or city employment time series.

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