Appendix to

"How Well Does the Real Business Cycle Model Fit Postwar U.S. Data?

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I. The Model

This section presents a simple version of the New Keynesian model with staggered price and wage setting, price and wage indexation, and habit formation in consumption. As in Ireland (2003), there are permanent technology shocks. There are also stationary preference shocks, price and wage markup shocks and a monetary policy shock.

A. Households

Households obtain utility from consuming the final good (C_t^j) and disutility from supplying hours of labor (N_t^j).

The lifetime utility function is:

$$E_0 \sum_{t=0}^{\infty} \beta^t [G_t \log(C_t^j - bC_{t-1}) - \zeta \frac{\left(N_t^j\right)^{1+\varphi}}{1+\varphi}]. \tag{1}$$

E_0 denotes the rational expectations operator using information up to time $t=0$. $\beta \in [0,1]$ is the discount factor. The utility function displays *external* habit formation. $b \in [0,1]$ denotes the importance of the habit stock, which is last period's aggregate consumption. $\eta>0$ is inverse elasticity of labor supply with respect to the real wage. G_t is a preference shifter shock, that follows an AR(1) process in logs:

$$\log(G_t) = \rho_g \log(G_{t-1}) + \varepsilon_t^g.$$

Households maximize their utility subject to the following budget constraint:

$$C_t^j + \frac{B_t^j}{P_t R_t} = \frac{W_t^j N_t^j}{P_t} + \frac{B_{t-1}^j}{P_t} + \int_0^1 \Pi_t^j(i)di, \tag{2}$$

where P_t is the price of the final good, W_t^j is the nominal wage earned by each household, and B_t^j denotes holdings of a riskless bond that costs the inverse of the gross nominal interest rate ($R_t>1$) and pays one unit of currency next period. The last term of the right hand side of the previous expression denotes the profits from the monopolistically competitive intermediate goods producers firms, which are ultimately owned by households.

As is costumary in this class of models, we assume that there exist state-contingent securities that insure households against variations in household-specific labor income. In order to keep notation simple, the structure of the complete asset markets is not explicitly introduced. However, it is a well known result that with complete markets, households' wealth is insured, and the consumption/savings decisions and the labor supply decision can be separated.

The first order condition for consumption/savings is as follows:

$$\frac{G_t}{(C_t^j - bC_{t-1})} = \beta E_t \left[\frac{G_{t+1}}{(C_{t+1}^j - bC_t)} \frac{R_t P_t}{P_{t+1}} \right]. \tag{3}$$

B. Final Good Producers

There is a continuum of final good producers, operating under perfect competition. The technology to produce the aggregate final good is:

$$Y_t = \left[\int_0^1 \left(Y_t^i\right)^{\frac{\varepsilon_t^p - 1}{\varepsilon_t^p}} di \right]^{\frac{\varepsilon_t^p}{\varepsilon_t^p - 1}}, \tag{4}$$

where ε_t>1 is a time-varying elasticity of substitution between types of goods, Y_t is the final good, and Y_t^i are the intermediate goods. The price markup shock is:

$$\log\left(\frac{\varepsilon_t^p}{\varepsilon_t^p - 1} \right) = \log\left(\frac{\bar{\varepsilon}^p}{\bar{\varepsilon}^p - 1} \right) + u_t$$

Profit maximization from the final goods producers delivers the following demand for each type of intermediate good:

$$Y_t^i = \left(\frac{P_t^i}{P_t} \right)^{-\varepsilon_t} Y_t, \text{ for all } i \in [0,1], \tag{5}$$

where P_t, the price of the final good, is obtained from the zero profit condition in the final goods sector:

$$P_t = \left[\int_0^1 \left(P_t^i\right)^{1-\varepsilon_t} di \right]^{\frac{1}{1-\varepsilon_t}},$$

and P_t^i are the prices of all intermediate goods.

C. Intermediate Goods Producers

The production function for intermediate goods producers is:

$$Y_t^i = A_t N_{i,t}, \tag{6}$$

where A_t is an economy wide technology factor, that follows the following process:

$$\log(A_t) = \log(A_{t-1}) + \varepsilon_t^a.$$

In order to obtain one effective unit of labor, firms employ all types of labor from households, which are aggregated the following way:

$$N_{i,t} = \left[\int_0^1 \left(N_{i,t}^j\right)^{\frac{\varepsilon_t^w - 1}{\varepsilon_t^w}} dj\right]^{\frac{\varepsilon_t^w}{\varepsilon_t^w - 1}}. \tag{7}$$

Each firm chooses the optimal labor mix taking all wages as given. Aggregating across firms delivers the following downward sloping demand for each type of labor *j*:

$$N_t^j = \left(\frac{W_t^i}{W_t}\right)^{-\varepsilon_t^w} N_t, \text{ for all } j \in [0,1], \tag{8}$$

where N_t is an index of aggregate labor, and the aggregate wage index is defined as:[1]

$$W_t = \left[\int_0^1 \left(W_t^j\right)^{1-\varepsilon_t^w} dj\right]^{\frac{1}{1-\varepsilon_t^w}}. \tag{9}$$

And the wage markup shock is:

$$\log\left(\frac{\varepsilon_t^w}{\varepsilon_t^w - 1}\right) = \log\left(\frac{\varepsilon^w}{\varepsilon^w - 1}\right) + v_t$$

D. Price and Wage Setting under Staggered Contracts

Prices and wages are set by intermediate goods producers and households in a staggered way. As in the model of Calvo (1983), it is assumed that prices and wages are set at random intervals. Agents can only adjust prices or wages whenever they receive a stochastic signal to do so. The probability of receiving this signal is independent of the past history of signals and across agents. This assumption greatly simplifies the aggregation of price and wage setting decisions.

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[1] The aggregate wage index comes from using $\int_0^1 W_t^j N_{i,t}^j dj = W_t N_t^j$, which can be viewed as a "zero profit condition" in the labor aggregating activity.

We denote by θ_p the probability of the Calvo lottery for price setters. Then, the average duration of price contracts is $1/(1-\theta_p)$. When firms face a Calvo-type restriction, they set prices maximizing the discounted sum of profits taking into account that the price that they fix today might not be reset optimally for some time, and taking as given the demand for their type of good. We assume that there is a degree $\eta_p \in [0,1]$ of indexation to last period's inflation rate (P_{t-1}/P_{t-2}), whenever firms are not allowed to reset prices optimally.

Hence, whenever intermediate-goods producers are allowed to reset their price, they maximize the following profit function, which discounts future profits by the probability of not being able to reset their prices every period:

$$\underset{P_t(i)}{Max}\, E_t \sum_{k=0}^{\infty} \theta_p^k \Lambda_{t,t+k} \left[\frac{P_{t,t+k}(i)}{P_{t+k}} - MC_{j,t+k} \right] Y_{t,t+k}^d(i), \tag{10}$$

where $P_{t,t+k}(i)$ is the price prevaling at $t+k$ assuming that the firm last reoptimized at time t, and whose evolution will deppend on whether the firm indexes its price to last period's inflation rate or not. $Y_{t,t+k}^d(i)$ is the associated demand. Profits are discounted with the stochastic discount factor: $\Lambda_{t,t+k} = \beta^k \left[\frac{C_{t+k} - bC_{t+k-1}}{C_t - bC_{t-1}} \right]^{-1}$.

Households face the same restriction to set their wages. Let θ_w denote the probability of the Calvo lottery for the wage setters. The associated average duration of wage contracts is $1/(1-\theta_w)$. Therefore, households' labor supply schedule comes from choosing their wage to maximize utility facing a downward sloping demand for their type of labor. Parallel to the price setting case, we assume that a fraction $\eta_p \in [0,1]$ of wage setters, whenever they are not allowed to reoptimize, index their wages to last period's inflation rate.

E. Monetary Policy and Market Clearing

As in Taylor (1993), it is assumed that the monetary authority conducts monetary policy with an interest rate rule.

The market clearing condition simply states that all output is consumed.

$$Y_t = C_t. \tag{11}$$

F. Symmetric Equilibrium

These are the equations in the symmetric equilibrium.

The first order conditions for consumption is

$$\frac{G_t}{(C_t - bC_{t-1})} = \beta E_t \left[\frac{G_{t+1}}{(C_{t+1} - bC_t)} \frac{R_t P_t}{P_{t+1}} \right]. \tag{12}$$

The optimal price and the evolution of the price level are:

$$P_t^* = \frac{\varepsilon_t^p}{\varepsilon_t^p - 1} E_t \left[\frac{\sum_{k=0}^{\infty} (\theta_p)^k \Lambda_{t,t+k} MC_{t+k}^* Y_{t,t+k}^*}{\sum_{k=0}^{\infty} (\theta_p)^k \Lambda_{t,t+k} \frac{Y_{t,t+k}^*}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\eta_p}} \right], \tag{13}$$

$$P_t = \left\{ (1-\theta_p)(P_t^*)^{1-\varepsilon_t^p} + \theta_p \left[P_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\eta_p} \right]^{1-\varepsilon_t^p} \right\}^{\frac{1}{1-\varepsilon_t^p}}. \tag{14}$$

The wage setting decision and the aggregate wage level is:

$$E_t \sum_{k=0}^{\infty} (\theta_w \beta)^k \left\{ \left[\frac{1}{C_{t+k} - bC_{t+k-1}} \frac{W_t^*}{P_{t+k}} \left(\frac{P_{t+k-1}}{P_{t-1}} \right)^{\eta_w} - \frac{\varepsilon_t^w}{\varepsilon_t^w - 1} \zeta \left(\overline{N}_{t+k}^j \right)^{\varphi} \right] \left(\overline{N}_{t+k}^j \right)^{\varphi} \right\} = 0, \tag{15}$$

and

$$W_t = \left\{ (1-\theta_w)(W_t^*)^{1-\phi_t} + \theta_w \left[W_{t-1} \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\eta_w} \right]^{1-\varepsilon_t^w} \right\}^{\frac{1}{1-\varepsilon_t^w}}. \tag{16}$$

The production function is

$$Y_t = A_t N_t, \tag{17}$$

The real marginal cost is

$$MC_t = \frac{W_t}{P_t} \frac{1}{A_t}. \tag{18}$$

The resource constraint is:

$$Y_t = C_t.$$

The model is closed with a Taylor rule, that we directly specifiy in log-linear form in the text.