Comment on Dewachter and Lyrio's "Learning, Macroeconomic Dynamics, and the Term Structure of Interest Rates"[1]

by

Jordi Galí
(CREI , MIT, and NBER)

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The present paper by Dewachter and Lyrio (henceforth, DL) is part of a small but growing literature that seeks to understand the yield curve and its evolution over time by combining two different modelling approaches: the arbitrage-free relations familiar from the finance literature and the dynamic general equilibrium approach of modern macroeconomic theory. DL's specific objective (and that of other recent papers cited by the authors) is to reconcile the observed behavior of the term structure with a fully articulated model of inflation, monetary policy and economic activity. As discussed by DL, an important requirement in order to achieve that objective is the introduction of a "level factor", i.e. variations in long-term expectations of short-term rates, that can account for the high volatility of the long-term yield. The main novelty of the present DL paper lies in the endogenous modelling of that level factor, whose variations result from the evolving perceptions by private agents on the endpoint short-term rate (or, more precisely, its two components: the real rate and inflation), brought about by the assumed learning dynamics.

1 A Simple Model of the Term Structure

In order to illustrate the basic point of the DL paper consider the following model of the term structure, generally referred to as the "expectations hypothesis" (EH) model

$$i_t^{(n)} = \frac{1}{n} \sum_{k=0}^{n-1} E_t\{i_{t+k}\} \tag{1}$$

where $i_t^{(n)}$ is the yield on an n-period bond, and i_t denotes the short-term nominal rate on a (nominally) riskless one-period bond held between period

t and $t+1$. Let me assume the following exogenous stationary process for the short rate:

$$i_t - i^* = \phi\ (i_{t-1} - i^*) + \varepsilon_t \tag{2}$$

where i^* is the unconditional mean of the short term rate, and $\phi \in (0,1)$. Then, under rational expectations we have

$$i_t^{(n)} = (1-\Theta_n)\ i^* + \Theta_n\ i_t \tag{3}$$

where $\Theta_n \equiv \frac{1}{n}\frac{1-\phi^n}{1-\phi}$.

2 An Empirical Puzzle

As (3) makes clear, the EH model implies a very tight relation between short-term and long-term rates, one which is clearly violated in U.S. data. In particular, the EH model implies that long-term rates should be much less volatile than they actually are. To see this note that applying OLS to (2) using quarterly data on the three-month Treasury Bill (TB) rate over the sample period 1954:I-2005:IV, yields an estimate $\widehat{\phi} = 0.96$. The latter in turn implies a value $\Theta_{40} \simeq 0.5$, where $n = 40$ corresponding to a 10 year maturity. Hence, the model predicts that the yield on a 10-year bond should have a standard deviation roughly half the size the standard deviation of the TB rate. That prediction is clearly rejected by the data: the ratio of standard deviations is approximately 0.9 rather than 0.5. In other words, the long-term rate appears to be excessively volatile relative to the predictions of the EH model.

3 A Proposed Solution: Endpoint Learning

Let me define, following DL, the *perceived endpoint* for the short-rate as the subjective long-run expectation

$$i_t^{*P} \equiv \lim_{k\to\infty} E_t^P\{i_{t+k}\}$$

where E_t^P is the subjective expectations operator. Agents' *perceived law of motion* for that endpoint is assumed to be given by the random walk model

$$i_t^{*P} = i_{t-1}^{*P} + v_t \tag{4}$$

Deviations from the endpoint are assumed to follow a stationary AR(1) process analogous to the rational expectations model described above:

$$i_t - i_t^{*P} = \phi \ (i_{t-1} - i_{t-1}^{*P}) + \varepsilon_t$$

given that the perceived endpoint is not observed, agents estimate it using the Kalman filter learning algorithm:

$$i_{t|t}^{*P} = i_{t-1|t-1}^{*P} + K \ (i_t - E_{t-1}^P\{i_t\})$$

where $K \in (0, 1)$.

Note that, while agents in this economy believe the endpoint for the short term rate to vary over time, we assume that the short term rate fluctuates around a constant mean value i^* according to the process

$$i_t - i^* = \phi \ (i_{t-1} - i^*) + \varepsilon_t$$

By combining the previous equations one can show that, in equilibrium, agents' estimate of the endpoint follows the stationary AR(1) process

$$i_{t|t}^{*P} - i^* = (1 - K(1 - \phi)) \ (i_{t-1|t-1}^{*P} - i^*) + K \ \varepsilon_t \tag{5}$$

Finally, one can combine the previous equation with the EH model of the term structure (1) to yield the following expression for the n-period bond yield under learning:

$$\begin{aligned} i_t^{(n)} &= \frac{1}{n} \sum_{k=0}^{n-1} E_t^P\{i_{t+k}\} \\ &= (1 - \Theta_n) \ i_{t|t}^{*P} + \Theta_n \ i_t \end{aligned} \tag{6}$$

A comparison of (6) to (3) makes clear that variations in the estimated endpoint $i_{t|t}^{*P}$ in the model under learning provide an additional source of volatility for long-term yields, and one whose relative importance rises with the maturity on the bond (since Θ_n is decreasing in n). Furthermore, since the time series properties of $i_{t|t}^{*P}$ depend on some unobservables (e.g., the variance of v_t in the model above–which measures the extent of the departure from rational expectations), the model with learning gives the researcher some room to improve on the fit of its rational expectations counterpart

4 Dewachter and Lyrio's Contribution

The simple learning model of the previous subsection conveys the essence of DL's proposed framework for understanding the term structure dynamics. Needless to say, DL's model is richer in several dimensions, some of which are likely to be important. First, and most noticeably, DL's model is a general equilibrium one. Thus, and in contrast with the framework above, the short-term rate does not follow an exogenous process but instead is determined according to a Taylor-type rule that has the output gap and inflation as arguments. The output gap and inflation are in turn determined (simultaneously with the short-term rate) by a hybrid new-Keynesian Phillips curve and a dynamic IS equation, which in combination with the interest rate rule, constitute the macro block of DL's model.

Secondly, DL use a pricing kernel consistent with the macro model in order to derive an affine model for the yield curve. This is in contrast with the simple (though pedagogically useful) expectations hypothesis model shown above. As a result, the yields for different maturities are not only a function of the current short-term rate and its perceived endpoint, but of also of inflation, the output gap as well as agents' current estimates of all those variables' endpoints.

The different models estimated by DL (four versions of the rational expectations model and four of the learning model) and their implied fit of the time series for bond yields of different maturities, lead a number of interesting insights, many of which are discussed in detail in DL's paper. Most importantly given the paper's objectives, and as summarized graphically by Figure 6 in that paper, DL's findings point to a potentially large explanatory role of learning dynamics as a source of the low frequency movement in long-term yields. While the estimated versions of the rational expectations (RE) model that allow for chairman-specific interest rate rules and time-varying price of risk (RE II and RE III) do a much better job than the simple bare-bones RE model (RE macro), they fall well short of the learning model once term structure data are used to estimate the latter (as in Learning I through III). Furthermore, much of the improvement in fit is due to a "level factor" generated by variations in the estimated inflation endpoint, which is reflected one for one in variations in the short-term rate endpoint. That feature of DL's learning model is shown to be largely consistent with the observed evolution of survey-based long-term inflation expectations.(which, in turn, display more variation than any model with chairman-specific inflation targets–but no learning about the latter–is bound to entail).

Of particular interest to monetary economists (even to those who may not care so much about the term structure) are the implications for the estimated deep parameters of DL's "macro block" resulting from the need to fit the term structure data, as well as the allowance for learning dynamics. Two findings are worth emphasizing. First, the importance of the backward-looking component of the hybrid new Keynesian Phillips curve goes down substantially when learning dynamics are allowed for. Secondly, the variances of the innovations in the perceived inflation and real rate endpoints tend to be smaller under Greenspan than under previous Fed chairmen, possibly suggesting an enhanced transparency of monetary policy over the past two decades.(since the true endpoints are indeed constant during each chairman's tenure).

5 Open Issues and Caveats

The present paper by DL constitutes an important contribution to the macro-finance literature on term structure dynamics. It is well written, and it contains a careful and extensive empirical analysis. Naturally, the paper leave a number of issues unexplained. It also relies on a number of assumptions that are not fully appealing. Let me turn to those briefly.

5.1 Do we need a full-fledged DSGE model to explain the term structure dynamics?

A simpler alternative to the full-fledged macro model developed and analyzed by DL, would consist of a partial equilibrium model of the term structure (e.g., the affine model used by DL) that takes as given the joint process for the short-term rate i_t and the price kernel m_t. (e.g. a reduced form VAR). That process could be augmented with a perceived law of motion for the short-term endpoint, as well as a learning algorithm similar to the one proposed by DL The use of a full-fledged model may impose unnecessary structure for the purpose at hand.

On the other hand, one can think of a possible justification for the DSGE approach pursued in the DL paper: to explore the macroeconomic implications of versions of a framework whose structure (including the embedded endpoint learning model) and estimated parameters are successful at fitting the term structure data. Among the questions one could ask based on that framework are the following: How does endpoint learning affect the transmission of monetary policy shocks? How does endpoint learning affect the desirability of alternative monetary policy rules? These are interesting and possibly important questions, but ones that fall beyond the scope of the DL paper.

5.2 A strong departure from rational expectations

A persistent gap between the perceived dynamics for some macro variables (or driving forces) and the actual equilibrium dynamics is a natural feature of models with constant-gain learning. DL's framework is no exception in that regard. Yet, in DL's model the gap between the perceived law of motion and the actual law of motion is particularly large. In particular, DL assumptions imply that agents believe the law of motion for the inflation and real rate endpoints corresponds to two independent random walks. By contrast, in all the equilibria considered by DL, the estimated endpoints follow a stationary process, with an unconditional mean that corresponds to the deterministic steady state of the rational expectations equilibrium. A similar gap emerges in the simple model of the term structure analyzed above, as a comparison of (4) and (5) reveals. In my opinion, a perceived law of motion that shares with the actual law of motion the latter's order of integration would seem to be among the desiderata to be fulfilled by non-rational expectations models.

5.3 Two competing models.

One can think of two alternative competing models that are likely to account for the observed behavior of bond yields equally well. The first class of models, exemplified by the present paper, takes the "true" inflation and real rate endpoints to be constant, while letting agents learn about those endpoints using some constant-gain learning algorithm. The second class of models, exemplified by Hordahl et al. (2006), among others, assumes rational expectations, combined with time-varying endpoints for the real rate and/or inflation. The latter could in turn be justified by changes in the central bank's inflation target and/or changes in trend productivity growth. Sorting out the empirical merits of both families of models is likely to be non-trivial, and is task that also falls beyond the scope of DL's present paper. Yet, the use of information on survey-based long-run expectations, as done in the present paper, may provide useful in achieving that objective.