Abstract
The standard New Keynesian model with staggered wage setting is shown to imply a simple dynamic relation between wage inflation and unemployment. Under some assumptions, that relation takes a form similar to that found in empirical wage equations—starting from Phillips’ (1958) original work—and may thus be viewed as providing some theoretical foundations to the latter. The structural wage equation derived here is shown to account reasonably well for the comovement of wage inflation and the unemployment rate in the US economy, even under the strong assumption of a constant natural rate of unemployment. (JEL: E24, E31, E32)

1. Introduction
The past decade has witnessed the emergence of a new popular framework for monetary policy analysis, the so-called New Keynesian (NK) model. The new framework combines some of the ingredients of Real Business Cycle theory (for example dynamic optimization, general equilibrium) with others that have a distinctive Keynesian flavor (for example monopolistic competition, nominal rigidities).

Many important properties of the NK model hinge on the specification of its wage-setting block. While basic versions of that model, intended for classroom exposition, assume fully flexible wages and perfect competition in labor markets, the larger, more realistic versions (including those developed in-house at different central banks and policy institutions) typically assume staggered nominal wage setting which, following the lead of Erceg, Henderson, and Levin (2000), are modeled in a way symmetric to

The editor in charge of this paper was Fabrizio Zilibotti.

Acknowledgments: I have benefited from comments by David López-Salido, Larry Christiano, Per Krusell, an anonymous referee, and participants in seminars at the CREI Macro Lunch, the Reserve Bank of Australia, Reserve Bank of New Zealand, U. Rovira i Virgili, NBER Summer Institute, Kiel EES Workshop, New York Fed, Columbia, NYU, Oxford University, ESSIM 2010, and the IIES. Tomaz Cajner and Lien Laureys provided excellent research assistance. I am grateful to the Fundación BBVA, the European Research Council, the Ministerio de Ciencia e Innovación, and the Government of Catalonia for financial support.

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price setting. The degree of nominal wage rigidities and other features of wage setting play an important role in determining the response of the economy to monetary and other shocks. Furthermore, the coexistence of price and wage rigidities has important implications for the optimal design of monetary policy. Yet, and despite the central role of the wage-setting block in the NK model, the amount of work aimed at assessing its empirical relevance has been surprisingly scant. This is in stark contrast with the recent but already large empirical literature on price inflation dynamics and firms’ pricing patterns, which has been motivated to a large extent by the desire to evaluate the price-setting block of the NK model.

This paper seeks to fill part of that gap, by providing evidence on the NK model’s ability to account for the observed patterns of wage inflation in the US economy. In order to do so, I reformulate the standard version of the NK wage equation in terms of a (suitably defined) unemployment rate. The main advantage of that reformulation is the observability of the associated driving force (the unemployment rate), which contrasts with the inherent unobservability of the wage markup or the output gap, which are the driving forces in standard formulations of the NK wage inflation equation.

The staggered wage setting model à la Calvo (1983) embedded in standard versions of the New Keynesian framework is shown to imply a simple dynamic relation between wage inflation and the unemployment rate, which I refer to as the New Keynesian Wage Phillips Curve. Under certain assumptions, that relation takes the same form as the original equation of Phillips (1958). Furthermore, in the presence of wage indexation to past inflation—an assumption often made in extensions of the basic model—the resulting wage dynamics are consistent with a specification often used in applied work (for example Blanchard and Katz 1999). The analysis developed here can thus be seen as providing some theoretical foundations for those specifications, as well as a structural interpretation to its coefficients.

In the second part of the paper I turn to the empirical evidence, and show how the NK Wage Phillips Curve accounts reasonably well for the behavior of wage inflation in the US economy, even under the strong assumption of a constant natural rate of unemployment. In particular, the model can account for the strong negative correlation between wage inflation and the unemployment rate observed since the mid-1980s. On the other hand, the lack of a significant correlation between the same variables for the postwar period as a whole can be explained as a consequence of the large fluctuations

3. A recent exception is Sbordone (2006).
5. Further applications of the same framework can be found in Galí (2011b) and Galí, Smets, and Wouters (2011).
in price inflation in and around the 1970s, in combination with wage indexation to past CPI inflation.

The remainder of the paper is organized as follows. Section 2 describes the basic model of staggered nominal wage setting. Section 3 introduces the measure of unemployment latent in that model, and reformulates the wage inflation equation in terms of that variable. Section 4 provides an empirical assessment of the model’s implied relation between wage inflation and unemployment using postwar US data. Section 5 concludes.

2. Staggered Wage Setting and Wage Inflation Dynamics

This section introduces a variant of the staggered wage setting model originally developed in Erceg, Henderson, and Levin (2000; henceforth, EHL). That model (and extensions thereof) constitutes the core of the monetary DSGE frameworks that have become part of the toolkit for policy analysis in both academic and policy circles. The variant presented here assumes that labor is indivisible, with all variations in hired labor input taking place at the extensive margin (that is in the form of variations in employment). The assumption of indivisible labor leads to a definition of unemployment consistent with its empirical counterpart.

The model assumes a (large) representative household with a continuum of members represented by the unit square and indexed by a pair \((i, j) \in [0, 1] \times [0, 1]\). The first dimension, indexed by \(i \in [0, 1]\), represents the type of labor service in which a given household member is specialized. The second dimension, indexed by \(j \in [0, 1]\), determines his disutility from work. The latter is given by \(\chi_t j^\varphi\) if he is employed and zero otherwise, where \(\varphi \geq 0\) determines the elasticity of the marginal disutility of work, and \(\chi_t > 0\) is an exogenous preference shifter which, due to its impact on labor supply (discussed below), is referred to as a labor supply shock in what follows. Furthermore, utility is logarithmic in consumption and there is full risk sharing among household members, as in Merz (1995) and much of the subsequent literature.

The household period utility corresponds to the integral of its members’ utilities, and is thus given by

\[
U(C_t, \{N_t(i)\}, \chi_t) \equiv \log C_t - \chi_t \int_0^1 \int_0^{N_t(i)} j^\varphi \, dj \, di
\]

\[
= \log C_t - \chi_t \int_0^1 N_t(i)^{1+\varphi} \frac{di}{1 + \varphi},
\]

where \(C_t\) denotes household consumption, and \(N_t(i) \in [0, 1]\) is the fraction of members specialized in type \(i\) labor who are employed in period \(t\). In what follows I discuss the robustness of the main findings to a generalization of the previous utility function that is consistent with (empirically more plausible) smaller wealth effects on labor supply.
The relevant decision unit is the household. The latter seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \{N_t(i)\}, \chi_t),$$

subject to a sequence of budget constraints

$$P_t C_t + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(i) N_t(i) \, di + \Pi_t,$$  \hspace{1cm} (1)

where $P_t$ is the price of the consumption bundle, $W_t(i)$ is the nominal wage for labor of type $i$, $B_t$ represents purchases of a nominally riskless one-period bond (at a price $Q_t$), and $\Pi_t$ is a lump-sum component of income (which may include, among other items, dividends from ownership of firms). The above sequence of period budget constraints is supplemented with a solvency condition which prevents the household from engaging in Ponzi schemes.

As in EHL, and following the formalism of Calvo (1983), workers supplying a labor service of a given type (or a union representing them) get to reset their (nominal) wage with probability $1 - \theta_w$ each period. That probability is independent of the time elapsed since they last reset their wage, in addition to being independent across labor types. Thus, a fraction of workers $\theta_w$ keep their wage unchanged in any given period, making that parameter a natural index of nominal wage rigidities. Once the wage has been set, the quantity of workers employed is determined unilaterally by firms, with households willingly meeting that demand (to the extent that the wage remains above the disutility of work for the marginal worker), by sending its specialized workers with the lowest work disutility.

When reoptimizing their wage in period $t$, workers choose a wage $W_t^*$ in order to maximize household utility (as opposed to their individual utility), subject to a sequence of isoelastic demand schedules for their labor type, and the usual sequence of household flow budget constraints.\(^6\) The first-order condition associated with that problem can be written as

$$\sum_{k=0}^{\infty} (\beta \theta_w^k) E_t \left\{ \frac{N_{t+k}^\psi}{C_{t+k}} \left( \frac{W_t^*}{P_{t+k}} - \mathcal{M}_t^w \text{MRS}_{t+k} \right) \right\} = 0,$$

where $N_{t+k}^\psi$ denotes the quantity demanded in period $t + k$ of a labor type whose wage is being reset in period $t$, $\text{MRS}_{t+k} = \chi_{t+k} C_{t+k} N_{t+k}^\psi$ is the relevant marginal rate of substitution between consumption and employment in period $t + k$, and $\mathcal{M}_t^w \equiv \epsilon_w / (\epsilon_w - 1)$ is the desired (or flexible wage) markup, with $\epsilon_w$ denoting the (constant) wage elasticity of demand for the services of each labor type.\(^7\)

Log-linearizing the above optimality condition around a perfect foresight zero inflation steady state, and using lower case letters to denote the logs of the

\(^6\) For details of the optimal wage setting condition derivation see EHL (2000).

\(^7\) See Galí, Smets, and Wouters (2011c) for an application of a similar framework allowing for time variations in the desired wage markup.
corresponding variable, we obtain the approximate wage setting rule

\[ w_t^* = \mu^w + (1 - \beta \theta^w) \sum_{k=0}^{\infty} (\beta \theta^w)^k E_t \{ mrs_{t+k|t} + p_{t+k} \}, \tag{2} \]

where \( \mu^w \equiv \log M^w \). Note that in the absence of nominal wage rigidities (\( \theta^w = 0 \)) we have \( w_t^* = w_t = \mu^w + mrs_t + p_t \), implying a constant markup \( \mu^w \) of the wage \( w_t \) over the price-adjusted marginal rate of substitution, \( mrs_t + p_t \). When nominal wage rigidities are present, new wages are set instead as a constant markup \( \mu^w \) over a weighted average of current and expected future price-adjusted marginal rates of substitution.

Letting \( mrs_t \equiv c_t + \varphi n_t + \xi_t \) denote the economy’s average (log) marginal rate of substitution, where \( \xi_t \equiv \log \chi_t \), we can write

\[ mrs_{t+k|t} = mrs_{t+k} + \varphi (n_{t+k|t} - n_{t+k}) = mrs_{t+k} - \epsilon^w \varphi (w_t^* - w_{t+k}). \tag{3} \]

Furthermore, log-linearizing the expression for aggregate wage index around a zero inflation steady state we obtain

\[ w_t = \theta^w w_{t-1} + (1 - \theta^w) w_t^*. \tag{4} \]

As in EHL (2000), we can combine equations (2) through (4) and derive the baseline wage inflation equation

\[ \pi^w_t = \beta E_t \{ \pi^w_{t+1} \} - \lambda^w \left( \mu^w_t - \mu^w \right), \tag{5} \]

where \( \pi^w_t \equiv w_t - w_{t-1} \) is wage inflation, \( \mu^w_t \equiv w_t - p_t - mrs_t \) denotes the (average) wage markup, and

\[ \lambda^w \equiv \frac{(1 - \theta^w)(1 - \beta \theta^w)}{\theta^w(1 + \epsilon^w \varphi)} > 0. \]

In words, wage inflation depends positively on expected one period ahead wage inflation and negatively on the deviation of the average wage markup from its desired value.\(^8\) Equivalently, and solving (5) forward, we have

\[ \pi^w_t = -\lambda^w \sum_{k=0}^{\infty} \beta^k E_t \{ (\mu^w_{t+k} - \mu^w) \}; \tag{6} \]

that is, wage inflation is proportional to the discounted sum of expected deviations of current and future average wage markups from their desired levels. Intuitively, if

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8. Note that the previous equation is the wage analog to the price inflation equation resulting from a model with staggered price setting à la Calvo. See Galí and Gertler (1999) and Sbordone (2002) for a derivation and empirical assessment.
average wage markups are below (above) their desired level, workers that have a chance to reset their wage will tend to adjust it upward (downward), thus generating positive (negative) wage inflation.

Estimated versions of the model above found in the literature generally allow for automatic indexation to price inflation of the wages that are not reoptimized in any given period. Here I assume the following indexation rule:

$$w_{t+k|t} = w_{t+k-1|t} + \gamma \bar{\pi}_t^p + (1 - \gamma)\pi^p + g,$$

for $k = 1, 2, 3, \ldots$, where $w_{t+k|t}$ is the period $t + k$ (log) wage for workers who last re-optimized their wage in period $t$ (with $w_{t|t} \equiv w^*_t$), $\bar{\pi}_t^p$ is the measure of price inflation to which wages are indexed, $\pi^p$ denotes steady state price inflation, and $g$ is the rate of growth of productivity (and real wages) in the steady state. In that case the following wage inflation equation can be derived:

$$\pi_t^w - \gamma \bar{\pi}_t^p = \alpha + \beta E_t \{\pi_{t+1}^w - \gamma \bar{\pi}_t^p\} - \lambda_w (\mu_t^w - \mu^w),$$

where $\alpha \equiv (1 - \beta)((1 - \gamma)\pi^p + g)$.

While in existing applications it is often assumed $\bar{\pi}_t^p \equiv \pi^p \equiv p_t - p_{t-1}$ (for example, Smets and Wouters 2003, 2007), it is important to note that the model allows for inflation measures other than the one-period lagged inflation as an indexing variable. In particular, some of the estimates reported below use the moving average $\bar{\pi}_t^p \equiv (1/4)(\pi_t^p + \pi_{t-1}^p + \pi_{t-2}^p + \pi_{t-3}^p)$ as a smoother alternative indexing variable.

3. Wage Inflation and Unemployment: A New Keynesian Wage Phillips Curve

3.1. The Extended Model

Next I introduce unemployment explicitly into the model. Consider household member $(i, j)$, specialized in type $i$ labor and with disutility of work $\chi_t j^\psi$. Using household welfare as a criterion, and taking as given current labor market conditions (as summarized by the prevailing wage for his labor type), he will find it optimal to participate in the labor market in period $t$ if and only if

$$\frac{W_t(i)}{P_t} \geq \chi_t C_t j^\psi;$$

that is, whenever the real wage prevailing in his trade is above his disutility from working, expressed in terms of consumption using the household’s marginal valuation of the latter.

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Thus, the marginal supplier of type $i$ labor (employed or unemployed), which I denote by $L_t(i)$, is implicitly given by

$$\frac{W_t(i)}{P_t} = \chi_t C_t L_t(i)\varphi.$$

Taking logs and integrating over $i$ we obtain

$$w_t - p_t = c_t + \varphi l_t + \xi_t,$$

where $l_t \equiv \int_0^1 l_t(i) \, di$ can be interpreted as the model’s implied aggregate participation or labor force, and $w_t \equiv \int_0^1 w_t(i) \, di$ is the average wage, both expressed in logs.

I define the unemployment rate $u_t$ as

$$u_t \equiv l_t - n_t,$$

which, for rates of unemployment of the magnitude observed in the postwar US economy, is a close (and algebraically convenient) approximation to the more conventional measure $(L_t - N_t)/L_t$. Note that the latter corresponds to the official definition of the unemployment rate in the United States under the assumption that active search (that is, signaling oneself as available for work) is needed in order to be hired, with that search activity generating a negligible utility cost.

Combining (9) and (10) with the expression for the average wage markup $\mu_t^w \equiv (w_t - p_t) - (c_t + \varphi n_t + \xi_t)$ used above yields the following simple linear relation between the wage markup and the unemployment rate:

$$\mu_t^w = \varphi u_t.$$

Let us define the natural rate of unemployment, $\un_t^w$, as the rate of unemployment that would prevail in the absence of nominal wage rigidities. It follows from the assumption of a constant desired wage markup that $\un_t^w$ is constant and given by

$$\un_t^w = \frac{\mu_t^w}{\varphi}.$$

Thus, in the framework above unemployment is a consequence of workers’ market power (that is, of the wage being above their perfectly competitive level), while unemployment fluctuations result from the slow adjustment of wages. Figure 1 represents the relation between the wage markup and the unemployment rate using a simple diagram of the labor market.

Finally, combining (5), (11), and (12) we obtain the following NK Wage Phillips curve (NKWPC, for short):

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w \varphi (u_t - \un_t).$$
FIGURE 1. The wage markup and the unemployment rate.

Note that the simple linear relation between the wage markup and unemployment derived in this section holds irrespective of the details of the wage setting process. In particular, it also holds in the presence of wage indexation as described in equation (7). In that case we can derive the implied wage Phillips curve by combining equations (8) and (11) to obtain

\[
\pi_w t = \alpha + \gamma \bar{\pi}_p t - 1 + \beta E_t \{\pi_w t + 1 - \gamma \bar{\pi}_p t\} - \lambda_w \varphi (u_t - u^n),
\]

(14)

which I refer to henceforth as the augmented NKWPC.

3.2. Relation to the Original Wage Phillips Curve

In his seminal paper, Phillips (1958) uncovered the existence of a strong inverse empirical relation between wage inflation and the unemployment rate in the United Kingdom over the period 1861–1957. His analysis was subsequently replicated using US data by Samuelson and Solow (1960), who showed that a similar empirical relation had been prevalent in the United States, with the exception of the New Deal period and the early years of World War I. Much of subsequent empirical work turned its focus instead to the relation between price inflation and unemployment, usually in the context of a discussion of NAIRU and its changes (for example, Gordon 1997; Staiger, Stock, and Watson 1997).10

10. Gordon (1997) claims that the shift in focus towards price inflation has been “deliberate”. In his words, “... [t]he earlier fixation on wages was a mistake. The relation of prices to wages has changed over time ... The Fed’s goal is to control inflation, not wage growth, and models with separate wage growth and price markup equations do not perform as well as the [price inflation] equation ... in which wages are only implicit ...”
Note that, like the original Phillips (1958) curve, the NKWPC establishes a relationship between wage inflation and the unemployment rate. But two key differences with respect to Phillips’ original curve (and some of its subsequent amendments) are worth emphasizing.

Firstly, (13) is a microfounded structural relation between wage inflation and unemployment, with coefficients that are functions of parameters that have a structural interpretation, and which are independent of the policy regime. In particular, the steepness of the slope of the implied wage inflation-unemployment curve (given expected wage inflation) is decreasing in the degree of wage rigidity $\theta_w$ (which is inversely related to $\lambda_w$). In the limit, as $\theta_w$ approaches zero (the case of full wage flexibility), the curve becomes vertical. Also, the slope of the $(\pi^w, u)$ relation is decreasing in the size of the Frisch labor supply elasticity (which corresponds to the inverse of $\phi$). That structural nature of (13) stands in contrast with the purely empirical basis of Phillips (1958) original curve, whose only theoretical underpinning was the plausibility of the principle that “when demand for labour is high and there are very few unemployed we should expect employers to bid wage rates up quite rapidly . . .”.

Secondly, note that (13) implies that wage inflation is a forward looking variable, which is inversely related to current unemployment but also to its expected future path. This feature, which reflects the forward looking nature of wage setting, is immediately seen by solving (13) forward to obtain

$$\pi^w_t = -\lambda_w \phi \sum_{k=0}^{\infty} \beta^k E_t \{(u_{t+k} - u^p)\},$$  

which contrasts with the static, contemporaneous nature of the original Phillips curve, in which expectations play no role.

Next I briefly discuss two extensions of the previous framework. The first one allows for changes over time in desired markups, whereas the second introduces a specification of preferences that allows for limited short-run wealth effects on labor supply.

### 3.3. An Extension with Time-Varying Desired Wage Markups

Estimated, medium-scale versions of the NK model often allow for a time-varying, exogenous desired wage markup, $\{\tilde{\mu}^w_t\}$ (see, for example, Smets and Wouters 2003, 2007). In that case, the wage inflation equation (shown in its version without indexation, for simplicity) is given by

$$\pi^w_t = \beta E_t \{\pi^w_{t+1}\} - \lambda_w (\mu^w_t - \tilde{\mu}^w_t),$$  

11. Needless to say this is only true to the extent that one is willing to take the assumptions of the Calvo formalism literally, including the exogeneity of parameter $\theta_w$.

12. Phillips (1958) also emphasized the likely existence of nonlinearities due to workers’ reluctance “to offer their services at less than the prevailing rates when the demand for labour is low and unemployment is high, so that wage rates fall only very slowly.” In the analysis of the present paper, as in standard versions of the NK model, the possible existence of such asymmetries is ignored.
while the corresponding NKWPC now takes the form

$$\pi^w_t = \beta E_t \{\pi^w_{t+1}\} - \lambda_w \varphi (u_t - u^n_t)$$

where $u^n_t \equiv \overline{\mu}^w_t / \varphi$ denotes the (now time-varying) natural rate of unemployment. Variations in the latter variable, resulting from changes in desired wage markups, may thus potentially shift the relation between wage inflation and the unemployment rate.

Note that we can also write

$$\pi^w_t = \beta E_t \{\pi^w_{t+1}\} - \lambda_w \varphi u_t + v_t,$$

(17)

where $v_t \equiv \lambda_w \bar{\mu}^w_t$. The previous specification can be compared against one often used in the literature which relies on (16) combined with the definition of the average wage markup, and which takes the form

$$\pi^w_t = \beta E_t \{\pi^w_{t+1}\} - \lambda_w (w_t - p_t - c_t - \varphi n_t) + v'_t,$$

where now the error term is given by $v'_t \equiv \lambda_w \bar{\mu}^w_t + \lambda_w \xi_t$, that is it is influenced by both wage markup shocks and preference shocks. That property contrasts with (17), whose error term captures exclusively wage markup shocks, but not preference shocks (even though the latter have been allowed for in the model). This feature should in principle allow one to overcome the identification problem raised by Chari, Kehoe, and McGrattan (2009) in their critique of current NK models. The potential advantage of the present formulation is discussed in detail in Galí, Smets, and Wouters (2011), who re-estimate the Smets and Wouters (2007) model using an unemployment-based wage equation, and re-assess some of the findings therein in light of the new estimates.

3.4. Robustness to a Specification of Preferences with Limited Short-Run Wealth Effects

The specification of preferences assumed above, while analytically convenient, has implications on labor supply that are rather implausible from an empirical viewpoint. In particular, the strong wealth effects implied by the logarithmic specification, while seemingly needed in order to remain consistent with balanced growth, are likely to be counterfactual. This becomes clear by looking at the labor participation equation (9), which implies that the wage-consumption ratio $(w_t - p_t - c_t)$ should be positively correlated to labor participation, at least conditional on shocks other than preference shocks being the source of fluctuations. In postwar US data, and possibly due to the wage rigidities of the kind emphasized in the present paper, $w_t - p_t - c_t$ is clearly countercyclical, while participation is procyclical (albeit moderately so). Thus, and unless one is willing to attribute a dominant weight to preference shocks as a source of cyclical fluctuations in wages, consumption and participation, equation (9) provides an unsatisfactory account of fluctuations in the labor force.\footnote{The previous observation is closely related to other implausible predictions of macro models that rely on similar preferences, including the negative impact on activity of higher anticipated productivity, as emphasized by the recent literature on news shocks.}

13. The previous observation is closely related to other implausible predictions of macro models that rely on similar preferences, including the negative impact on activity of higher anticipated productivity, as emphasized by the recent literature on news shocks.
Here I consider a simple extension of the preferences assumed above that can in principle overcome that problem, by allowing for arbitrarily small short-run wealth effects while remaining consistent with a balanced growth path.\textsuperscript{14} Thus, individual utility from consumption is assumed to be given by
\[ U(C_t) \equiv \Theta_t^t \log C_t(i, j), \]
where \( \Theta_t \) is a preference shifter taken as exogenously given by each household, but determined by the ratio of aggregate consumption \( \bar{C}_t \) to a measure of its trend level. More precisely, I assume
\[ \Theta_t = \bar{C}_t / Z_t \]
where
\[ Z_t \equiv Z_{\vartheta} - c_t - \vartheta = \vartheta \]
with \( \vartheta \in [0, 1) \) is a (one-sided) long-run trend for aggregate consumption. Implicit in the specification above is the assumption that the marginal utility of individual consumption rises in response to a boom in aggregate consumption (relative to its trend), possibly due to a “keeping up with the Joneses” effect.

Under the above assumptions, aggregation of individual utilities yields the following period utility for the household:
\[ U(C_t, \{N_t(i)\}, \chi_t) \equiv \Theta_t \log C_t - \chi_t \int_0^1 N_t(i)^{1+\varphi} \frac{1}{1+\varphi} di. \]

The derivation of the wage inflation equation (5) (or (8), in the presence of indexation) carries over to this case, with the relevant marginal rate of substitution in the optimal wage setting problem now being given by
\[ \frac{MRS_{t+k|t} \equiv (\chi_{t+k} / \Theta_{t+k})C_{t+k}N_{t+k|t}}{MRS_{t+k|t}}. \]
Note, however, that in a symmetric equilibrium \( C_t = \bar{C}_t \) for all \( t \), which allows us to write the equilibrium (log) marginal rate of substitution as
\[ mrs_t = z_t + \varphi n_t + \xi_t, \]
where
\[ z_t \equiv \log Z_t \]
evolves over time according to
\[ z_t = \vartheta z_{t-1} + (1 - \vartheta) c_t. \]
The equations for the average wage markup and participation are now respectively given by
\[ \mu^w_t \equiv (w_t - p_t) - (z_t + \varphi n_t + \xi_t), \]
\[ w_t - p_t = z_t + \varphi l_t + \xi_t, \]
which can be combined to yield the same simple proportional relation between the wage markup and the unemployment rate as above, that is
\[ \mu^w_t = \varphi u_t. \]
Note that the previous specification is still consistent with a balanced growth path since, in the long run, \( z_t \) will grow at the same rate as consumption. In the short run, however, the impact of changes in consumption on the marginal rate of substitution may be rendered arbitrarily small by increasing parameter \( \vartheta \), thus yielding a more plausible labor supply model. Yet, and most importantly for the purposes of the present paper, the specification of the NKWPC in (13) (or (14)) remains unaffected.\textsuperscript{15}

\textsuperscript{14} Christiano, Trabandt, and Walentin (2009) follow a similar approach to deal with the strong wealth effect problem. In a subsequent paper (Christiano, Trabandt, and Walentin 2010) the same authors propose an alternative model, where the probability of finding a job is increasing in search effort, and where imperfect risk sharing among individuals is a consequence of the unobservability of effort. In that framework the labor force comoves positively with employment and negatively with unemployment.

\textsuperscript{15} An identical robustness result can be shown to obtain under a period utility function of the sort assumed by Jaimovich and Rebelo (2009) with a similar motivation, namely
\[ U(C_t, \{N_t(i)\}, Z_t) = \frac{1}{1-\sigma} \left( C_t - \chi_t Z_t \int_0^1 N_t(i)^{1+\varphi} \frac{1}{1+\varphi} di \right)^{1-\sigma}. \]
3.5. A Reduced Form Representation for the NKWPC

In this section, and in order to obtain some additional intuition about the joint dynamics for wage inflation and unemployment, I derive a simple reduced form representation of the NKWPC. Let us thus assume that the unemployment rate follows a stationary AR(2) process. As discussed in section 4.2, this process turns out to provide a reasonably good approximation to the behavior of US unemployment in the postwar period. Formally,

$$\hat{u}_t = \phi_1 \hat{u}_{t-1} + \phi_2 \hat{u}_{t-2} + \epsilon_t,$$

(18)

where $\hat{u}_t = u_t - u^n$ and $\{\epsilon_t\}$ is white noise. Combining (18) with (14) yields the wage inflation equation

$$\pi^w_t = \alpha + \gamma \bar{\pi}^{p}_{t-1} + \psi_0 \hat{u}_t + \psi_1 \hat{u}_{t-1},$$

(19)

where

$$\psi_0 = -\frac{\lambda_w \varphi}{1 - \beta(\phi_1 + \beta \phi_2)},$$

$$\psi_1 = -\frac{\lambda_w \varphi \beta \phi_2}{1 - \beta(\phi_1 + \beta \phi_2)},$$

or, equivalently,

$$\pi^w_t = \alpha' + \gamma \bar{\pi}^{p}_{t-1} - \delta \hat{u}_t - \psi_1 \Delta \hat{u}_t,$$

(20)

where $\delta \equiv -(\psi_0 + \psi_1)$.

Estimates of the unemployment process (18) using postwar US data unambiguously point to the following properties: $\phi_1 > 1$, $-1 < \phi_2 < 0$ and $0 < \phi_1 + \phi_2 < 1$, the latter being a requirement for stationarity. If those inequalities hold then we have $\psi_0 < 0$, $\psi_1 > 0$, and $\delta > 0$. Thus, under the previous assumptions and conditional on the relevant lagged price inflation measure used for indexation purposes, wage inflation should respond negatively to both the level and the first difference of the unemployment rate, with the size of that response being a well defined function of structural parameters, including those characterizing the process for the unemployment rate.

Interestingly, a specification like (20) has often been proposed and used in empirical applications (for example, Blanchard and Katz 1999), as well as in mainstream undergraduate textbooks (though the latter typically omit lagged unemployment). In fact, in his seminal paper Phillips (1958) himself argued that it was plausible that wage inflation would depend negatively on both the level and the change of the unemployment rate, since both captured important dimensions of the unemployment.

where $Z_t = Z^{\text{non}}_t C_t^{-\delta}$. This is the specification adopted in Galí, Smets, and Wouters (2010). The non-separability of the previous specification for household utility, however, prevents one from interpreting it as the aggregation of the utilities of individual household members.
degree of tightness or excess demand in labor markets, and tried to uncover their joint influence on the unemployment rate.

The following section revisits and updates estimates of equations (19) and (20), and reinterprets them through the lens of the New Keynesian model developed above. In addition, it provides an empirical assessment of the NKWPC (14) that does not rely on the assumption of an exogenous univariate process for the unemployment rate.

4. Empirical Evidence

The present section provides an empirical assessment of the NKWPC developed above. More specifically, I want to evaluate to what extent a version of the NKWPC with a constant natural rate can account for the joint behavior of unemployment and wage inflation in the US economy.

First, I use simple statistics and graphical tools to seek evidence of a prima facie negative relationship between wage inflation and unemployment of the sort predicted by the theory. Secondly, I compare the observed behavior of wage inflation with that predicted by an estimated version of the model above, conditional on the unemployment rate. I do so using two alternative approaches. First, I estimate the single equation model for wage inflation implied by assumption of a simple univariate process for the unemployment rate. Then I relax the latter assumption, and use an empirical strategy based on Campbell and Shiller (1987) that imposes no restrictions on the information set used to predict the unemployment rate.

The empirical analysis relies on quarterly postwar US data drawn from the Haver database. I use the civilian unemployment rate as my measure of unemployment. All measures of price inflation are constructed using the consumer price index. There are two main alternative sources for average wage data that one may use to construct measures of wage inflation: the earnings data for production and non-supervisory workers from the Establishment Survey (starting in 1964Q1), and the compensation data from the Productivity and Costs publication of the Bureau of Labor Statistics (available from 1948Q1 onwards). Aside from their time span, the main difference between the two is that compensation is a more encompassing measure of the cost to the employer, and as such it includes employer contributions to employee-benefit plans or irregular bonuses, whereas earnings is restricted to wage income proper (including premium pay for overtime). Unfortunately, and as noted by several authors, the discrepancy between their implied inflation measures is far from negligible. This is made clear by Figure 2, which plots wage inflation based on both measures. Note that in this and subsequent figures—though not in the formal econometric work that follows—wage inflation is measured as the centered four-quarter difference of the log nominal wage expressed in percentage terms (that is, \(100 \times (w_{t+2} - w_{t-2})\)), in order to smooth the high volatility associated with quarter-to-quarter log-differences.

16. The observability of the unemployment rate, may be viewed as an advantage of the present framework relative to Sbordone (2006), who focuses instead on a parameterized version of (5).

17. See, for example, Abraham, Spletzer, and Stewart 1999.
Three features stand out in Figure 2. First, the two measures display very similar medium-run patterns, both picking up during the Great Inflation episode, dropping during the Volcker disinflation, and fluctuating around a low and stable mean during the Great Moderation years. This is reflected in their high contemporaneous correlation (0.82). Secondly, their mean values in the common sample period are noticeably different: 4.44% for the earnings-based measure, 5.36% for the compensation-based one. Finally, and perhaps more strikingly, the compensation-based measure appears far more volatile than the earnings-based measure, especially over the past two decades. A possible explanation for the difference in volatility lies in the inclusion of the payoffs from the exercise of employee stock options in the compensation measure. As argued by Mehran and Tracy (2001), the impact of changes in that (increasingly important) component of compensation on wage inflation is substantial, even if it only accounts for a small fraction of overall compensation.

Interestingly, and those differences notwithstanding, both measures are used in empirical macro applications, often with little or no discussion regarding the choice. While the compensation-based measure has an advantage in its longer span, its sizable high-frequency variations render it somewhat suspicious, and suggest the presence of substantial measurement error. Thus, in the empirical analysis in what follows I will use the earnings-based wage inflation as a baseline measure, but will check and discuss the robustness of the main findings to the use of the alternative, compensation-based, wage inflation measure. Perhaps surprisingly, given the seemingly large differences observed in Figure 1, none of the key qualitative findings in the empirical analysis hinge on the choice of wage inflation measure.

### 4.1. A Quick Glance at the Data

The New Keynesian Wage Phillips curve derived in Section 3 implies an inverse relation between wage inflation and the unemployment rate, albeit not a simple or
contemporaneous one, especially in the presence of indexation. As a first pass in the empirical assessment of the model it seems natural to check whether the raw data hint at any such inverse relation.

Figure 3 displays two scatterplots of wage inflation and the unemployment rate for the US economy, using both measures of wage inflation. The scatterplots reveal the absence of a stable negative relation between the two variables. Similar graphs, though typically focusing on price inflation, have often been used to demonstrate “the empirical failure of the Phillips curve”. That visual impression is confirmed by the correlation between the two series, which is as low as 0.02 and −0.01, respectively.

Figure 4 displays the evolution of unemployment and wage inflation over time, using the earnings-based measure of the latter. While no stable relation seems evident
at a first glance, a more careful examination points to a strong inverse relation starting sometime around the mid-1980s and prevailing up to the end of the sample. That inverse relation is confirmed by the scatterplots of Figure 5, which are now restricted to the post-1984 period, that is the so-called Great Moderation era. For the restricted sample period the correlations are $-0.76$ and $-0.27$, respectively.

Figure 6 adds a temporal dimension to the Phillips curve scatterplots initially shown in Figure 3, now restricted to the earnings-based wage inflation. It suggests
that the paths of US wage inflation and unemployment have completed a full circle, returning in recent years to the same downward locus that characterized the 1960s. The evidence thus points to the presence of a stable negative relation between wage inflation and unemployment during periods of relatively low and stable price inflation. That relation is broken during transitions from low to high inflation (early 1970s), or from high to low inflation (the early 1980s), leading to an overall lack of correlation, as suggested by Figure 3.

Thus, it seems clear from the previous quick glance at the data that any model that implies a simple inverse relation between wage inflation and unemployment will be at odds with the behavior of those two variables during the long 1970–1985 episode. Yet, one cannot rule out that extensions of such a model which allow for indexation to price inflation may be consistent with the evidence. I explore that hypothesis in the next section, using the augmented version of the NK Wage Phillips curve.

Why has the re-emergence of a stable negative relation between wage inflation and unemployment over the past two decades gone unnoticed among academic economists? A possible explanation lies in the focus on price inflation and away from wage inflation in much of the empirical research of recent years, combined with a lack of a significant empirical relation between price inflation and unemployment. The correlation between those two series over the post-1984 period is low and insignificant (−0.13), and its negative value is due exclusively to the most recent observations: if I end the sample period in 2007Q4 the correlation becomes even smaller in absolute value and has the wrong sign (0.08). Of course, the theory developed above has nothing to say, by itself, about the relation between price inflation and the unemployment rate, since that relation is likely to be influenced by factors other than wage setting, including features of price setting and the evolution of labor productivity, among others.18

Next I turn to a more formal empirical assessment of the NK Wage Phillips curve.

### 4.2. Estimates of the Reduced Form NK Wage Phillips Curve

In this section I report estimates of the reduced form wage equation (19). The focus on (19) is motivated by the good fit of an AR(2) process for the unemployment rate. Using data starting in 1948Q1 I estimate the following model:

\[
    u_t = 0.22^{**} + 1.66^{**} u_{t-1} - 0.70^{**} u_{t-2} + \varepsilon_t,
\]

with any further lags of the unemployment rate not being significant.

For the empirical analysis I augment equation (19) with an error term that reflects the likely measurement error in the wage inflation data, and which I assume to be

---

18. See, for example, Blanchard and Galí (2010), Galí (2011a) and Thomas (2009) for an analysis of the relation between price inflation and unemployment in a model with labor market frictions.
Table 1. Estimated wage inflation equations: earnings-based.

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
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<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
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</thead>
<tbody>
<tr>
<td>$u_t$</td>
<td>-0.001</td>
<td>-0.030*</td>
<td>-0.079**</td>
<td>-0.177</td>
<td>-0.377**</td>
<td>-0.334**</td>
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<td></td>
<td>(0.019)</td>
<td>(0.017)</td>
<td>(0.015)</td>
<td>(0.083)</td>
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<tr>
<td>$u_{t-1}$</td>
<td>0.153</td>
<td>0.304**</td>
<td>0.294**</td>
<td>0.453**</td>
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<td>(0.112)</td>
<td>(0.078)</td>
<td>(0.095)</td>
<td>(0.073)</td>
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<tr>
<td>$\pi_{t-1}$</td>
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<td>0.503**</td>
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<tr>
<td></td>
<td>(0.043)</td>
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<tr>
<td>$\pi_{t-1}^{(4)}$</td>
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<td>0.687**</td>
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<tr>
<td></td>
<td>(0.038)</td>
<td>(0.041)</td>
<td>(0.038)</td>
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<td>$p-value$</td>
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<td>0.67</td>
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<td>$\theta_w (\phi = 1)$</td>
<td>0.788**</td>
<td>0.629**</td>
<td>0.755**</td>
<td>0.607**</td>
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<td></td>
<td>(0.083)</td>
<td>(0.059)</td>
<td>(0.061)</td>
<td>(0.063)</td>
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<tr>
<td>$\theta_w (\phi = 5)$</td>
<td>0.896**</td>
<td>0.805**</td>
<td>0.879**</td>
<td>0.792**</td>
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<tr>
<td></td>
<td>(0.044)</td>
<td>(0.036)</td>
<td>(0.033)</td>
<td>(0.039)</td>
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</tbody>
</table>

Note: The table reports OLS estimates of several specifications of the wage inflation equation, each of which can be seen as a restricted version of (19). In all cases robust standard errors are reported in brackets. The wage inflation data is based on the earnings series, and the sample period starts in 1964Q1. The first three columns report estimates including only the current value of the unemployment rate, augmented in the case of columns (2) and (3) with lagged quarter-to-quarter price inflation (column (2)) and year-on-year price inflation (column (3)), with the latter expressed as a quarterly rate to facilitate comparison of coefficients. Note that when price inflation is not controlled for, the coefficient on unemployment is very close to zero and statistically insignificant. When lagged inflation is added as a regressor its coefficient is highly significant, while the coefficient on unemployment increases in absolute value and becomes significant (though only at the 10% level when quarter-to-quarter inflation is used as a regressor). Columns (5) and (6) include the lagged unemployment rate, and are thus consistent with the specification implied by the model. In both cases the coefficients on the unemployment rates have the sign predicted by the theory, though they are only significant when year-on-year price inflation is used as the indexing variable. Also, as

independent of unemployment at all leads and lags. The estimated equation is thus,

$$\pi_t^w = \alpha + \gamma \pi_{t-1}^p + \psi_0 u_t + \psi_1 u_{t-1} + \zeta_t$$

where $\zeta_t$ is a zero mean, possibly autocorrelated error term. Note that, under the assumptions above, the wage inflation equation should have a negative coefficient on the current unemployment rate and a positive one on its lag, in addition to a positive coefficient on lagged price inflation in the presence of indexation. Furthermore, note that the model (19) implies the cross-equation restriction $\psi_1 = \psi_0 \beta \phi_2$, which can be easily tested conditional on a value for $\beta$, which I set to 0.99.

Table 1 reports OLS estimates of several specifications of the wage inflation equation, each of which can be seen as a restricted version of (19). In all cases robust standard errors are reported in brackets. The wage inflation data is based on the earnings series, and the sample period starts in 1964Q1. The first three columns report estimates including only the current value of the unemployment rate, augmented in the case of columns (2) and (3) with lagged quarter-to-quarter price inflation (column (2)) and year-on-year price inflation (column (3)), with the latter expressed as a quarterly rate to facilitate comparison of coefficients. Note that when price inflation is not controlled for, the coefficient on unemployment is very close to zero and statistically insignificant. When lagged inflation is added as a regressor its coefficient is highly significant, while the coefficient on unemployment increases in absolute value and becomes significant (though only at the 10% level when quarter-to-quarter inflation is used as a regressor). Columns (5) and (6) include the lagged unemployment rate, and are thus consistent with the specification implied by the model. In both cases the coefficients on the unemployment rates have the sign predicted by the theory, though they are only significant when year-on-year price inflation is used as the indexing variable. Also, as

19. The error term will also capture any random variations in the desired wage markup, which has been assumed constant throughout. To the extent that the unemployment rate is correlated with that desired markup the OLS estimates will not be consistent. See Galí, Smets, and Wouters (2011) for a full-information approach to estimation in that case, with the wage inflation equation embedded in a complete DSGE model.
shown in the row labeled \( p \)-value, the cross-equation restriction specified above cannot be rejected when both equations are estimated jointly. The final two rows report the implied estimates of the Calvo wage rigidity parameter \( \theta_w \), conditional on calibrated values for \( \varphi \) and \( \epsilon_w \), since the three parameters are not separately identified. Note that \( \varphi \) is the inverse Frisch labor supply elasticity, a controversial parameter. I consider two alternative calibrations, \( \varphi = 1 \) and \( \varphi = 5 \), which span the range of values often assumed in the literature. Given \( \varphi \), I use (12) to set \( \epsilon_w \) to a value consistent with a natural rate of unemployment of 5\%, which is roughly the average unemployment rate over the sample period considered. The point estimates range from 0.62 to 0.89, suggesting substantial wage rigidities, with those associated with the \( \varphi = 5 \) calibration implying average durations that may be viewed as implausibly long (though the non-negligible size of the standard errors allow for more plausible underlying degrees of wage rigidity).

A more detailed analysis of the fit of the estimated wage equations reported in columns (5) and (6), suggests a poor fit during the recent recession. The reason is simple: the rapid increase in the unemployment rate and the very low levels of price inflation (which became deflation for some quarters), lead the fitted wage equation to predict substantial nominal wage deflation. While actual wage inflation was brought down by the recession, it has always remained positive. The presence of downward nominal wage rigidities, which are ignored in the standard wage setting model developed above, could in principle account for that poor fit. Motivated by that observation, columns (7) and (8) in Table 1 report estimates of the wage equation using data up to 2007Q4, thus avoiding any distortion resulting from the use of recent data. Note that for both specifications, the coefficients on current and lagged unemployment increase substantially and now become highly significant even when quarter-to-quarter price inflation is used as a regressor. Figure 7 displays actual and fitted wage inflation, using the estimates shown in column (8). While the estimated model misses much of the high-frequency variations, it appears to capture well most movements at medium-term frequencies, with the exception of the spikes in 1971–1972 and 1976–1977. The correlation between the two series is 0.83.

Table 2 reports estimates of wage inflation equations using the measure based on compensation, and with the sample period starting in 1948Q1. Most of the findings in Table 1 appear to be robust to the use of the alternative wage inflation measure, which is indeed surprising given the large discrepancies between the two inflation measures. Note, however, that the exclusion of post-2007 data does not have much of an impact now, possibly because of its reduced weight in the longer sample period. In particular, the coefficients on unemployment and its lagged value in column (5) are now significant, in addition of having the right pattern of signs and relative magnitude.

The estimates in Tables 1 and 2 can be used to determine the implied coefficients on the level and the change in the unemployment rate, corresponding to the reduced form specification (20). Interestingly, the size of the coefficient on the change in

---

Note that \( \epsilon_w = (1 - \exp(-\varphi u^*))^{-1} \). This implies setting \( \epsilon_w = 20.5 \) when \( \varphi = 1 \), and \( \epsilon_w = 4.52 \) when \( \varphi = 5 \).


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<th>(1)</th>
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<th>(6)</th>
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<th>(8)</th>
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<tr>
<td>$u_t$</td>
<td>-0.015</td>
<td>-0.046*</td>
<td>-0.087**</td>
<td>-0.227**</td>
<td>-0.387**</td>
<td>-0.241**</td>
<td>-0.397**</td>
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<td>(0.028)</td>
<td>(0.024)</td>
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<td>(0.099)</td>
<td>(0.125)</td>
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<td>(0.101)</td>
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<tr>
<td>$u_{t-1}$</td>
<td>0.189**</td>
<td>0.310**</td>
<td>0.189**</td>
<td>0.301**</td>
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<tr>
<td>(0.096)</td>
<td>(0.116)</td>
<td>(0.090)</td>
<td>(0.098)</td>
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<tr>
<td>$\pi_{t-1}$</td>
<td>0.500**</td>
<td>0.505**</td>
<td>0.522**</td>
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<tr>
<td>(0.046)</td>
<td>(0.046)</td>
<td>(0.046)</td>
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</tr>
<tr>
<td>$\pi_{t-1}^{(4)}$</td>
<td>0.578**</td>
<td>0.610**</td>
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<tr>
<td>(0.058)</td>
<td>(0.060)</td>
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</tr>
<tr>
<td>$p-value$</td>
<td>0.57</td>
<td>0.79</td>
<td>0.54</td>
<td>0.66</td>
<td></td>
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<tr>
<td>$\theta_w (\varphi = 1)$</td>
<td>0.709**</td>
<td>0.637**</td>
<td>0.675**</td>
<td>0.569**</td>
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<tr>
<td>(0.086)</td>
<td>(0.059)</td>
<td>(0.069)</td>
<td>(0.057)</td>
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</tr>
<tr>
<td>$\theta_w (\varphi = 5)$</td>
<td>0.853**</td>
<td>0.780**</td>
<td>0.833**</td>
<td>0.767**</td>
<td></td>
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<tr>
<td>(0.049)</td>
<td>(0.043)</td>
<td>(0.040)</td>
<td>(0.037)</td>
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</table>

unemployment is between three and seven times larger than that on the level. That finding suggests that a high unemployment rate will, in itself, put limited downward pressure on wage inflation, relative to the larger impact of the initial increase in unemployment.

4.3. A Measure of Fundamental Wage Inflation in the Spirit of Campbell–Shiller

In the previous section I reported estimates of a reduced-form wage inflation equation implied by the NKWPC, under the assumption that the unemployment rate follows an exogenous, univariate AR(2) process. While the previous assumption provides a fairly good approximation to the dynamics of unemployment in the postwar period and
leads to a reduced-form specification which makes contact with that used in Phillips (1958) and in subsequent applied work, one may legitimately wonder whether the favorable empirical assessment of the NKWPC hinges on that assumption. Relaxing that assumption has an additional justification: simple Granger-causality tests reject the null of no-Granger causality from wage and price inflation to unemployment. Thus, in particular, the four lags of (earnings-based) wage inflation and price inflation are significant at the 1% level in a regression of the unemployment rate on its own four lags and the lags of the two inflation measures over the 1964Q1–2009Q3 sample period. An analogous test using the compensation-based measure of wage inflation and extended over the sample period 1948Q1–2009Q3 only rejects the null of no-Granger causality at the 7% significance level.

Motivated by the previous observation, and in the spirit of Campbell and Shiller’s (1987) proposed assessment of present value relations, I start by defining the following measure of fundamental or model-based wage inflation:

$$\tilde{\pi}_t^w(\Theta) \equiv \gamma \tilde{\pi}_{t-1}^p - \lambda_w \varphi \sum_{k=0}^{\infty} \beta^k E \{u_{t+k}|z_t\},$$

where vector $\Theta \equiv [\gamma, \theta_w, \beta, \epsilon_w, \varphi]$ collects the exogenous parameters of the model and where $z_t = [u_t, \pi_t^w - \gamma \tilde{\pi}_{t-1}^p, \ldots, u_{t-q}, \pi_{t-q}^w - \gamma \tilde{\pi}_{t-1-q}^p]$ for some finite $q$. Under the null hypothesis that model (14) holds exactly (that is, in the absence of measurement error and with a constant natural unemployment rate), it is easy to check that $\tilde{\pi}_t^w(\Theta) = \pi_t^w$, for all $t$. In other words, given the structure of the conditioning variable $z_t$ the use of a limited information set is not restrictive under the null.

Next I estimate $\tilde{\pi}_t^w(\Theta)$ and plot it against actual wage inflation, to evaluate the extent to which the simple model developed here can explain observed fluctuations in that variable. I assume that the joint dynamics of unemployment and wage inflation are well captured by the first-order vector autoregressive model

$$z_t = A z_{t-1} + \varepsilon_t,$$

where $E \{\varepsilon_t|z_{t-1}\} = 0$ for all $t$. Thus, and letting $e_i$ denote the $i$th unit vector in $\mathbb{R}^{2q}$, we have $E \{u_{t+k}|z_t\} = e_i^t A^k z_t$, implying

$$\tilde{\pi}_t^w(\Theta) = \gamma \tilde{\pi}_{t-1}^p - \lambda_w \varphi e_i^t (I - \beta A)^{-1} z_t. \quad (21)$$

I exploit the previous result to construct a time series for fundamental inflation $\tilde{\pi}_t^w(\Theta)$ using a minimum distance estimator. Since not all structural parameters in $\Theta$ are separately identified, I calibrate three of them ($\beta, \epsilon_w, \varphi$) and estimate the remaining two ($\gamma, \theta_w$), which define the degree of rigidities and indexation. As previously, I set $\beta = 0.99$ and report results for both $\varphi = 1$ and $\varphi = 5$, with $\epsilon_w$ set to imply a natural unemployment rate of 5% in each case. I estimate the two remaining parameters, $\theta_w$ and $\gamma$, by minimizing $\sum_{t=0}^{T}(\pi_t^w - \tilde{\pi}_t^w(\Theta))^2$ subject to (21), over all possible values $(\theta_w, \gamma) \in [0, 1] \times [0, 1]$, and given the calibrated values for ($\beta, \epsilon_w, \varphi$) and the OLS estimate for matrix $A$ (with $q = 4$). As in the empirical analysis above I use lagged
TABLE 3. Estimated parameters using minimum distance estimator.

<table>
<thead>
<tr>
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<th>Earnings</th>
<th>Compensation</th>
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<tr>
<td></td>
<td>$\pi_{t-1}$</td>
<td>$\pi_{t-1}^{(4)}$</td>
</tr>
<tr>
<td>$\varphi = 1$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\gamma$</td>
<td>0.52 (0.013)</td>
<td>0.83 (0.130)</td>
</tr>
<tr>
<td>$\theta_w$</td>
<td>0.65 (0.013)</td>
<td>0.52 (0.080)</td>
</tr>
<tr>
<td>$\rho(\hat{\pi}_w^n, \pi_w^n)$</td>
<td>0.82 (0.013)</td>
<td>0.82 (0.013)</td>
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<tr>
<td>$\varphi = 5$</td>
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<tr>
<td>$\gamma$</td>
<td>0.52 (0.017)</td>
<td>0.81 (0.110)</td>
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<td>$\theta_w$</td>
<td>0.82 (0.008)</td>
<td>0.74 (0.020)</td>
</tr>
<tr>
<td>$\rho(\hat{\pi}_w^n, \pi_w^n)$</td>
<td>0.91 (0.017)</td>
<td>0.91 (0.017)</td>
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</tbody>
</table>

quarterly and annual inflation as an indexing variable, and both earnings-based and compensation-based measures of wage inflation.

Table 3 reports the main results for the exercise. The estimates of $\gamma$, the degree of indexation, are always highly significant and lie between 0.52 and 0.83, depending on the specification, values which are slightly higher than those obtained in the previous section. The point estimates for the Calvo parameter $\theta_w$ are also highly significant in all cases. Under the $\varphi = 1$ assumption they lie between 0.52 and 0.65, implying an average duration between two- and three-quarters. When $\varphi = 5$ is assumed, the estimates are substantially higher (between 0.75 and 0.82), but still within the range of plausibility, given the evidence uncovered by micro-studies. Interestingly, my estimates for $\gamma$ and $\theta_w$ are very close to those obtained by Smets and Wouters (2007) using a very different approach (and one that does not use information on the unemployment rate, among other differences): 0.58 and 0.7, respectively.

The multivariate model analyzed here implies some restrictions that can be subject to formal testing. In particular, note that if model (14) holds exactly (that is with neither measurement error nor variations in the natural unemployment rate), the set of restrictions $e_2^\prime + \lambda w \varphi e_1 = \beta e_2^\prime A$ must be satisfied. Unfortunately the latter can be rejected at low significance levels for our sample and baseline calibration. This may not be surprising, given the simplicity of the model and the likely importance of measurement error. But, following Campbell and Shiller (1987), I seek a more informal evaluation of the model by comparing actual and fundamental wage inflation. The last row of Table 3 displays the correlation between the four-quarter centered moving averages of both variables: the correlations are positive and high (above 0.75) in all cases, suggesting a good fit of the model. This is also illustrated in Figure 8, which displays actual (earnings-based) wage inflation and fundamental wage inflation, where the latter is based on the estimates using year-on-year price inflation as an indexing variable and $\varphi = 1$. While the fit is far from perfect, it is clear that the model-based series captures pretty well the bulk of the low and medium frequency fluctuations in

21. Standard errors are obtained by drawing from the empirical distribution of $A$, and re-estimating $\theta_w$ and $\gamma$ for each draw.

22. See, for example, Taylor (1999). Note, however, that alternative specifications of the wage-setting block—as in Schmitt-Grohé and Uribe (2006)—would imply a larger value of $\theta_w$ (and hence a longer average duration of wage contracts) corresponding to the same estimate of $\lambda_w$. 
actual wage inflation (with the exception of some episodes, including the 2008–2009 recession). The fact that such a good fit is obtained using a model for wage inflation that assumes a constant natural rate of unemployment makes that finding perhaps even more surprising.

Given the large fluctuations in price inflation over the sample period considered and the well-known positive correlation between price and wage inflation, one may wonder to what extent the high correlation between actual and fundamental inflation is largely a consequence of indexation to past price inflation. In order to address that question I construct a measure of the cyclical component of fundamental inflation, by subtracting from the latter the inflation indexation component, $\gamma \bar{\pi}_{t-1}^p$. The cyclical component is thus driven exclusively by current and anticipated future unemployment rates. Figure 9 displays the cyclical component of fundamental inflation thus constructed, together with actual wage inflation. It is clear from that evidence that while the cyclical component did not have a dominant role in accounting for the fluctuations in wage inflation in the Great Inflation era, one can still detect several episodes in which it shapes the shorter-term fluctuations in observed wage inflation, including the 1968–1969 hump and the 1974–1975 trough. It is not until the advent of the Great Moderation period and the associated stability in price inflation that the cyclical component of fundamental wage inflation emerges as a central factor behind fluctuations in wage inflation, as Figure 9 makes clear.

5. Concluding Comments

In his seminal 1958 paper, A. W. Phillips uncovered a tight inverse relation between unemployment and wage inflation in the United Kingdom. That relation was largely
abandoned on both theoretical and empirical grounds. From a theoretical viewpoint, it was not clear why the rate of change of the nominal wage (as opposed to the level of the real wage) should be related to unemployment. From an empirical viewpoint, economists’ attention shifted to the relation between price inflation and unemployment, but hopes of establishing a stable relationship between those variables faded with the stagflation of the 1970s.

The present paper has made two main contributions. First, it provides some theoretical foundations to a Phillips-like relation between wage inflation and unemployment. It does so not by developing a new model but, instead, by showing that such a relation underlies a standard NK framework with staggered wage setting, even though versions of the latter found in the literature do not explicitly incorporate or even discuss unemployment. Secondly, the implied wage equation is shown to account reasonably well for the comovement of wage inflation and the unemployment rate in the US economy, even under the strong assumption of a constant natural rate of unemployment. In particular, that equation can explain the strong negative comovement between wage inflation and unemployment observed during the past two decades of price stability.

It is far from the objective of the present paper to claim that the staggered wage setting model of Erceg, Henderson, and Levin (2000) provides an accurate description of the US labor market. It is clear that some of its underlying assumptions—most noticeably, the unilateral setting of the wage by a monopoly union—are at odds with arrangements prevailing in most sectors. Yet, as a matter of fact, the EHL structure underlies most of the medium-scale DSGE models that have been developed in recent years, by both academics and institutions. Identifying and testing further predictions coming out of those models would seem a worthy undertaking and a source of guidance in any effort to improve the frameworks available for policy analysis.
In that spirit, and in work based on the same framework as the present paper (Galí 2011b), I have shown that a calibrated version of the standard NK model generates fluctuations in the unemployment rate that display properties similar to those observed in the data. In addition, I show how the unemployment rate can be used to construct a measure of the model-based output gap (that is, the log deviation between output and its efficient level), and discuss the desirability of having the central bank respond to the unemployment rate (in addition to inflation).

References


