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# Unemployment in dynamic general equilibrium economies

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**Abstract**

A tractable approach to modelling unemployment fluctuations in dynamic general equilibrium economies with non-Walrasian labor markets is presented and discussed.

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## 1. Introduction

Though the use of dynamic stochastic general equilibrium models (henceforth, DSGE models) in business cycle theory may have proved useful at understanding some aspects of aggregate fluctuations, the assumption of a Walrasian labor market embedded in most of the examples found in the literature has rendered them incapable of accounting for a key macroeconomic phenomenon: the large and persistent fluctuations in unemployment observed in market economies.

In response to that shortcoming, a number of researchers have been exploring alternative ways to enrich those models by introducing elements of imperfect competition in the labor market, in a way which allows them to generate

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unemployment (and unemployment fluctuations) as an equilibrium outcome. [1] That line of research can be interpreted as an attempt to bridge the gap between recent business cycle modelling strategies – based on the use of DSGE models – and the traditional literature on unemployment. The latter focuses on the existence of a non-Walrasian labor market structure, in which wages are *set* (by firms, workers, or the government) somewhere above their market-clearing level, thus generating unemployment, i.e., the inability of (some) individual workers to sell as much labor services as they wish to supply, given the prevailing wages and other labor market conditions. [2] Unfortunately, that literature has been largely restricted to static or partial equilibrium models, falling short of adopting an explicit DSGE framework.

In my view, the challenge ahead lies in developing, analyzing, and evaluating examples of DSGE economies with non-Walrasian labor markets, which can be easily compared to their perfectly competitive counterparts and which, in contrast with the latter, can account for some key stylized facts regarding the cyclical behavior of labor markets observed in most market economies, including (a) the procyclicality of employment and labor force measures, (b) the high variability of employment relative to the labor force, and (c) the persistence and highly countercyclical behavior of unemployment rates.

In the present paper I present a tractable and relatively general DSGE framework that can encompass a broad class of departures from perfect competition in the labor market. Since the idea is not to analyze and evaluate a *specific* model of the labor market, a detailed description of the latter is deliberately omitted. The proposed framework maintains the abstraction of a representative household which is key to the tractability (and, arguably, the popularity) of recent business cycle models. The main contribution is the introduction of a measure of unemployment that is well defined in the class of economies considered (taking the form, under standard assumptions, of an invariant function of aggregate state variables), and which appears conceptually close to its empirical counterpart.

## 2. People

I assume a continuum of identical infinite-lived households. Each household seeks to maximize an objective function $\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t, n_t)$, subject to a se-

[1] See, e.g., Danthine and Donaldson (1985), and Galí (1995), as well as references therein. An alternative approach to modelling unemployment in a dynamic setup is provided by the search literature (see, e.g., Layard et al., 1991, Ch. 5). In the latter, unemployment results from technological/physical constraints which prevent those who quit/lose a job from finding employment immediately, as opposed to the non-Walrasian wage-setting process emphasized here.

[2] Work in that tradition includes models with unions, efficiency wages, and insider–outsider models, among others. See Blanchard and Fischer (1989) or Layard et al. (1991) for a review of that literature.

quence of budget constraints

$$k_{t+1} = [(1-\delta) + q_t]k_t + w_{jt}n_t + \pi_t - c_t$$

and non-negativity constraints $k_t \geq 0$, $n_t \geq 0$, $c_t \geq 0$, for $t = 0,1,2,\ldots$. Here $k$ denotes capital holdings, $c$ is consumption, $q$ is the rental cost of capital, and $\pi$ is profits from stock holdings. All those variables are in terms of a single (or well defined composite) final good. $w_j$ is the wage per unit of labor services, with $j$ being an index for the wage setting unit (which could be a firm, a sector, etc.) the household belongs to. $n$ denotes the quantity of labor services sold by the household. If the household is viewed as consisting of a single agent, $n$ has the natural interpretation of work hours. At the other extreme, we can think of each household as being made up of a continuum of individuals, in which case $n$ would represent a measure of the subset of household members that are employed (each working a fixed number of hours). Intermediate interpretations are also possible, under appropriate assumptions.

Irrespective of how $w_j$ and $n$ are determined, each household's consumption/savings decision must satisfy the Euler equation

$$\beta \mathrm{E}_t\left[\left(\frac{U_c(c_{t+1}, n_{t+1})}{U_c(c_t, n_t)}\right)R_{t+1}\right] = 1, \quad t = 0,1,2,\ldots, \tag{1}$$

where $R_{t+1} \equiv 1 - \delta + q_{t+1}$ denotes the return on capital holdings.

## 3. Firms

The market structure is left largely unspecified, though an assumption of full symmetry (and aggregability) is implicit throughout (across firms, industries, etc.). A typical firm (say, firm $j$) has access to a production function $y_{jt} = \exp(\phi_t)F(k_{jt}, n_{jt})$. $\{\phi_t\}$ is an exogenous technology process common to all firms. We assume $\phi_t = h^{\phi}(\phi_{t-1}, \epsilon_t)$, with $\{\epsilon_t\}$ white noise. Regardless of how employment and the wage are determined (possibly at the firm level), each period the firm rents the profit-maximizing level of capital in a centralized competitive market, at a rental cost $q$. Thus the firm's choice of $k_{jt}$ solves

$$\max_{k_{jt}} p_{jt}y_{jt} - w_{jt}n_{jt} - q_t k_{jt},$$

subject to the production function above, and the (inverse) demand schedule $p_{jt} = p(y_{jt}, s_t)$, where $s_t = [k_t, \phi_t]'$ represents the vector of aggregate state variables. The latter will determine in a symmetric equilibrium (s.e.) the industry and/or economy-wide variables (taken as given by the individual firm) which shift the firm's demand schedule, thus justifying their inclusion as an argument of the demand function. The first-order condition, evaluated at a s.e., is

$$q_t = \left(1 - \frac{1}{\xi(s_t)}\right)\exp(\phi_t)F_k(k_t, n_t), \quad t = 0,1,2,\ldots, \tag{2}$$

where

$$\xi_t \equiv \left( \frac{\partial p}{\partial y} \frac{y_t}{p_t} \right)^{-1} = \xi(s_t)$$

is the price-elasticity of demand for the firm's product, which is assumed to depend (at least indirectly) on the current state. Such a formulation allows for price taking behavior (e.g., $p \equiv 1$), as well as a broad class of (symmetric) imperfectly competitive models of the goods market. In what follows, we normalize the price set (or faced) each period by all firms in a symmetric equilibrium to unity (i.e., $p_{jt} = p_t \equiv 1$, all $t$ and $j$). By normalizing the set of firms to have measure one we can conveniently interpret firm-level variables in a symmetric equilibrium as their corresponding aggregates. [3]

## 4. Wage and employment determination

The details on how wage and employment decisions come about are deliberately left unspecified. Instead I postulate a general wage equation:

$$w_t = \lambda_t \left( \frac{-U_n(c_t, n_t)}{U_c(c_t, n_t)} \right), \quad t = 0,1,2,\dots, \tag{3}$$

where $\lambda_t$ represents the wedge between the wage and the marginal rate of substitution between labor and consumption, which I refer to as a *wage markup*. Under perfect competition and continuous clearing of the labor market $\lambda_t = 1$, all $t$ (i.e., the household is always 'on its competitive labor supply schedule'). On the other hand, as long as households (individually or through collusion) enjoy (and exploit) some monopoly power and/or experience quantity constraints (resulting from, say, indivisibilities or rationing) in the labor market, we should expect $\lambda_t \neq 1$.

Employment is (implicity) determined by each firm according to

$$w_t = \varphi_t \left( 1 - \frac{1}{\xi(s_t)} \right) \exp(phi_t) F_n(k_t, n_t) \tag{4}$$

where $\varphi_t$ is an indicator of short-run departures from static profit maximization resulting from adjustment costs, efficient contracts, etc. A particular case is given by the right-to-manage assumption which implies $\varphi_t = 1$, for all $t$.

A model of the labor market (i.e., of wage and employment determination) is a specification of the environment which allows us to determine the symmetric

[3] This will also be the case in the presence of entry and exit, as long as $F$ is homogeneous of degree one. See Galí (1995) for an illustration of that case.

equilibrium values $\lambda_t$ and $\varphi_t$ as a function of the aggregate state variables, i.e., $\lambda_t = \lambda(s_t)$ and $\varphi_t = \varphi(s_t)$.

*Example* (minimum wages).

$$\lambda_t = \max\left( -\overline{w}(s_t)\frac{U_c(c_t, n_t)}{U_n(c_t, n_t)}, 1 \right),$$

where $\overline{w}(s_t)$ is a minimum wage set by the government as a function of the state. Employment could then be chosen to maximize static profits, i.e. according to (4) with $\varphi_t = 1$.

## 5. Equilibrium

We restrict ourselves to economies that, fitting the above framework, have a determinate equilibrium represented by (i) a first-order difference equation for the vector of state variables

$$s_t = h(s_{t-1}, \epsilon_t)$$

where $h(s_{t-1}, \epsilon_t) \equiv [h^k(s_{t-1}), h^{\phi}(\phi_{t-1}, \epsilon_t)]'$, and (ii) a set of equilibrium functions $y_t = y(s_t)$, $c_t = c(s_t)$, $n_t = n(s_t)$, $w_t = w(s_t)$, $R_t = R(s_t)$, $\lambda_t = \lambda(s_t)$, $\varphi_t = \varphi(s_t)$ and $\pi_t = \pi(s_t)$, satisfying $h^k(s_t) = (1-\delta)k_t + y(s_t) - c(s_t)$, and $y(s_t) = \exp(\phi_t)F(k_t, n(s_t))$, and $\pi(s_t) = y(s_t) - w(s_t)n(s_t) - (R(s_t) - 1 + \delta)k_t$, in addition to the Euler equation (1), profit maximizing condition (2), wage and employment equations (3) and (4), and a transversality condition $\lim_{T \to \infty} \beta^T \mathrm{E}_t[U_c(c(s_T), n(s_T))k_T] = 0$.

## 6. Unemployment

We define the *unemployment rate* in period $t$, denoted by $u_t$, as the (log) difference between (i) the level of employment-denoted by $n_t^*$ – that an individual household wishes to supply, *taking as given the equilibrium law of motion for the wage (as well as for the interest rate and profits)* and (ii) the equilibrium level of employment $n_t$. Given the law of motion for $n_t$, determining the equilibrium process for $u$ requires, in a first stage, solving the partial equilibrium dynamic optimization problem faced by the individual wage-taking household. Formally, this involves maximizing $\mathrm{E}_0 \sum_{t=0}^{\infty} \beta^t U(c_t^*, n_t^*)$, subject to

$$k_{t+1}^* = R(s_t)k_t^* + w(s_t)n_t^* + \pi(s_t) - c_t^*,$$

the equilibrium law of motion for the aggregate state variables $s_t = h(s_{t-1}, \epsilon_t)$ introduced above, and the nonnegativity constraints $k_t^* \geq 0$, $n_t^* \geq 0$, $c_t^* \geq 0$, for $t = 0,1,2,\ldots$, where we let variables with a '$*$' superscript denote the choice variables of the wage-taking household. The solution to the previous problem must satisfy

$$\beta \mathrm{E}_t \left[ \left( \frac{U_c(c_{t+1}^*, n_{t+1}^*)}{U_c(c_t^*, n_t^*)} \right) R(s_{t+1}) \right] = 1,$$

$$w(s_t) = \frac{-U_n(c_t^*, n_t^*)}{U_c(c_t^*, n_t^*)}$$

and the transversality condition $\lim_{T\to\infty} \mathrm{E}_t \beta^T U_c(c_T^*, n_T^*) k_T^* = 0$. The solution to that program includes a labor supply function $n_t^* = n^*(k_t^*, s_t)$. Given the definition above, the *unemployment rate u* is given, in equilibrium, by

$$u_t = u(s_t) \equiv \log\left( \frac{n^*(k_t, s_t)}{n(s_t)} \right)$$

where the symmetry assumption $k_t^* = k_t$ is imposed so that $n_t^*$ can be interpreted as the optimal labor supply choice of the representative household, given his current capital holdings. Positive values of $u_t$ can be interpreted as involuntary unemployment, insofar as they point to the inability of individual agents to sell as much labor services as they wish to supply (*given* wages and other prices). Whether unemployment takes place at the intensive or extensive margin (or both) will depend on the interpretation of a household in the model (as discussed above).

## 7. Applications

Though only technology shocks have been considered, the framework above could be easily augmented to allow for other sources of fluctuations, including shocks to tastes, fiscal policy, etc. Given a complete parametric description of the goods and labor market structure, technologies, preferences, etc. the equilibrium of the model can be solved using any of the methods available for non-optimal economies. For given parameter values, a number of model predictions can be generated, including impulse responses, dynamic cross-correlations, etc., which can be contrasted with actual observation. In Galí (1995) I provide an example of a model that fits the framework above, be embedding a 'monopoly union plus right-to-manage' structure in an RBC model with imperfect competition in both goods and labor markets, and entry and exit. In that model we have $\lambda(k_t) = \eta(k_t)/(\eta(k_t) - 1)$, where $\eta(k_t)$ denotes the wage-elasticity of the labor demand schedule facing a monopoly union operating at the firm level, evaluated at a symmetric equilibrium. The wage-elasticity is an increasing function of the price-elasticity in the goods market. The latter is given by a linear function of the number of firms, which is in turn increasing in the aggregate capital stock. As a result, $\eta'(k_t) > 0$. A calibrated version of that model succeeds in replicating, at least qualitatively, stylized labor market facts (a), (b), and (c), in addition to other aspects of business cycles. Among other results, the model points to the presence of countercyclical markups in the *goods* market as a key factor in generating countercyclical unemployment rates. I view that finding as an illustration of how efforts to explain unemployment dynamics by means of full-fledged DSGE models may also help us understand the mechanisms underlying other aspects of economic fluctuations.

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