The Basic New Keynesian Model

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Evidence on Monetary Policy, Output, and Prices:

- Short run effects of monetary policy shocks
  - (i) persistent effects on real variables
  - (ii) slow adjustment of aggregate price level
  - (iii) liquidity effect
- Micro evidence on price and wage-setting behavior: significant rigidities

Failure of Classical Monetary Models

Key ingredients of the NK Model

- monopolistic competition
- nominal rigidities
Estimated Dynamic Response to a Monetary Policy Shock

Source: Christiano, Eichenbaum and Evans (1999)
Micro Evidence on Price Rigidity: An Illustration

Source: Dhyne et al. (JEP 2006)
Introduction

Evidence on Monetary Policy, Output, and Prices:

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  - (i) persistent effects on real variables
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Failure of Classical Monetary Models

Key ingredients of the NK Model

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The Basic New Keynesian Model: Key Blocks

- **Assumptions:**
  - monopolistic competition in the goods market
  - staggered price setting

- **New Keynesian Phillips Curve**
  \[ \pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa_p \tilde{y}_t \]
  
  where \( \pi_t \equiv p_t - p_{t-1} \) and \( \tilde{y}_t \equiv y_t - y^n_t \)

- **Dynamic IS Equation**
  \[ \tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - r^n_t) + E_t \{ \tilde{y}_{t+1} \} \]

- **Monetary Policy Rule**
  \[ i_t = \rho + \phi_{\pi} \pi_t + \phi_y \hat{y}_t + \nu_t \]
  
  where \( \hat{y}_t \equiv y_t - y \)
Representative household solves

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t)
\]

where \( \beta \equiv \frac{1}{1+\rho} \)

\[
C_t \equiv \left[ \int_0^1 C_t(i)^{\frac{1-\frac{1}{\epsilon}}{\epsilon-1}} \, di \right]^{\frac{\epsilon}{\epsilon-1}}
\]

subject to

\[
\int_0^1 P_t(i) C_t(i) \, di + B_t \leq B_{t-1}(1 + i_{t-1}) + W_t N_t + D_t
\]

for \( t = 0, 1, 2, \ldots \) plus solvency constraint.
1. Optimal allocation of expenditures

\[ C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon} C_t \]

where

\[ P_t \equiv \left[ \int_0^1 P_t(i)^{1-\epsilon} di \right]^{\frac{1}{1-\epsilon}} \]

Implication:

\[ \int_0^1 P_t(i) C_t(i) \, di = P_t C_t \]

2. Other optimality conditions

\[ - \frac{U_{n,t}}{U_{c,t}} = \frac{W_t}{P_t} \]

\[ U_{c,t} = \beta(1 + i_t) E_t \left\{ U_{c,t+1} \frac{P_t}{P_{t+1}} \right\} \]
Specification of utility:

\[ U(C_t, N_t; Z_t) = \left( \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t \]

where \( z_t \equiv \log Z_t = \rho_z z_{t-1} + \epsilon_t^Z \)

Log-linearized optimality conditions

\[ w_t - p_t = \sigma c_t + \varphi n_t \equiv mrs_t \]

\[ c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi^p_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_z)z_t \]
Firms

- Continuum of firms, indexed by $i \in [0, 1]$
- Each firm produces a differentiated good
- Identical technology
  
  $$Y_t(i) = A_t N_t(i)$$

  where $a_t \equiv \log A_t = \rho_a a_{t-1} + \varepsilon_t$

- Isoelastic demand $\Rightarrow$ constant desired markup $\mathcal{M} \equiv \frac{\varepsilon}{\varepsilon - 1}$ under flexible prices
- Probability of being able to reset price in any given period: $1 - \theta$, independent across firms (Calvo (1983))
- $\theta \in [0, 1]$: index of price stickiness
- Implied average price duration $\frac{1}{1-\theta}$
The New Keynesian Phillips Curve

- **Price level dynamics**

\[ P_t = \left[ \theta (P_{t-1})^{1-\epsilon} + (1 - \theta) (P_t^*)^{1-\epsilon} \right]^{\frac{1}{1-\epsilon}} \]

Log-linearized version:

\[ p_t = \theta p_{t-1} + (1 - \theta) p_t^* \]

- **Optimal Price Setting**

\[
\max_{P_t^*} \sum_{k=0}^{\infty} \theta^k E_t \left\{ \Lambda_{t,t+k} \left( \frac{1}{P_{t+k}} \right) \left( P_t^* Y_{t+k|t} - C_{t+k} (Y_{t+k|t}) \right) \right\}
\]

subject to

\[ Y_{t+k|t} = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \]

for \( k = 0, 1, 2, \ldots \) where \( \Lambda_{t,t+k} \equiv \beta^k U_{c,t+k} / U_{c,t} \).
Optimal Price Setting (cont.)

Optimality condition:

$$\sum_{k=0}^{\infty} \theta^k E_t \{\Lambda_{t,t+k} Y_{t+k|t} (1/P_{t+k}) (P_t^* - \mathcal{M} \Psi_{t+k})\} = 0$$

Log-linearized version:

$$p_t^* = \mu + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t \{\psi_{t+k}\}$$
The New Keynesian Phillips Curve

- **Inflation and the Markup Gap**

\[ \pi_t = \beta E_t \{ \pi_{t+1} \} - \lambda (\mu_t - \mu) \]

where \( \mu \equiv \log M \), \( \mu_t \equiv p_t - \psi_t \) and \( \lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \)

- **Average Markup**

\[ \mu_t = p_t - (w_t - a_t) \]

- **Labor Market Clearing**

\[ w_t - p_t = \sigma c_t + \varphi n_t \]

\[ n_t = y_t - a_t \]
- **Goods Market Clearing**

\[ y_t = c_t \]

- **Average Markup and the Output Gap**

\[ \mu_t = (1 + \varphi) a_t - (\sigma + \varphi) y_t \]

Under flexible prices:

\[ \mu = (1 + \varphi) a_t - (\sigma + \varphi) y_t^n \]

Combining both:

\[ \mu_t - \mu = - (\sigma + \varphi) \tilde{y}_t \]

- **The New Keynesian Phillips Curve**

\[ \pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t \]

where \( \kappa \equiv \lambda (\sigma + \varphi) \)
Some properties:

(i) Forward-looking

\[ \pi_t = \kappa \sum_{k=0}^{\infty} \beta^k E_t \{ \tilde{y}_{t+k} \} \]

⇒ no role for past inflation

(ii) No tradeoff between output gap and inflation stabilization

⇒ "the Divine Coincidence"

⇒ costless disinflations

(iii) Model-based vs. traditional output gap

\[ \hat{y}_t = y_t - f(t) \]

\[ \tilde{y}_t \equiv y_t - y_t^n \]

⇒ may distort empirical assessments
Euler equation + goods market clearing

\[ y_t = E_t \{ y_{t+1} \} - \frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - \rho) + \frac{1}{\sigma} (1 - \rho z) z_t \]

Combined with \( \tilde{y}_t \equiv y_t - y_t^n \)

\[ \tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - r_t^n) + E_t \{ \tilde{y}_{t+1} \} \]

where

\[ r_t^n \equiv \rho + \sigma E_t \{ \Delta y_{t+1}^n \} + (1 - \rho_z) z_t \]

\[ = \rho - \frac{\sigma (1 + \varphi)(1 - \rho_a)}{\sigma + \varphi} a_t + (1 - \rho_z) z_t \]
Interest rate rule

\[ i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t + \nu_t \]

The role of monetary aggregates

- money demand (ad hoc):

\[ m_t - \rho_t = y_t - \eta i_t \]

- implied money growth:

\[ \Delta m_t = \pi_t + \Delta y_t - \eta \Delta i_t \]
The Basic New Keynesian Model: Key Blocks

- **New Keynesian Phillips Curve**
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- **Dynamic IS Equation**
  \[ \tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1} \} - r_t^n) + E_t \{ \tilde{y}_{t+1} \} \]
  where
  \[ r_t^n = \rho - \frac{\sigma(1 + \varphi)(1 - \rho_a)}{\sigma + \varphi} a_t + (1 - \rho_z) z_t \]

- **Monetary Policy Rule**
  \[ i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + \nu_t \]
  where \( \tilde{y}_t \equiv y_t - y \)
Equilibrium Dynamics

- **System of difference equations:**

  \[
  \begin{bmatrix}
  \tilde{y}_t \\
  \pi_t
  \end{bmatrix} = A_T \begin{bmatrix}
  E_t\{\tilde{y}_{t+1}\} \\
  E_t\{\pi_{t+1}\}
  \end{bmatrix} + B_T \ u_t
  \]

  where

  \[
  u_t \equiv \hat{r}_t^n - \phi_y \tilde{y}_t^n - \nu_t
  \]

  \[
  = - \frac{(1 + \varphi)(\phi_y + \sigma(1 - \rho_a))}{\sigma + \varphi} a_t + (1 - \rho_z)z_t - \nu_t
  \]

  \[
  A_T \equiv \Omega \begin{bmatrix}
  \sigma & 1 - \beta \phi\pi \\
  \sigma \kappa & \kappa + \beta(\sigma + \phi_y)
  \end{bmatrix}; \quad B_T \equiv \Omega \begin{bmatrix}
  1 \\
  \kappa
  \end{bmatrix}
  \]

  with \( \Omega \equiv \frac{1}{\sigma + \phi_y + \kappa \phi\pi} \)

- **Uniqueness condition (Bullard and Mitra):**

  \[
  \kappa(\phi\pi - 1) + (1 - \beta)\phi_y > 0
  \]

- **Analytical solution (method of undetermined coefficients)**
Equilibrium uniqueness under the simple interest rate rule
Calibration (Galí (2015))

Households: $\sigma = 1 ; \varphi = 5 ; \beta = 0.99 ; \epsilon = 9 ; \eta = 4 ; \rho_z = 0.5$

Firms: $\alpha = 1/4 ; \theta = 3/4 ; \rho_a = 0.9$

Policy rules: $\phi_{\pi} = 1.5, \phi_y = 0.125 ; \rho_v = 0.5$

Dynamic Responses to Exogenous Shocks

(i) Monetary policy
(ii) Discount factor
(iii) Technology
Dynamic responses to a monetary policy shock: Interest rate rule
Dynamic responses to a discount rate shock: Interest rate rule
Dynamic responses to a technology shock: Interest rate rule
Estimated Effects of Technology Shocks

Figure 4. Estimated Impulse Responses from a Five-Variable Model: U.S. Data, First-Differenced Hours
(Point Estimates and ±2 Standard Error Confidence Intervals)

Source: Galí (1999)
Estimated Effects of Technology Shocks