Appendix B for "The Lumpy Job Separation Rate"

Regis Barnichon

B1. Wage bargaining

This section presents the general derivation of the wage paid by a representative firm. As is usual in the search literature, workers and the firm bargain individually about the real wage and split the surplus in shares determined by an exogenous bargaining weight γ (see e.g. Krause and Lubik, 2007 and Trigari, 2009).

On the firm's side, the surplus $J(w_t)$ obtained from a marginal worker equals his marginal contribution to profits so

$$\begin{aligned} J(w_t) &= \frac{\partial \left(y_t - w_t n_t\right)}{\partial n_t} + E_t \beta_{t+1}(1-\bar{\rho}-\rho_{t+1})J(w_{t+1}) \\ &= \frac{h_t}{\alpha}\frac{\partial w_t}{\partial h_t} - w_t + E_t \beta_{t+1}(1-\bar{\rho}-\rho_{t+1})J(w_{t+1}) \end{aligned} \tag{1}$$

with w_t the wage, $\beta_{t+1} = \beta\frac{\lambda_{t+1}}{\lambda_t}$ the stochastic discount factor, and λ_t the marginal utility of consumption. Because the firm faces an exogenous demand stream, its revenue is independent of n_t. Therefore, the contribution of the marginal worker to flow profits is given, not by the marginal revenue product of the worker ($\frac{\partial y_t}{\partial n_t} = 0$), but by the marginal reduction in the wage bill ($-\frac{\partial (w_t n_t)}{\partial n_t} = -n_t \frac{\partial w_t}{\partial h_t}\frac{\partial h_t}{\partial n_t} - w_t = \frac{h_t}{\alpha}\frac{\partial w_t}{\partial h_t} - w_t$ since $y_t^d = (1-\bar{\rho}-\rho_t)n_t h_t^{\alpha}$). If the worker walked away from the job, given the impossibility of hiring a replacement immediately, the firm would need to increase the number of hours of (and therefore the wage payments to) all other workers in order to meet its demand.

A vacancy is filled with probability $q(\theta_t)$ and remains open otherwise. With c_t the cost of keeping a vacancy open at date t, the value V_t of posting a vacancy in terms of current consumption is given by

$$V_t = -c_t + E_t\beta_{t+1}[q(\theta_t)\,(1 - JS_{t+1})\,J(w_{t+1}) + (1 - q(\theta_t))V_t] \tag{2}$$

with $c_t = \frac{c}{\lambda_t}$.Note that the firm will post vacancies as long as the value of a vacancy is greater than zero. In equilibrium, $V_t = 0$ so that

$$\frac{c_t}{q(\theta_t)} = E_t\beta_{t+1}(1 - \bar{\rho} - \rho_{t+1})J(w_{t+1}) \tag{3}$$

Turning to the worker's problem, denote $W(w_t)$ and U_t the value of being respectively employed and unemployed in units of consumption goods. The worker's asset value of being matched to the firm is

$$W(w_t) = w_t - \frac{1}{\lambda_t}\frac{\lambda_h}{1+\sigma_h}h_t^{1+\sigma_h} + E_t\beta_{t+1}[(1 - JS_{t+1})W(w_{t+1}) + JS_{t+1}U_{t+1}] \tag{4}$$

and the value of being unemployed U_t is

$$U_t = b_t + E_t\beta_{t+1}\left[\theta_t q(\theta_t)\,(1 - JS_{t+1})\,W(w_{t+1}) + (1 - \theta_t q(\theta_t))U_{t+1} + \theta_t q(\theta_t) JS_{t+1} U(w_{t+1})\right] \tag{5}$$

with $b_t = \frac{b}{\lambda_t}$ the value of home production or unemployment benefits. A worker receives wage payments minus the disutility of labor, and has a probability λ of becoming unemployed next period. When unemployed, a worker receives b_t, has a probability $1 - \theta_t q(\theta_t)$ to remain unemployed and has a probability $\theta_t q(\theta_t)$ to find a job next period (unless its match is destroyed at the beginning of next period).

The equilibrium wage w_t satisfies $w_t = \underset{w_t}{\text{argmax}}\ (W(w_t) - U_t)^\gamma\, J(w_t)^{1-\gamma}$ so that the surplus-sharing rule implies

$$W(w_t) - U_t = \frac{\gamma}{1-\gamma}J(w_t). \tag{6}$$

Denoting the worker's surplus $S_t = W(w_t) - U_t$, I can write

$$\begin{aligned}
S_t &= w_t - \frac{1}{\lambda_t}\frac{\lambda_h}{1+\sigma_h}h_t^{1+\sigma_h} - b_t \\
&\quad + E_t\beta_{t+1}\left(1-\theta_t q(\theta_t)\right)\left(1-JS_{t+1}\right)S_{t+1} \\
&= w_t - \frac{1}{\lambda_t}\frac{\lambda_h}{1+\sigma_h}h_t^{1+\sigma_h} - b_t \\
&\quad + E_t\beta_{t+1}\frac{\gamma}{1-\gamma}\left[1-\theta_t q(\theta_t)(1-\bar{\rho}-\rho_{t+1})J(w_{t+1})\right] \quad \text{using (6)} \\
&= w_t - \frac{1}{\lambda_t}\frac{\lambda_h}{1+\sigma_h}h_t^{1+\sigma_h} - b_t \\
&\quad + \frac{\gamma}{1-\gamma}\frac{c_t}{q(\theta_t)}(1-\theta_t q(\theta_t)) \text{ with (3)} \qquad (7)
\end{aligned}$$

Combining (7) with (6), (1) and (3), the equilibrium wage satisfies

$$\begin{aligned}
&w_t - \tfrac{1}{\lambda_t}\tfrac{\lambda_h}{1+\sigma_h}h_t^{1+\sigma_h} - b_t \\
&\qquad + \tfrac{\gamma}{1-\gamma}\tfrac{c_t}{q(\theta_t)}(1-\theta_t q(\theta_t)) = \tfrac{\gamma}{1-\gamma}\left(-w_t + \tfrac{h_t}{\alpha}\tfrac{\partial w_t}{\partial h_t} + \tfrac{c_t}{q(\theta_t)}\right)
\end{aligned}$$

or after rearranging,

$$w_t = \gamma\left(\frac{h_t}{\alpha}\frac{\partial w_t}{\partial h_t} + c_t\theta_t\right) + (1-\gamma)\left(b_t + \frac{1}{\lambda_t}\frac{\lambda_h}{1+\sigma_h}h_t^{1+\sigma_h}\right). \qquad (8)$$

While the wage equation (8) is a weighted average of both parties surpluses and is similar to other bargained wages derived in e.g. Krause and Lubik (2007) or Trigari (2009), the firm's surplus is *not* given by the marginal product of labor. Indeed, once the firm has chosen its price, it is demand constrained and a marginal worker will not increase the firm's revenue. Instead, the first term of (8) is given by $-\frac{\partial w_t}{\partial n_t} = \frac{h_t}{\alpha}\frac{\partial w_t}{\partial h_t}$, the change in the wage bill caused by substituting the intensive margin (hours and effort) with the extensive one (employment).

A solution to (8) is given by

$$w_t = \gamma c_t\theta_t + (1-\gamma)b_t + (1-\gamma)\varkappa\frac{h_t^{1+\sigma_h}}{\lambda_t} \qquad (9)$$

with $\varkappa = \frac{\lambda_h}{(1+\sigma_h)} \frac{1}{1-\frac{\gamma}{\alpha}(1+\sigma_h)}$.

B.2 Simulation with a higher hours per worker elasticity

In this section, I reproduce the simulation of the main text but allowing for a higher hours per worker elasticity, $\sigma_h = 2$. Table A2 presents the results. While most of the results are little changed, the main differences come from the lower volatility of the job separation rate (reduced by about 50 percent) and higher volatility of hours per worker. Indeed, with a higher hours per worker elasticity, the cost of increasing hours per worker is lower and this tilts the intensive magin-extensive margin tradeoff towards the intensive margin: hours per workers is more volatile and employment is less volatile. The inaction band for the job separation rate is larger as it takes bigger negative shock to generate bursts of job separation. As a result, the contribution of the job separation rate to unemployment fluctuations is lower at 12 percent. With $\sigma_h = 2$, unemployment is slightly less volatile than in the data.

Table B2: Search model with demand constraints, Aggregate Demand shocks, σ_h=2

		u	*v*	*θ*	*jf*	*js*	*h*	*y*
Standard deviation		0.154	0.496	0.621	0.175	0.029	0.009	0.020
Quarterly autocorrelation		0.85	0.60	0.70	0.70	0.20	0.30	0.81
Correlation matrix	*u*	1	-0.78	-0.87	-0.87	0.43	-0.43	-0.95
	v	-	1	0.99	0.99	-0.52	0.89	0.93
	θ	-	-	1	1.00	-0.52	0.81	0.97
	jf	-	-	-	1	-0.52	0.81	0.97
	js	-	-	-	-	1	-0.41	-0.51
	h	-	-	-	-	-	1	0.66
	y	-	-	-	-	-	-	1

Notes: Standard errors -the standard deviation across 500 model simulations over 600 months- are reported in parentheses.