On the Coexistence of Fiat Money and Real Asset Bubbles: A Note

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Consider an overlapping generations model with two period-lived consumers. A consumer born in period t seeks to maximize

$$\log C_{1,t} + \beta \log C_{2,t+1}$$

subject to the budget constraints

$$C_{1,t} + \frac{M_t}{P_t} + Q_t^B = W + U + \frac{T_t}{P_t}$$

$$C_{2,t+1} = 1 - W + \frac{M_t}{P_{t+1}} + B_{t+1}$$

where $C_{1,t}$ and $C_{2,t+1}$ respectively denote consumption when young and old, P_t is the price of the single good, M_t/P_t and Q_t^B denote, respectively, real balances and real bubble holdings at the end of period t, $U \geq 0$ is the value of the new bubbles introduced by each cohort, B_{t+1} is the value in period t of the real bubble purchased in period t, T_t is a monetary injection (in the form of a lump-sum transfer to the young), and W and $1-W$ are, respectively, the endowments of the young and old. There is no uncertainty. For simplicity, I restrict myself to perfect foresight paths.

The consumer's optimality conditions are

$$1 = \beta \left(\frac{C_{1,t}}{C_{2,t+1}} \right) \left(\frac{P_t}{P_{t+1}} \right) \tag{1}$$

$$Q_t^B = \beta \left(\frac{C_{1,t}}{C_{2,t+1}} \right) B_{t+1} \tag{2}$$

The money supply grows at a constant rate k, i.e.

$$\frac{M_t}{M_{t-1}} = 1 + k$$

thus implying transfers $T_t = kM_{t-1}$.

The following conditions must hold in equilibrium

$$C_{1,t} = W - (B_t + m_t) \tag{3}$$

$$C_{2,t} = 1 - W + (B_t + m_t) \tag{4}$$

$$Q_t^B = B_t + U \tag{5}$$

where $m_t \equiv M_{t-1}/P_t$.

I consider three cases in turn: pure monetary bubbles, pure real bubbles and coexistence of monetary and real bubbles.

Case #1: Pure monetary bubbles ($B_t = U = 0$, all t).

Combining equilibrium conditions (1), (3) and (4) above and making use of the identity $m_{t+1} = m_t(1+k)(P_t/P_{t+1})$, we see that an equilibrium path for $\{m_t\}$ must satisfy:

$$\frac{m_t}{W - m_t} = \left(\frac{\beta}{1+k}\right)\left(\frac{m_{t+1}}{1 - W + m_{t+1}}\right)$$

with $0 \leq m_t \leq W$ for all t. Equivalently,

$$m_{t+1} = \frac{(1+k)(1-W)m_t}{\beta W - (1+\beta+k)\, m_t} \equiv H(m_t)$$

A necessary and sufficient condition for existence of bubbly equilibria (i.e. equilibrium paths where $m_t > 0$) is given by $H'(0) < 1$ or equivalently

$$W\left(1 + \frac{\beta}{1+k}\right) > 1 \tag{6}$$

In that case there are two steady states: a bubbleless steady state ($m = 0$) and a bubbly steady state ($m = W - \frac{1+k}{1+k+\beta}$). The bubbly steady state is unstable. There is a continuum of equilibrium paths $\{m_t\}$ such that $m_t < W - \frac{1+k}{1+k+\beta}$ and $\lim_{t\to\infty} m_t = 0$, i.e. they all converge to the bubbleless steady state.

Case #2: Pure real bubbles ($m_t = 0$, all t).

Combining equilibrium conditions (1), (3), (4) and (5), we can derive the following difference equation that any equilibrium path $\{B_t\}$ must satisfy:

$$\frac{B_t + U}{W - B_t} = \beta\left(\frac{B_{t+1}}{1 - W + B_{t+1}}\right)$$

with $0 \leq B_t \leq W$ for all t, for some $U \geq 0$. Equivalently,

$$B_{t+1} = \frac{(1-W)(B_t + U)}{\beta W - (1+\beta)\, B_t - U} \equiv H(B_t, U)$$

A necessary and sufficient condition for existence of bubbly equilibria (i.e. equilibrium paths where $B_t > 0$) is given by $\partial H(0,0)/\partial B_t < 1$ or equivalently

$$W(1+\beta) > 1$$

Consider first the case of $U = 0$. In that case there are two steady states: a bubbleless steady state ($B = 0$) and a bubbly steady state ($B = W - \frac{1}{1+\beta}$) The bubbly steady state is unstable. In addition to the steady states, there is a continuum of bubbly equilibrium paths $\{B_t\}$ such that $B_t \in \left(0, W - \frac{1}{1+\beta}\right)$ for $t = 0, 1, 2...$and $\lim_{t\to\infty} B_t = 0$, i.e. they all converge to the bubbleless steady state.

Let $\overline{U} \equiv \beta + (1+\beta)(1-W) + 2\sqrt{\beta(1+\beta)(1-W)}$. If $U \in (0, \overline{U})$ there are two bubbly steady states: a stable steady state (B^S) and an unstable one (B^U), satisfying $0 < B^S < B^U < W - \frac{1}{1+\beta}$. In addition, there is a continuum of equilibrium paths $\{B_t\}$ such that $B_t \in (0, B^U)$ for $t = 0, 1, 2...$and $\lim_{t\to\infty} B_t = B^S$, i.e. they all converge to the stable bubbly steady state.

Case #3: Coexistence of monetary and real bubbles

Using equilibrium conditions (1), (2), (3), (4) and (5). An equilibrium path for $\{B_t, m_t\}$ such that $B_t > 0$ and $m_t > 0$ for all t must satisfy

$$\frac{m_t}{W - (B_t + m_t)} = \left(\frac{\beta}{1+k}\right)\left(\frac{m_{t+1}}{1 - W + (B_{t+1} + m_{t+1})}\right) \quad (7)$$

$$\frac{m_t}{B_t + U} = \left(\frac{1}{1+k}\right)\left(\frac{m_{t+1}}{B_{t+1}}\right) \quad (8)$$

Hence, any path $\{B_t, m_t\}$ satisfying (7) and (8), $B_t > 0$, $m_t > 0$.and $B_t + m_t \leq W$ for all t and for some $U \geq 0$ constitutes a bubbly equilibrium.in which real and monetary bubbles coexist.

A steady state in which monetary and real bubbles coexist is defined by

$$\Pi = 1 + k$$

$$kB = U$$

$$B + m = W - \frac{1+k}{1+k+\beta}$$

where $B > 0$ and $m > 0$. Note that this requires that $0 \leq k < \frac{W(1+\beta)-1}{1-W}$, i.e. a non-negative money supply growth below a certain bound. Note that condition (6) remains necessary in this case, but it is no longer sufficient.

The case of $k=0$ renders itself to a simple analysis. In that case, a steady state with both real and monetary bubbles requires that $U=0$, i.e. no new real bubbles. Any bubbly equilibrium requires a constant ratio between real balances and the value of real bubble, i.e. $B_t = \lambda m_t$, for some λ. The equilibrium path for $\{m_t\}$ must satisfy

$$\frac{m_t}{W-(1+\lambda)m_t} = \beta\left(\frac{m_{t+1}}{1-W+(1+\lambda)m_{t+1}}\right)$$

or, equivalently,

$$m_{t+1} = \frac{(1-W)m_t}{\beta W-(1+\beta)(1+\lambda)m_t} \equiv H(m_t)$$

A necessary and sufficient condition for the existence of a bubbly equilibrium is given by $H'(0)<1$ or equivalently

$$W(1+\beta)>1$$

In that case there are two steady states: a bubbleless steady state ($B = m = 0$) and a bubbly steady state ($B+m = W-\frac{1}{1+\beta}$) The bubbly steady state is unstable. There is a continuum of bubbly equilibrium paths $\{B_t+m_t\}$ such that $B_t = \lambda m_t$, $B_t+m_t < W-\frac{1}{1+\beta}$ and $\lim_{t\to\infty}\{B_t+m_t\} = 0$, i.e. they all converge to the bubbleless steady state. Note however that parameter λ, which determines the relative weight of the real and monetary components of the aggregate bubble B_t+m_t cannot pinned down by the equilibrium conditions, implying indeterminacy in the *composition* of the aggregate bubble.