Nominal Debt as a Burden on Monetary Policy*

Javier Díaz-Giménez Giorgia Giovannetti Ramon Marimon
Pedro Teles†

This version: March 5, 2006

Abstract

We study the effects of nominal debt on the optimal sequential choice of monetary policy. When the stock of debt is nominal, the incentive to generate unanticipated inflation increases the cost of the outstanding debt even if no unanticipated inflation episodes occur in equilibrium. Without full commitment, to deplete the outstanding stock of debt progressively until these extra costs disappear is part of a sequential optimal policy. Nominal debt is therefore a burden on monetary policy, not only because it must be serviced, but also because it creates a time inconsistency problem that distorts interest rates. The introduction of alternative forms of taxation may lessen this burden, if there is enough commitment to fiscal policy. Full commitment for the fiscal authority can override any commitment problem of the monetary authority.

JEL Classification Numbers: E40, E50, E58, and E60

---

*We would like to thank Jośe-Victor Ríos-Rull, Jaume Ventura, Juan Pablo Nicolini and Isabel Correia for their comments, as well as the participants in seminars and conferences where this work has been presented. Díaz-Giménez, Giovannetti and Marimon gratefully acknowledges the financial support of the Spanish Ministerio de Ciencia y Tecnología (Grants: SEC2002-004318,). Corresponding author: Ramon Marimon; Universitat Pompeu Fabra; Ramon Trias Fargas, 25-27; 08005 Barcelona (Spain); <ramon.marimon@upf.edu>.

†J. Díaz-Giménez: Universidad Carlos III and CAERP; G. Giovannetti: Università di Firenze; R. Marimon: Universitat Pompeu Fabra, CREi, CREA, CEPR and NBER, and P. Teles : Bank of Portugal and CEPR.

1 Introduction

Fiscal discipline has often been seen as a precondition to sustain price stability. Such is, for example, the rationale behind the Growth and Stability Pact in Europe. The policy discussion shows the concern with a time inconsistency problem associated with high levels of nominal debt that could be monetized. In this paper we take a simple model where a benevolent government has an incentive to monetize nominal debt, even if it is costly. We characterize the dynamic time consistent optimal policy when policy choices are made sequentially and we compare it to the optimal policy in an economy where debt is indexed.

We use a cash-in-advance model where agents start the period with predetermined money balances which they use for consumption during the period, as in Svensson (1985). The problem of the government is to finance exogenous government expenditures in the least distortionary manner. In this economy, an increase in the price level will deplete the real value of outstanding money and nominal debt, therefore reducing the need for distortionary taxation. However, this will also induce a decrease in present consumption. As shown by Nicolini (1998), the incentives to inflate, or deflate, depend on preferences and on whether debt is nominal or real.

We consider a version of the model in Nicolini (1998) where, if debt is real, policy is time consistent, while if it is nominal there is always an incentive to inflate. As we show in this paper, this introduces dynamic distortions when optimal policy is chosen sequentially. Therefore, if the government could issue both real and nominal debt, only real debt would be issued. Instead, if there is only nominal debt, these distortions can only be mitigated by reducing the debt. We show that reducing asymptotically the debt is part of an optimal sequential policy.

Related work includes Chari and Kehoe (1999), Ellison and Rankin (2005), and Obstfeld (1997). These last two papers are the closest to ours. Both, however, assume that debt is real, and they focus only on monetary policy. They analyze Markov perfect equilibria when the source of the time inconsistency of monetary policy is related to the depletion of the real value of money balances. Obstfeld (1997) uses a model where money balances are not predetermined and therefore must consider an ad-hoc cost of a surprise inflation. Ellison and Rankin (2005) use the model in Nicolini (1998) with a class of preferences for which the level of real debt matters for the direction of the time inconsistency problem.

There is a related literature on how optimal policies under commitment can be made time consistent by properly managing the portfolio of government assets and liabilities. The closest paper to ours is Persson, Persson and Svensson (2006)[1]. They use the same structure as Nicolini (1998) and assume that the government can use both nominal and real debt and there are no restrictions on debt being positive or negative. A straightforward application of their result to our framework is that policy can be made

[1]See also Alvarez, Kehoe and Neumeyer (2004) and Lucas and Stokey (1983).

time consistent by issuing only real debt.

In this paper, we start by solving for the optimal policy when debt is nominal and there is full commitment. In this case, it is optimal to monetize part of the initial nominal debt. This incentive to deplete the real value of nominal debt is present every period if there is no commitment. We then study optimal policy without commitment, so that policies are decided sequentially. We confine attention to Markov perfect equilibria. We call this equilibrium *recursive* as in Cole and Kehoe (1996), and in Obstfeld (1997). We characterize and compute numerically the equilibrium. The optimal inflation tax is non-stationary, and converges to the inflation tax that obtains when there is no government debt. We show that nominal debt is indeed a burden for monetary policy not only because it has to be serviced, but also because of the dynamic distortion associated with the time inconsistency. The full characterization and computation of the optimal policy in a recursive equilibrium with a state variable is an additional contribution of this paper[2].

Having characterized the recursive monetary equilibrium with nominal debt, then, as a benchmark, we characterize the optimal policy that obtains when debt is indexed. The solution, being stationary, is the same with and without commitment. Next, we compare the welfare properties of the equilibria in the different economies. We discuss how to make these comparisons meaningful, taking into account that in the nominal economies the initial conditions are in nominal terms and that in the indexed economy the initial condition for debt is in real terms. When the initial real liabilities are the same the economy with indexed debt has the highest welfare.

Finally, we test for the robustness of the results to the introduction of additional taxes. This is important since, in advanced economies, seigniorage is a minor source of tax revenues, and we want to know if our results still hold when government outlays are financed with other taxes. Specifically, we study the case of consumption taxes. First, we impose the natural assumption that taxes are chosen before the monetary policy decisions are made. In particular they are assumed to be chosen one period in advance. We find that the same equilibria result when there are both seigniorage and consumption taxes than when there is only seigniorage, provided that the optimal monetary policy distortions can be supported with strictly positive nominal interest rates.

When, instead, there is enough fiscal commitment, the fiscal authority can constrain the monetary authority to follow the Friedman rule, of zero nominal rates, from the outset. In this case, since negative interest rates cannot be sustained in equilibrium, the monetary authority has no incentive to monetize the debt and, as a result, it implements the optimal equilibrium with commitment.[3]

[2]In this respect, our work is closely related to the recent work of Krusell, Martín and Ríos-Rull (2003) who characterize the recursive equilibria that obtain in an optimal labor taxation problem.

[3]See also Marimon, Nicolini and Teles, 2003.

2 The model economy

In our model economy there is a representative household and a government. In each period $t \geq 0$, the government issues currency M^g_{t+1} and nominal debt B^g_{t+1}, to finance an exogenous and constant level of public consumption g.[4] Initially, we abstract from all other sources of public revenues. The sequence of government budget constraints is the following:

$$M^g_{t+1} + B^g_{t+1} \geq M^g_t + B^g_t(1 + i_t) + p_t g, \ t \geq 0 \tag{1}$$

where i_t is the nominal interest rate paid on debt issued by the government at time $t-1$, and p_t is the price of one unit of the date t composite good in units of money. The initial stock of currency, M^g_0, and initial debt liabilities, $B^g_0(1 + i_0)$, are given. A government policy is, therefore, a specification of $\{M^g_{t+1}, B^g_{t+1}, g\}$ for $t \geq 0$.

We assume that the household's preferences over consumption and labor can be represented by the following utility function:

$$\sum_{t=0}^{\infty} \beta^t [u(c_t) - \alpha n_t] \tag{2}$$

where $c_t > 0$ denotes consumption at time t, n_t denotes labor at time t, and $0 < \beta < 1$ is the time discount factor. We assume that the utility of consumption satisfies the standard assumptions of being strictly increasing and strictly concave. In most of this article we assume that the utility is logarithmic in consumption, i.e., $u(c) = \log(c)$.

We assume that consumption in period t must be purchased using currency carried over from period $t-1$ as in Svensson (1985). This timing of the cash-in-advance constraint implies that the representative household takes both M_0 and $B_0(1 + i_0)$ as given when solving its maximization problem, and it is crucial to obtain the results that we report here. The specific form of the cash-in-advance constraint faced by the representative household is:

$$p_t c_t \leq M_t \tag{3}$$

for every $t \geq 0$.

To simplify the production side of this economy, we assume that labor can be transformed into either the private consumption good or the public consumption good on a one-to-one basis. The economy's resource constraint is:

$$c_t + g \leq n_t \tag{4}$$

[4] We assume that government expenditures, g, are given, although our analysis can easily be extended to the case of endogenous government expenditures.

for every $t \geq 0$.

Each period the representative household faces the following budget constraint:

$$M_{t+1} + B_{t+1} \leq M_t - p_t c_t + B_t(1 + i_t) + p_t n_t \tag{5}$$

where M_{t+1} and B_{t+1} denote, respectively, the stock of money and the stock of nominal government debt that the household carries over from period t to period $t+1$. Finally, we assume that the representative household faces a no-Ponzi games condition:

$$\lim_{T \longrightarrow \infty} \beta^T \frac{B_{T+1}}{p_T} \geq 0 \tag{6}$$

2.1 A competitive equilibrium

Definition 1 *A competitive equilibrium for an economy with nominal debt is a government policy,* $\{M^g_{t+1},\ B^g_{t+1}, g, \}_{t=0}^{\infty}$, *an allocation* $\{M_{t+1}, B_{t+1}, c_t, n_t\}_{t=0}^{\infty}$, *and a price vector,* $\{p_t, i_{t+1}\}_{t=0}^{\infty}$, *such that:*

(i) given M^g_0 *and* $B^g_0(1+i_0)$, *the government policy and the price vector satisfy the government budget constraints described in expression (1);*

(ii) when households take M_0, $\mathrm{B}_0(1+\mathrm{i}_0)$ *and the price vector as given, the allocation maximizes problem (2), subject to the cash-in-advance constraints (3), the household budget constraints (5), and the no-Ponzi games condition (6); and*

(iii) all markets clear, that is: $M^g_{t+1} = M_{t+1}$, $B^g_{t+1} = B_{t+1}$, *and* g *and* $\{c_t, n_t\}_{t=0}^{\infty}$ *satisfy the economy's resource constraint (4), for every* $t \geq 0$.

Given our assumptions on the utility of consumption u, it is straightforward to show that the competitive equilibrium allocation of this economy satisfies both the economy's resource constraint (4) and the household's budget constraint (5) with equality, and that the first order conditions of the Lagrangian of the household's problem are both necessary and sufficient to characterize the solution to the household's problem. The cash in advance constraint, (3), will be binding in every period $t \geq 0$ if $\frac{u'(c_t)}{\alpha} > 1$. Since $\frac{u'(c_{t+1})}{\alpha} = 1 + i_{t+1}$, $t \geq 0$, this will be the case whenever $i_{t+1} > 0$. In period zero the cash in advance constraint will be binding whenever $c_0 \leq c_{t+1}$, $t \geq 0$. This will be a feature of the equilibria that we characterize.

The competitive equilibrium allocation of an economy with nominal debt can be completely characterized by the following conditions that must hold for every $t \geq 0$:

$$\frac{u'(c_{t+1})}{\alpha} = 1 + i_{t+1}, \tag{7}$$

$$1+i_{t+1}=\beta^{-1}\frac{p_{t+1}}{p_t}, \tag{8}$$

$$c_t=\frac{M_t}{p_t}, \tag{9}$$

the government budget constraints (1), the resource constraints (4), and the transversality condition (6)

$$\lim_{T\longrightarrow\infty}\beta^T\left(\frac{M_{T+1}+B_{T+1}}{p_T}\right)=0 \tag{10}$$

2.2 Implementability

When choosing its policy the government takes into account the above equilibrium conditions. These conditions can be summarized with implementability conditions in terms of the real allocations. In particular, the government budget constraint (1) with equality can be written as the implementability condition

$$c_{t+1}u'(c_{t+1})\frac{\beta}{\alpha}+\beta z_{t+1}c_{t+1}=c_t+z_tc_t+g,\ \ t\geq 0 \tag{11}$$

where

$$z_t\equiv\frac{B_t^g(1+i_t)}{M_t^g}$$

To see this, notice that the constraint (1) with equality can be written in real terms as

$$\frac{M_{t+1}^g}{p_t}+\frac{B_{t+1}^g}{p_t}=\frac{M_t^g}{p_t}+\frac{B_t^g(1+i_t)}{p_t}+g$$

and, using the first order conditions of the households problem, (7), (8) and (9), $\frac{M_t^g}{p_t}=c_t$; $\frac{M_{t+1}^g}{p_t}=\frac{M_{t+1}^g}{p_{t+1}}\frac{p_{t+1}}{p_t}=c_{t+1}\beta(1+i_{t+1})=c_{t+1}u'(c_{t+1})\frac{\beta}{\alpha}$; $\frac{B_t^g(1+i_t)}{p_t}=\frac{B_t^g(1+i_t)}{M_t^g}\frac{M_t^g}{p_t}=z_tc_t$, and $\frac{B_{t+1}^g}{p_t}=\frac{B_{t+1}^g(1+i_{t+1})}{M_{t+1}^g}\frac{M_{t+1}^g/p_{t+1}}{p_t(1+i_{t+1})/p_{t+1}}=\beta z_{t+1}c_{t+1}$.

From the transversality condition (10) we have that $\lim_{T\longrightarrow\infty}\beta^T(c_{T+1}u'(c_{T+1})\frac{\beta}{\alpha}+$ $+\beta z_{T+1}c_{T+1})=0$, which implies that the present value government budget constraint takes the form

$$\sum_{t=0}^{\infty}\beta^t\left(c_{t+1}u'(c_{t+1})\frac{\beta}{\alpha}-(c_t+g)\right)=z_0c_0 \tag{12}$$

This condition summarizes the competitive equilibrium restrictions on the sequence of consumption $\{c_t\}_{t=0}^{\infty}$.

3 Optimal policy with nominal debt

3.1 The case of full commitment

We start the analysis by studying the optimal monetary policy that obtains when the stock of government debt is nominal and the government can fully commit. When debt is not indexed a higher initial price level depletes the real value of both money balances and debt, which is a seigniorage tax at $t = 0$ that can be levied without affecting the commitment to future interest rates. The fact that consumption must be purchased with currency carried over from the previous period means that there is a cost in reducing the real value of money balances. Even if this cost is present, there may be an incentive for a Ramsey government to increase the initial seigniorage tax at $t = 0$, and to use its proceeds to reduce the need for future distortionary taxation.

Definition 2 *A **full commitment Ramsey equilibrium** with nominal debt is a competitive equilibrium such that $\{c_t\}$ solves the following problem:*

$$Max \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \alpha(c_t + g)\right] \tag{13}$$

subject to the implementability condition (12).

The other competitive equilibrium variables which are the government policy $\{M^g_{t+1}, B^g_{t+1}, g, \}_{t=0}^{\infty}$, the allocation $\{M_{t+1}, B_{t+1}, n_t\}_{t=0}^{\infty}$, and the price vector, $\{p_t, i_{t+1}\}_{t=0}^{\infty}$, are obtained using the competitive equilibrium conditions.

In the log case where $u(c_t) = \ln(c_t)$, 12 reduces to

$$\sum_{t=0}^{\infty} \beta^t \left(\frac{\beta}{\alpha} - g - c_t\right) = z_0 c_0 \tag{14}$$

and the full commitment Ramsey equilibrium, denoted by superscript F, is characterized by the following conditions:

$$\frac{\frac{1}{c_0^F} - \alpha}{1 + z_0} = \frac{1}{c_1^F} - \alpha \tag{15}$$

$$c_{t+1} = c_1^F, \text{ for } t \geq 1. \tag{16}$$

Notice that, as long as $z_0 \equiv \frac{B_0(1+i_0)}{M_0} > 0$, c_0^F is smaller than c_1^F. The initial outstanding nominal debt is depleted at the expense of a lower real stock of money, and, therefore, lower consumption. In other words the planner finds it optimal for consumption to be lower in the initial period since that way it is able, by depleting the real value of initial outstanding nominal debt, to raise less distortionary inflation tax revenues in the future.

3.2 Recursive monetary equilibria

When the government cannot commit to its monetary policy, the incentive to monetize part of the debt discussed in the previous section arises every period. In other words, the special features of 'period zero' in the full commitment economy with nominal debt become the recurrent features of the economy without commitment.

In each period $t \geq 0$, the government chooses c_t and is not able to commit. We look at Markov perfect equilibria where c_t does not depend on the whole history up to period t but may only depend on the pay-off relevant state variable z_t. That is, the government policy is a function C such that the consumption path is sequentially defined by $c_t = C(z_t)$.

Definition 3 *A* ***recursive monetary equilibrium*** *with nominal debt is a value function V(z) and policy functions C(z) and Z(z) such that $c = C(z)$ and $z' = Z(z)$ solve*

$$V(z) = \max_{\{c,z'\}}\{u(c) - \alpha(c+g) + \beta V(z')\} \tag{17}$$

subject to

$$C(z')u'(C(z'))\frac{\beta}{\alpha} + \beta z' C(z') = zc + c + g \tag{18}$$

The recursive equilibrium can be obtained by solving the following dynamic programming problem

$$V(z) = \max_{\{c,z'\}}\{u(c) - \alpha(c+g) + \beta V(z')\} \tag{19}$$

s.t.

$$\overline{C}(z')u'(\overline{C}(z')) + \beta z'\overline{C}(z') = c(1+z) + g \tag{20}$$

for an arbitrary function $\overline{C}$. As a solution to this problem there is an optimal policy $\overline{\overline{C}} = \mathcal{F}(\overline{C})$, where $\mathcal{F}$ maps the –exogenously given– functions into optimal policy functions. A recursive equilibrium is then a fixed point $C = \mathcal{F}(C)$.

In the log case, this simplifies to finding a function C that is the optimal policy to the problem

$$V(z) = \max\{\log(c) - \alpha(c+g) + \beta V(z')\} \tag{21}$$

s.t.

$$\frac{\beta}{\alpha} + \beta z' C(z') = c(1+z) + g \tag{22}$$

To characterize the recursive monetary equilibrium, notice that the first order conditions of the problem described in expressions (21) and (22) are

$$\frac{1}{c} - \alpha = \lambda(1+z) \tag{23}$$

and

$$V'(z') = -\lambda C(z')[1 + \varepsilon_c(z')] \tag{24}$$

$\varepsilon_c(z) = \frac{zC'(z)}{C(z)}$ is the elasticity of $C(z)$. As we will compute below this elasticity is negative and less than one in absolute value. (23) and (24) imply

$$\frac{\frac{1}{c} - \alpha}{1+z} = -\frac{V'(z')}{C(z')}[1 + \varepsilon_c(z')]^{-1} \tag{25}$$

This condition equates the 'reduced' marginal gain of one additional unit of consumption to its 'amplified' marginal cost associated with higher debt needed to finance this consumption. The gain is 'reduced', by the factor $1 + z$, since increasing consumption, by decreasing the current price level, increases the real value of the outstanding stock of debt. The cost of the debt is 'amplified' since increasing the stock of debt makes future consumption more costly by facing the trade-off of monetizing a higher stock of debt. Notice that the first distortion ('reduction') is a price level effect, already present in the period zero of the Ramsey equilibrium (15), while the second distortion ('amplification') imbeds an interest rate effect, not present in the Ramsey equilibrium, reflecting the intertemporal distortions that appear along a recursive equilibrium path when nominal debts can be sequentially monetized.

Using the envelope condition,

$$V'(z) = -\lambda c$$

for $\frac{V'(z')}{c'}$, equation (25), translates into the following intertemporal condition

$$\frac{\frac{1}{c} - \alpha}{1+z} = \frac{\frac{1}{c'} - \alpha}{1+z'}[1 + \varepsilon_c(z')]^{-1} \tag{26}$$

which can be rewritten as

$$\frac{\frac{1}{c} - \alpha}{\left[1 + \frac{(1+i)B}{M}\right]} = \frac{\frac{1}{c'} - \alpha}{\left[1 + \frac{(1+i')B'}{M'}\right]}\left[1 + \varepsilon_c\left(z'\right)\right]^{-1} \tag{27}$$

This intertemporal equation reflects the different distortions present as a result of debt being nominal and policy decisions being sequential. The term $\left[1+\frac{(1+i)B}{M}\right]$ results from the discretionary incentive to reduce the real value of nominal debt. It is present in the problem with commitment only at time 0 (equation (15)). Higher consumption today means that the price level will have to be lower, which in turn means that the real value of outstanding nominal debt will be higher, and therefore future distortionary taxation will also be higher. Hence, the benefits for the benevolent government of higher consumption today are lower. The term $\left[1+\varepsilon_c\left(\frac{(1+i')B'}{M'}\right)\right]$ results from the dynamic nature of this problem. Higher consumption today, and a lower price today, also means that future debt will higher, and that will have a cost because it will exacerbate the time inconsistency problem of future decisions[5].

Neither the discretionary incentive to reduce the real value of debt, nor the dynamic effect on these incentives associated with the accumulation of debt are there if debt is indexed, as will be shown below.

While the equilibrium with full commitment, or with indexed debt, can be easily solved analytically, we recur to numerical solutions for the recursive equilibrium. Before showing the results we analyze the case with indexed debt.

3.3 The benchmark: Indexed debt

The case of indexed debt is the benchmark against which we compare the optimal policy that obtains when the stock of government debt is nominal —that is, not indexed— which is the main focus of this article.

In the economies analyzed in the previous two sections, the real value of the outstanding debt, b_t, is given by $b_t \equiv \frac{B_t(1+i_t)}{p_t} = \frac{B_t(1+i_t)}{M_t}\frac{M_t}{p_t} = z_t c_t$, where z_t is predetermined. In the economy where debt is indexed to the price level, b_t is predetermined. The nominal interest rate adjusts to movements in the price level, as to keep the real interest rate unchanged at $\beta^{-1}-1$, which is equivalent to z_t adjusting to movements in c_t, as to keep b_t unchanged.

A competitive equilibrium for an economy with indexed debt is defined as a government policy, $\{M^g_{t+1}, b^g_{t+1}, g,\}_{t=0}^{\infty}$, an allocation $\{M_{t+1}, b_{t+1}, c_t, n_t\}_{t=0}^{\infty}$, and a price vector, $\{p_t, i_{t+1}\}_{t=0}^{\infty}$, such that the conditions (i), (ii) and (iii) of Definition 1 are satisfied when nominal liabilities are replaced by real liabilities, according to $\frac{B_t(1+i_t)}{p_t} = b_t$, where b_t is predetermined.

The design of the government optimal policy consists in solving the problem:

$$\max_{\{c_t,n_t\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t[u(c_t) - \alpha(c_t+g)]$$

[5] Myopic governments that would not take into account the effect of their choices on the state variables of future government decisions would be solving a problem where only the first term would be present.

$$c_{t+1}u'(c_{t+1})\frac{\beta}{\alpha}+\beta b_{t+1}=c_t+g+b_t,\quad t\geq 0$$

This problem is recursive when $u(c)=\ln(c)$. In this case, as Nicolini (1998) has shown, the optimal monetary policy is time-consistent. This follows from the fact that the price elasticity is unitary and the implicit elasticity of the cash-in-advance constraint is also one. The problem can be written recursively as follows:

$$V(b)=\max_{c,b'}\{\log(c)-\alpha(c+g)+\beta V(b')\} \tag{28}$$

subject to:

$$\frac{\beta}{\alpha}+\beta b'=c+g+b \tag{29}$$

The first order condition for c is:

$$\frac{1}{c}-\alpha=-\beta V'(b') \tag{30}$$

That is, the marginal gain of increasing consumption is equated to the marginal cost of increasing future debt, without the price distortions present in (25). Using the envelope theorem, we have

$$V'(b)=V'(b'), \tag{31}$$

and, substituting expression (30) into this expression, we obtain that

$$\frac{1}{c}-\alpha=\frac{1}{c'}-\alpha \tag{32}$$

which implies that the optimal level of consumption, c^I(where superscript I stands for indexed) is constant and equal to:

$$c^I=\frac{\beta}{\alpha}-g-(1-\beta)\,b_0. \tag{33}$$

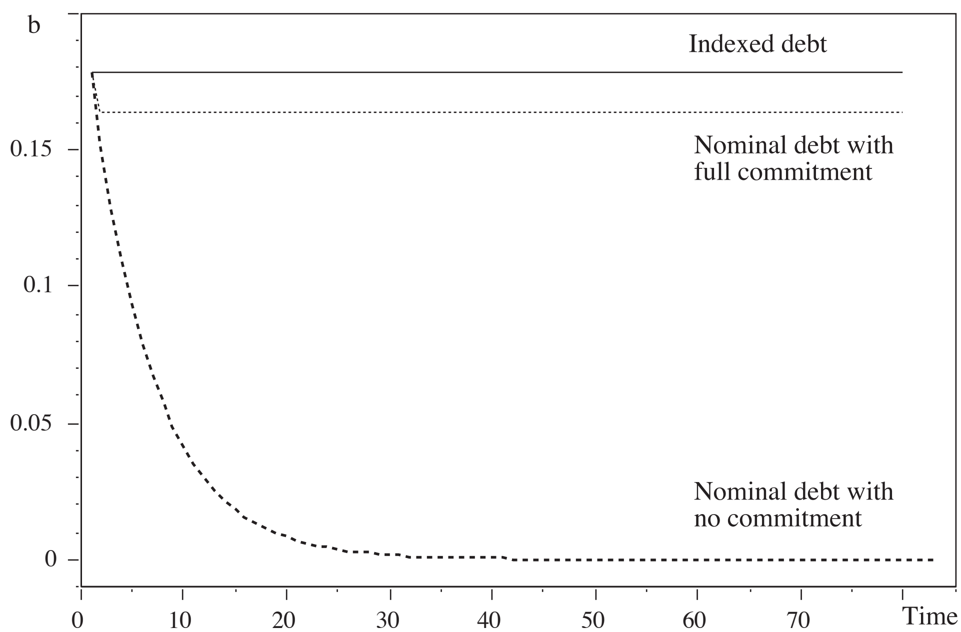
Notice that expression (31) implies that the real value of the government debt is stationary and, consequently, that $b^I=b_0$.

3.4 Computing a recursive equilibrium

Now we compute a recursive equilibrium and compare it to the equilibria when there is full commitment and when debt is indexed. Our model economies are characterized by three parameters: α, β and g. To make them comparable, we follow the strategy of equating the initial values of real debt, b_0. The equilibrium in the economy with indexed debt is computed directly using expression 33. In the economy with nominal debt and full commitment the equilibrium is computed solving a system of two equations,(34) and (35), in two unknowns, c_0^F and c_1^F. To find the initial condition z_0^F, we define $b_F(z) \equiv zc_0^F(z)$. This function is invertible and we use it to obtain $z_0^F = b_F^{-1}(b_0)$.

The algorithm that we use to compute the recursive competitive equilibrium is described in the Appendix. The output of this algorithm is the policy function $C(z)$ that we use to define $b_N(z) \equiv zC(z)$. This function is invertible and we use it to obtain the new initial condition $z_0^N = b_N^{-1}(b_0)$. Since $c_0^F(z_0) > c_0^N(z_0)$ for any z_0, it turns out that the implied $z_0^F < z_0^N$.

Figure 1: The optimal stocks of indexed debt and of nominal debt with full commitment and with no commitment



The values that we use in our computational exercise to identify the parameters of the model economies are $\alpha = 4.45$, $\beta = 0.98$, and $g = 0.00822$. To compute the time paths we choose the initial condition $b_0 = 0.17865$. Since our period corresponds to a year, this value of b_0 is intentionally chosen to be very high. Specifically, the value of the debt to government expenditures ratio is $b_0/g \simeq 22$. The results that we obtain for the time paths of real debts is reported in Figure 1.

Figure 1 shows that in the economies with indexed debt it is to optimal to keep the value of debt stationary, and that in the economies with nominal debt it is optimal to reduce its real value. Under full commitment this reduction is carried out only in the first period, while under no commitment, the real value of debt is depleted progressively until, asymptotically, it is fully depleted. Recall that in these economies the time paths of real debt are given by $b_t^F = z_t^F c_t^F$ and by $b_t^N = z_t^N c_t^N$.

We also find that the long-run nominal interest rate that obtains when debt is indexed is higher than the nominal interest rates that obtain when debt is nominal. Moreover, the long-run interest rate under full commitment is higher than the one that obtains than under no commitment. Notice that in the three economies the nominal interest rates in period $t = 0$ satisfy the condition $1 + i_0 \equiv 1/(\alpha c_0)$. These are economies in which the realized real return in period $t = 0$ is assumed to be $1 + r_0 = \beta^{-1}$. This is equivalent to choosing $B_0 = \frac{z_0 M_0}{1+i_0}$.

3.5 Welfare comparisons

We first compare two economies with identical initial nominal liabilities, M_0 and $B_0(1+i_0)$. The recursive equilibrium is less efficient than the full commitment Ramsey equilibrium. The full commitment Ramsey solution is the choice of a sequence of consumption $\{c_t\}_{t=0}^{\infty}$ that maximizes welfare in the set of competitive equilibrium sequences defined by (14). Since the utility function is strictly concave and the set of restrictions is convex, there is a single maximum. The recursive equilibrium is a competitive equilibrium, and therefore, being different, must give strictly lower welfare. More formally,

Proposition 1 *Assume that $u(c) = \log(c)$. Consider two identical economies with the same initial conditions, $B_0(1 + i_0) > 0$ and M_0. The full commitment Ramsey equilibrium gives higher welfare than the no-commitment Recursive equilibrium.*

The result is clear when we think of the recursive equilibrium as the equilibrium of a game between successive governments. The recursive equilibrium, imposes as additional restrictions the optimality of decisions by future governments.

We now extend the comparison to economies with indexed debt. Indexed debt can be viewed as an extreme form of commitment. We want to compare the full commitment Ramsey equilibrium allocation in the economy with nominal debt with the equilibrium allocation that obtains in a comparable economy when debt is indexed. Notice that in order for this comparison to be meaningful, we must choose the appropriate initial conditions. To do this, we require that the initial money holdings, M_0, and the real value of the initial debt liabilities in both economies is the same. This is not completely straightforward because the initial real liabilities, b_0, are exogenous in the indexed debt economy, but endogenous in the nominal debt economies.

If we start with the nominal economy with full commitment, with initial condition for nominal debt $B_0(1 + i_0)$, then the real value of the initial nominal liabilities is

$b_0 = z_0 c_0^F(z_0) = \frac{B_0(1+i_0)}{p_0^F}$.

Proposition 2 *Assume that $u(c) = \log(c)$. Consider two economies with initial money stock M_0. One of them has initial nominal debt $B_0(1+i_0) > 0$, and the other has initial indexed debt b_0. If $b_0 = \frac{B_0(1+i_0)}{p_0^F}$, then the welfare in the economy with indexed debt is higher than in the economy with nominal debt and full commitment.*

Proof: With indexed debt we have that consumption is constant over time $c^I = \frac{\beta}{\alpha} - g - (1-\beta)\, b_0$. With nominal debt and full commitment, using the implementability condition (14), we have

$$(1-\beta)\, c_0^F + \beta c_1^F = \frac{\beta}{\alpha} - g - (1-\beta)\, z_0 c_0^F \tag{34}$$

where

$$\frac{1}{c_0^F} - \alpha = \left[\frac{1}{c_1^F} - \alpha\right][1 + z_0] \tag{35}$$

If $b_0 = z_0 c_0^F$, then, notice that $c^I = (1-\beta)\, c_0^F + \beta c_1^F$. Since the utility function is strictly concave, by Jensen's inequality, welfare will be higher in the case with indexed debt.■

In the economy with nominal debt it is optimal to reduce the initial level of consumption because monetizing part of this debt creates no distortions, since there is no time zero indexation to internalize, and because it reduces the amount of future seigniorage. By imposing that $b_0 = \frac{B_0(1+i_0)}{p_0^F}$, we restrict our attention to the distortion introduced by the incentive to monetize the nominal debt, and we abstract from the gain introduced by the reduction in its real value.

4 Additional taxes

In most advanced economies, seigniorage is a minor source of revenue, and government liabilities are financed mostly through consumption and income taxes. In this section we show two basic results, regarding the introduction of taxes in our economy. First, we show that the introduction of taxes, while reducing the need to raise revenues through seigniorage, may not change the characterization of equilibria with respect to the economies with monetary policy only, analyzed in the previous sections. This is the case if the fiscal authority simply sets taxes one period in advance and, subsequently, the monetary authority sets its policy. This adds realism to the analysis of the interaction of fiscal and monetary policies. Second, we show that, in contrast, if there is full commitment on the part of the fiscal authority that makes its policy choices in the initial period, before the monetary authority does, then the full commitment outcome can be achieved even if there is no commitment on the part of the monetary authority. We show that it is part of such policy to finance all the outstanding government liabilities with the consumption tax, and to constrain the monetary authority to implement a zero nominal interest rate.

We show these results introducing consumption taxes, $\{\tau_t\}_{t=0}^{\infty}$. The analysis easily generalizes to the introduction of other taxes. However, it is not the purpose of the paper to provide a complete characterization of all possible fiscal instruments, neither to consider different games between fiscal and monetary authorities.

4.1 The model economy with consumption taxes

When the government levies consumption taxes (τ), the household problem becomes:

$$\max \sum_{t=0}^{\infty} \beta^t \left[u(c_t) - \alpha n_t\right] \tag{36}$$

subject to:

$$p_t(1+\tau_t)c_t \leq M_t \tag{37}$$

$$M_{t+1} + B_{t+1} \leq M_t - p_t(1+\tau_t)c_t + B_t(1+i_t) + p_t n_t \tag{38}$$

and to:

$$\lim_{T \longrightarrow \infty} \beta^T \frac{B_{T+1}}{p_T} \geq 0 \tag{39}$$

Now, the marginal conditions (7), (8) and (9) characterizing the households's optimal choice become:

$$\frac{u'(c_{t+1})}{\alpha} = (1+i_{t+1})(1+\tau_{t+1}) \tag{40}$$

$$1 + i_{t+1} = \beta^{-1}\frac{p_{t+1}}{p_t} \tag{41}$$

and

$$c_t = \frac{M_t}{p_t(1+\tau_t)} \tag{42}$$

These conditions must hold for every $t \geq 0$. Notice that (40) reflects the fact that the household makes plans based on expectations about both interest rates and taxes. The intertemporal condition (41) is exactly the same as expression (8) while the cash-in-advance constraint (42) now includes consumption taxes.

The sequence of government budget constraints in this economy is now given by:

$$p_t g + M_t^g + B_t^g(1+i_t) \leq p_t \tau_t c_t + M_{t+1}^g + B_{t+1}^g \tag{43}$$

while the feasibility conditions (4) do not change. The implementability conditions (11)can be written as

$$c_{t+1}u'(c_{t+1})\frac{\beta}{\alpha} + \beta z_{t+1}c_{t+1}(1+\tau_{t+1}) = c_t + z_t c_t(1+\tau_t) + g \tag{44}$$

together with the terminal condition

$$\lim_{T\longrightarrow\infty} \beta^T c_{T+1}u'(c_{T+1})\frac{\beta}{\alpha} + \beta z_{T+1}c_{T+1}(1+\tau_{T+1})) = 0 \tag{45}$$

4.2 Optimal monetary policy when the fiscal authority moves one period in advance

We now consider the case where tax decisions for some period t must be made one period in advance, and may depend only on the state at $t-1$. In this case we can define the new state variable $\widehat{z}_t \equiv z_t(1+\tau_t)$, and the problems are isomorphic to the problems in the previous sections, since the implementability condition (44) reduces to

$$c_{t+1}u'(c_{t+1})\frac{\beta}{\alpha} + \beta\widehat{z}_{t+1}c_{t+1} = c_t + \widehat{z}_t c_t + g \tag{46}$$

which is formally identical to (11).

There is an additional restriction that the nominal interest rate must be nonnegative. This constraint was satisfied when seigniorage was the only source of revenue, but it is not necessarily satisfied in this case.

We can model the interaction between the monetary and the fiscal authority in two ways. One way is to assume that they jointly decide as a single authority. In this case the analysis goes through as before and we obtain the same results in terms of allocations. The alternative is to assume some given strategy for taxes as a function of the state. As long as the nominal interest rates are away from the lower bound of zero nominal interest rates, the problem for the monetary authority has the same structure as before, and therefore the same results go through, even if part of the government liabilities are financed with taxes.

In summary, the monetary authority faces the same problem with consumption taxes than with only seigniorage, for any degree of monetary commitment. Therefore, the allocations for the various types of debt and monetary policy commitment technologies are exactly the same as those that obtained before. This result is established in the following subsections:

Consumption taxes, nominal debt and full commitment to monetary policy. In this case we obtain the Ramsey equilibrium allocation, characterized by

$$\frac{1}{c_0^F} - \alpha = \left[\frac{1}{c_1^F} - \alpha\right] \left[1 + z_0 \left(1 + \tau_0\right)\right] \tag{47}$$

$$c_{t+1} = c_1^F, \text{ for } t \geq 1. \tag{48}$$

where τ_0 is predetermined.

Consumption taxes, nominal debt and no commitment to monetary policy. In this case the recursive equilibrium allocation must satisfy the following intertemporal condition

$$\frac{\frac{1}{c} - \alpha}{\left[1 + \frac{(1+i)B}{M}(1+\tau)\right]} = \frac{\frac{1}{c'} - \alpha}{\left[1 + \frac{(1+i')B'}{M'}(1+\tau')\right]} \left[1 + \epsilon_c(\widehat{z}')\right]^{-1} \tag{49}$$

It follows that in the economy with nominal debt and no commitment to monetary policy, the path of depletion of the stock of debt in real terms coincides with the one characterized in the economy without taxes.

Consumption taxes and indexed debt. In this case policies are stationary and we obtain the stationary equilibrium allocation $c^I = \frac{\beta}{\alpha} - g - (1-\beta)\,\widehat{b}_0$ where $\widehat{b}_0 = b_0\,(1+\tau_0)$.

4.3 Optimal fiscal policy with commitment

In the three regimes discussed above we do not determine exactly how the equilibrium allocations are supported since the household only cares about the effective nominal rate of return, $(1+i)(1+\tau)$. For instance, it is always possible to set taxes in a way that the resulting monetary policy follows the Friedman rule of zero nominal interest rates, even though in our economy there is no efficiency gain from following such a rule.[6]

[6]This may not be true in a more general model economy. For instance, this is not true if we introduce a distinction between cash and credit goods. In this case, the Friedman rule would eliminate the distortion between cash and credit goods created by the cash-in-advance constraint. This notwithstanding, the distortions introduced by the presence of a positive stock of nominal debt would still be there, just as in the economy with only cash goods.

To see this, suppose that the stock of debt is nominal and that there is full commitment to fiscal policy. Let the fiscal authority set, for $t \geq 0, \tau_{t+1} = \tau(\widehat{z}_t) = \tau(\widehat{z}_0)$, where $\tau(\widehat{z}_0)$ corresponds to the tax rate that fully finances the government liabilities in the allocation that obtains with full commitment, from period one on. That is,

$$(1 + \tau(\widehat{z}_0)) = \frac{u'(c^F)}{\alpha} \tag{50}$$

If, at any $t > 0$, the monetary authority tries to monetize part of the existing stock of nominal debt and to use the resulting revenues to increase future consumption —say, maintaining a constant $\overline{c}$ from then on— then, it must be the case that $c_t < c^F < \overline{c}$. Given that

$$(1 + \tau(\widehat{z}_0))(1 + \overline{i}) = \frac{u'(\overline{c})}{\alpha} \tag{51}$$

and that (50) must be satisfied, the interest rate would have to be negative, $\overline{i} < 0$. Negative interest rates can not be an equilibrium in this economy since then the household would like to borrow unboundedly. Therefore, given that it is not possible to raise future consumption with negative taxes, there is no gain in partially monetizing the stock of nominal debt in period zero. In this case monetary policy is time consistent. Therefore if there is no commitment to monetary policy, a fully committed fiscal authority who wants to maximize utility (2), will set $\tau_{t+1} = \tau(\widehat{z}_0)$, $t \geq 0$.[7] The following proposition summarizes this result:

Proposition 3 *Assume that fiscal authorities maximize the welfare of the representative household and can fully commit to their policies. Then the equilibrium allocation is the optimal equilibrium allocation that obtains when there is a single Ramsey planner, regardless of the degree of commitment of the monetary authority.*

5 Concluding comments

This paper discusses the different ways in which nominal and indexed debt affect the sequential choice of optimal monetary and debt policies. To this purpose, we study a general equilibrium monetary model where the costs of an unanticipated inflation arise from a cash-in-advance constraint with the timing as in Svensson (1985), and where government expenditures are exogenous. In our environment, as in Nicolini (1998), when the utility function is logarithmic in consumption and linear in leisure and debt is indexed, there is no time-inconsistency problem. In this case, the optimal monetary policy is to maintain the initial level of indexed debt, independently of the level of commitment of a Ramsey government.

[7] Marimon, Nicolini, and Teles (2003) make a similar argument.

In contrast, for the same specification of preferences, when the initial stock of government debt is nominally denominated, a time inconsistency problem arises. In this case, the government is tempted to inflate away its nominal debt liabilities. When the government cannot commit to its planned policies, progressively depleting the outstanding stock of debt is part of an optimal sequential policy consists, so that such policy converges asymptotically to zero debt liabilities. Optimal nominal interest rates in this case are also decreasing and converge asymptotically.

Such equilibrium path is not chosen when the initial stock of government debt is nominally denominated and the government –as a Ramsey planner– can fully commit to its planned policies. In this case, it is optimal to increase the inflation tax in the first period, and to keep a lower and constant inflation tax for the rest of the future. This is also the case, even if there is no commitment, if the government can choose a portfolio of real and nominal debt, since then it is optimal to end period zero with only real debt liabilities and never issue nominal debt afterwards. That the Ramsey outcome can be made time consistent by having only real debt liabilities exemplifies, in our context, the existing results due to Persson, Persson and Svensson (2006) and others.

In the rational expectations equilibria of our economies there are no surprise inflations. Still, for a given initial real value of outstanding debt, the most efficient equilibrium is the one that obtains when debt is indexed, the equilibrium with nominal debt and full commitment comes second, and the equilibrium with nominal debt and no commitment is the least efficient. This result highlights the sense in which nominal debt is indeed a burden on optimal monetary policy.

It should be noted that the source of the inefficiencies and of the monetary policy distortions discussed in this paper is not the desire to run a soft budgetary policy that increases the debt liabilities of the government. Every policy discussed in this article is an optimal policy, subject to the appropriate institutional and commitment constraints, and it is implemented by a benevolent and far-sighted government who does not face either uncertainty or the need for public investment, and who would, therefore, prefer to reduce debt liabilities. The source of the inefficiencies is the distortion created by the lack of commitment that results from the mere existence of an outstanding stock of nominal debt. Therefore, our results highlight the need to implement policy and institutional arrangements that either guarantee high commitment levels, or that reduce the allowed levels of nominal debt.

The introduction of additional forms of taxation further clarifies the interplay between the various forms of debt and commitment possibilities. Under the natural assumption that fiscal policy choices are predetermined, we show that the optimal policy problem has the same characterization, provided that the revenues levied through seigniorage are enough to allow for an optimal monetary policy with non-negative interest rates. Instead, as in Marimon, Nicolini and Teles (2003), if there is full commitment to an optimal fiscal policy, the fiscal authorities, anticipating monetary policy distortions, choose to fully finance government liabilities, and the resulting monetary policy is the Friedman rule of zero nominal interest rates. Moreover, this policy results in the equilibrium that obtains in the economy with full commitment.

References

[1] Alvarez, F., P. J. Kehoe and P. A. Neumeyer, 2004, “The Time Consistency of Optimal Fiscal and Monetary Policies”. *Econometrica* 72, 2, 541-567.

[2] Chari, V.V. and P. J. Kehoe. 1999. “Optimal Fiscal and Monetary Policy”, in *Handbook of Macroeconomics*, vol 1C, edited by John Taylor and Michael Woodford, North-Holland, National Bureau of Economic Research.

[3] Cole, H. and T. Kehoe, 1996, “A self-fulfilling model of Mexico’s 1994-1995 debt crisis”, *Journal of International Economics* 41, 309-330.

[4] Ellison, M. and N. Rankin, 2005, ”Optimal Monetary policy when Lump-Sum Taxes are Unavailable: A Reconsideration of the Outcomes under Commitment and Discretion”, forthcoming *Journal of Economics, Dynamics and Control*.

[5] Krusell, P., F. Martin and J.-V. Ríos-Rull, 2003, “On the Determination of Government Debt,” mimeo, U. Pennsylvania.

[6] Lucas, R. E., JR. and N. L. Stokey, 1983, “Optimal Fiscal and Monetary Theory in an Economy without Capital”, *Journal of Monetary Economics* 12, 55-93.

[7] Marimon, R., J. P. Nicolini and P. Teles, 2003, “Inside-Outside Money Competition,” *Journal of Monetary Economics* 50, 1701-1718.

[8] Nicolini, J. P., 1998, “More on the Time Inconsistency of Optimal Monetary Policy”, *Journal of Monetary Economics* 41, 333-350.

[9] Obstfeld M., 1997, “Dynamic Seigniorage Theory: An Explanation”, *Macroeconomic Dynamics* 1, 3, 588-614.

[10] Persson, M. T. Persson and L. Svensson, 2006, “Time Consistency of Fiscal and Monetary Policy: A Solution”, *Econometrica* 74, 1, 193-.

[11] Svensson, L.E.O., 1985, “Money and Asset Prices in a Cash-in-Advance Economy”, *Journal of Political Economy* 93, 919–944.

Appendix: Computation

To compute the recursive monetary equilibrium defined in Section 3.2, we solve the following dynamic program:

$$V(z) = \max\{\log(c) - \alpha(c+g) + \beta V(z')\} \tag{52}$$

s.t.

$$\frac{\beta}{\alpha} - g + \beta z' C(z') = c(1+z) \tag{53}$$

To solve this problem, we use the following algorithm:

- Step 1: Define a discrete grid on z
- Step 2: Define a decreasing discrete function $\bar{C}(z)$
- Step 3: Iterate on the Bellman operator described in equation (52) until we find the converged $V^*(z)$, $Z'^*(z)$,$C^*(z)$
- Step 4: If $C^*(z) = \bar{C}(z)$, we are done. Else, let $\bar{C}(z) = C^*(z)$ and go to Step 3.