On the Timing of Balance of Payments Crises: Disaggregated Information and Interest Rate Policy*

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Abstract

This paper proposes a dynamic framework to study the timing of balance of payments crises. The model incorporates two main ingredients: (i) investors have private information; (ii) investors interact in a dynamic setting, weighing the high returns on domestic assets against the incentives to pull out before the devaluation. The model shows that the presence of disaggregated information delays the onset of BOP crises, giving rise to discrete devaluations. It also shows that high interest rates can be effective in delaying and possibly avoiding the abandonment of the peg. The optimal policy is to raise interest rates sharply as fundamentals become very weak. However, this policy is time inconsistent, suggesting a role for commitment devices such as currency boards or IMF pressure.

Keywords: Currency crises; timing; disaggregated information; interest rate defenses.

JEL Classification: D8, E43, E58, F31.

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1 Introduction

During the 1990's the world witnessed a large number of balance of payments (BOP) crises, including the EMS crisis in 1992, the Mexican crisis in early 1995, the Asian crisis in 1997, the Russian crisis in 1998, and the Brazilian crisis in 1999. The large, and rapidly growing, literature on BOP crises has provided many insights into the causes behind these crises. A consensus now exists about the importance of institutions (e.g. bank supervision, corporate governance), debt management, and consistency in the setting of monetary and fiscal policy. Despite this progress, however, economists still have a limited understanding of the dynamics and timing of crises.

Balance of payments crises are characterized by two seemingly contradictory features. On the one hand, BOP crises are usually "large," in that they involve massive asset reallocations, wild swings in asset prices, and heavy output losses. On the other hand, BOP crises are often triggered by shocks that seem too small to account for these effects.[1] Another characteristic of BOP crises is that governments often attempt to prevent them by raising domestic interest rates to contain capital outflows.[2]

Figure 1 illustrates these characteristics by showing the behavior of exchange rates and domestic interest rates in selected currency crisis episodes. The two series are constructed by taking averages of the series corresponding to each country, after shifting the series so that time 0 coincides with the abandonment of the peg. For the interest rate, we use overnight domestic currency rates, from which we subtracted the average crawling speed during the year preceding the crisis. Figure 1 shows both the large "discrete" devaluations after pegs are abandoned, and the increase in interest rates as conditions deteriorate.

In this paper, we propose a dynamic framework for studying the timing of BOP crises that accounts for the suddenness and size of the crises, while providing a rational for interest rate defenses. We emphasize the process through which investors learn from each other and its crucial interaction with interest rate policy, allowing us to characterize the optimal interest rate defense.

We model BOP crises as the equilibrium outcome of a dynamic game between a monetary authority, which attempts to keep a fixed exchange rate, and a set of investors that at each point in time decide how much of their capital to invest in the country. The model relies on two basic

[1] We do not mean that economic fundamentals are unable to account for the severity of the crises, but rather that the deterioration in fundamentals is too smooth to explain their sudden onset.

[2] Such policies are often advocated by international financial institutions, especially the IMF.

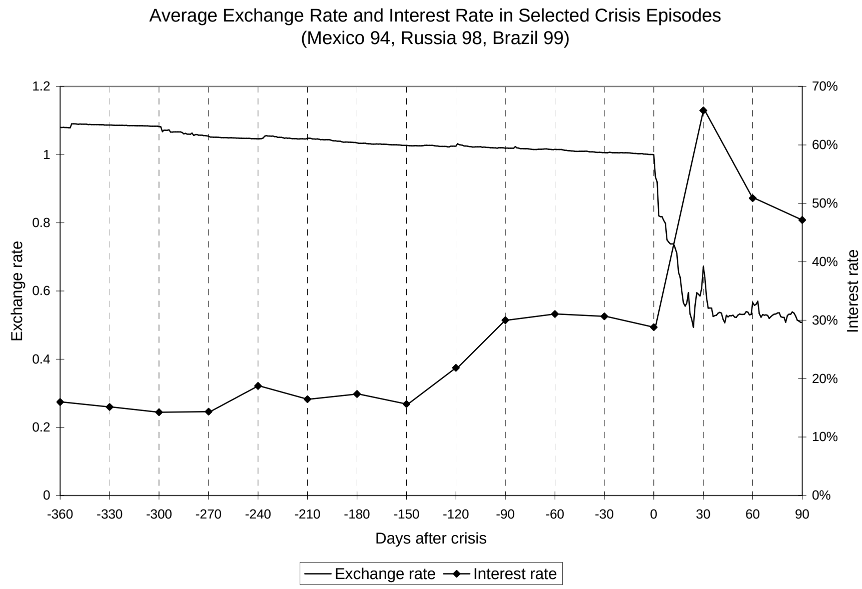

Figure 1: Behavior of exchange rates and short-term domestic-currency interest rates around episodes of currency crises. Day 0 is 12/19/94 for Mexico, 8/16/98 for Russia, and 1/12/99 for Brazil.

ingredients: (i) investors have private information about the state of the economy (in particular, about the level of the exchange rate in case the peg is abandoned); and (ii) investors interact in a dynamic setting, weighing the high returns they receive while holding domestic assets (high domestic interest rate) against the incentives to pull out before the crisis takes place. The crisis is triggered by some investors selling their domestic assets and starting a run on the central bank's reserves, with other investors following suit until reserves are exhausted. Crises are associated with discrete devaluations, as investors revise downwards their assessments about other investors private information. As a result, in the run up to the crisis investors' strategies incorporate an incentive to take their capital out before the crisis takes place (to avoid capital losses), which they weigh against the high domestic interest rate. To determine the expected return on domestic assets investors estimate a "crisis hazard rate," which depends on investors' assessment of other investors' information and is itself endogenously determined by the interaction of domestic interest rates and investors' learning process. Importantly, although the timing of the crisis is unpredictable based on public information, the model has a unique equilibrium.

Two versions of the model are presented. The first, in which the domestic interest rate is taken as exogenous, emphasizes investors' learning process and its implications for the timing of BOP crises. This version provides a number of insights into the behavior of asset prices during crises, as well as into the effects of interest rates and disaggregated information on their timing.

First, as mentioned above, "large shocks" are not necessary in order for crises to involve discrete drops in asset prices, even though the model has a single equilibrium.

Second, we show that the presence of private information makes the peg last longer than if the information were public. This result follows from two features of BOP crises that are captured by the model. The high returns on domestic assets in episodes of BOP crises create an incentive to wait past the point when the expected devaluation is zero. Without private information, however, investors cannot "coordinate" into staying past this point and leave when the size of the devaluation is zero. In addition, as an attack on a currency takes place, investors become more informed because they infer the private information of the investors who take their capital out first. The remaining investors then know if the attack would lead to a "revaluation," in which case they would prefer to stay in the country.[3,4]

Third, the fixed exchange rate lasts longer when domestic interest rates are high. This result follows from the fact that, conditional on other investors' actions, each investor has greater incentives to leave his capital in the country when interest rates are high. In addition, an indirect channel exists due to the presence of complementarities in investors' actions: if each investor stays longer the expected losses from devaluation decrease, further increasing the incentives not to pull out.

In the second version of the model, we study what the optimal interest rate policy is in such an environment. We assume that the monetary authority controls the domestic interest rate and tries to minimize a loss function. The loss function incorporates a (flow) cost of raising interest rates and a cost of abandoning the peg. We obtain a number of implications for interest rate policy during BOP crises.

First, the optimal interest rate policy is to raise interest rates sharply when fundamentals become very weak, as opposed to raising them earlier on by a smaller amount. This follows from the fact that raising interest rates when fundamentals are very weak is both "more effective," in that it is more likely to postpone the attack on the currency than when raising interest rates early

[3] In other words, if investors start leaving "too soon," they can recognize their mistake before reserves are exhausted, which gives rise to probing attacks. If, on the other hand, they start leaving "too late" there is a devaluation.

[4] The model can thus account for the observation that BOP crises often occur long after problems in the affected countries are recognized.

on,[5] and "cheaper," in that it is possible for the cost associated with raising the interest rate not to be incurred if either the attack takes place or the situation improves earlier on.

Second, there exists a problem of time inconsistency: the monetary authority would be better off if it could commit to raising interest rates as fundamentals deteriorate. This is due to the fact that interest rates at a point in time affect investors' strategies for previous times and, as a result, the benefits of high interest rates are "sunk" when that time is reached.[6]

Third, it is optimal to defend pegs more strongly (i.e. raise interest rates by a larger amount) in cases of liquidity crises than in cases of solvency crises.[7]

Fourth, although high interest rates can be an effective defense against speculative attacks, crises are more likely while interest rates are high, even conditioning on the level of fundamentals. As a result, empirical studies on the effectiveness of interest rate defenses should be careful in interpreting episodes in which interest rates are raised but the peg is abandoned as evidence against interest rate defenses.

Although there are no systematic studies of whether asymmetric information exists in the context of BOP crises, suggestive evidence exists. Evans and Lyons (1999) find a strong positive correlation between order flow[8] and price movements in the US$/DM exchange rate market, which is consistent with investors' trades revealing price-relevant private information. Garber (1998) argues that the existence of derivatives "obscures true risk positions and undermine the usefulness of balance-of-payments capital account categories." For example, according to IMF's International Capital Markets (1995), published 8 months after the Mexican devaluation, most of the Tesobonos outstanding at the time of the devaluation were held by foreigners (page 62). However, according to Garber, all of the US$ 16 billion worth of Tesobonos held by foreigners were involved in swaps with Mexican banks, so that all the risk was actually held by domestic banks. Furthermore, international investors do not share information on these types of trades, for they are considered proprietary.

[5]For any interest rate path, the effect of increasing the interest rate at a point in time increases the equilibrium probability that the crisis will occur at that point. (This corresponds approximately to uncovered interest parity.) Correspondingly, the amount of learning that takes place at that point increases, thereby "shifting back" the crisis distribution function for all previous times.

[6]This suggests a role for international organizations such as the IMF, or for commitment devices such as currency boards.

[7]In the context of this paper, a "liquidity crisis" is a crisis in which the probability that the peg survives increases when the attack is postponed.

[8]Evans and Lyons define order flow as "a measure of buying/selling pressure. It is the net of buyer-initiated orders and seller-initiated orders."

There exists an account of the events that led to the collapse in Mexico's bond market in which the crisis was triggered by investors' realization of the size of the total Tesobono swaps.[9] Johnson, Boone, Breach, and Friedman (1999) find that measures of corporate governance have a significant explanatory power for the size of devaluations and drops in local stock markets in a cross-section of countries during the Asian crisis. Under the assumption that investors have private information regarding the extent of corporate governance problems in the firms they invest, it is plausible that private information played a role in the crisis. Other "evidence" includes the fact that, in many cases, crises are triggered when an identifiable group of investors "pulls out," such as when domestic investors refused to roll over Russia's debt in August 1997.

The paper is organized as follows. Section (2) provides a brief review of the related literature. Section (3) describes the model under the assumption that the interest rate on domestic assets is constant. Section (4) solves and analyzes the model. Section (5) calculates the optimal interest rate policy. Section (6) describes the robustness of the results under alternative assumptions. Section (7) concludes and suggests some speculative applications of the theory presented in this paper for contagion, asset-market bubbles, and banking crises.

2 Related Literature

The large shifts in asset holdings during crises initially led observers to associate such episodes with investor irrationality. The so called first-generation approach to BOP crises, initiated by Salant and Henderson (1978), Krugman (1979), and Flood and Garber (1984), provided an alternative explanation. If crises mark a switch in regimes, with inflation higher after the fixed exchange rate is abandoned, the desired holdings of domestic currency should likely fall during crises. As a result, a "run" on the central banks's reserves could be interpreted as a rational portfolio reallocation. These models, though, also imply that crises should be fully predictable and, as a result, not associated with large movements in asset prices.[10]

The model in this paper shares important features with the first generation literature: fun-

[9]Another piece of evidence that suggests that investors learned about the situation of the Mexican banking system during the crisis is given by the fact that, in January 1995, the stock prices of the banks fell much more than that of other companies, even though banks' stock prices closely followed the stock market index throughout 1994.

[10]Flood and Garber (1984) and Dornbusch (1987) develop stochastic models of BOP crises that address this point by assuming the existence of large shocks. Rigobon (1999) presents an alternative argument: "small shocks," if unexpected, can give rise to large reassessments about a country's fundamentals.

damentals deteriorate monotonically and the timing of the crisis is determined by investors being indifferent between investing in the country and investing abroad. Figure 2 provides an illustration. The "shadow exchange rate" (i.e. the exchange rate that would be observed if a successful attack occurred at a point in time) depreciates over time as fundamentals deteriorate. In the absence of private information, the peg is abandoned at point A when the shadow exchange rate reaches the peg, since arbitrage eliminates all other possible times. The exchange rate is thus continuous.

Previous studies of interest rate defenses have mostly concentrated on the effect of interest rates on the timing of crises through their impact on fundamentals and the shadow exchange rate. For example, Calvo (1995) argues that even though high interest rates could induce capital inflows in the run up to the crisis, these would be compensated by a larger portfolio reallocation when the peg is abandoned. He also argues that, since the fiscal deficit (or expected future deficits) likely increases when interest rates are raised to defend a peg, fundamentals deteriorate faster and, as a result, it is possible for the "defense" to actually hasten the end of the peg. This corresponds to point B in Figure 2.[11] On the other hand, Lahiri and Végh (2000) focus on the value of interest rates *after* the peg is abandoned and how they affect the desired portfolio reallocation at the time of the crisis. They show that by promising to raise interest rates once the peg is abandoned, governments can shift *up* the shadow exchange rate and, as a result, delay the crisis.

Our approach is quite different, since we disregard the effect of interest rates on the shadow exchange rate. We rather show that when investors have disaggregated information, the peg is not abandoned when the shadow exchange rate reaches the peg, but at a point that is endogenously determined by the interaction between interest rates and investors' learning process. High interest rates delay the crisis and, as a result, lead to discrete devaluations when the crisis takes place, as in point C in Figure 2.

An alternative approach, which also accounts for the unpredictability of crises, is to assume the existence of multiple equilibria. Starting with Obstfeld (1984), second-generation models introduced the possibility that crises be self-fulfilling: if investors expect a crisis, they will act in a way such that a crisis occurs. However, these models have little to say about the *timing* of BOP crises, as a wide range of results can be obtained by assuming appropriate expectational dynamics. Furthermore, as Morris and Shin (1998) show in a generic second-generation model, the existence of multiple

[11] A similar point is made, in a more formal framework, by Flood and Jeanne (2000).

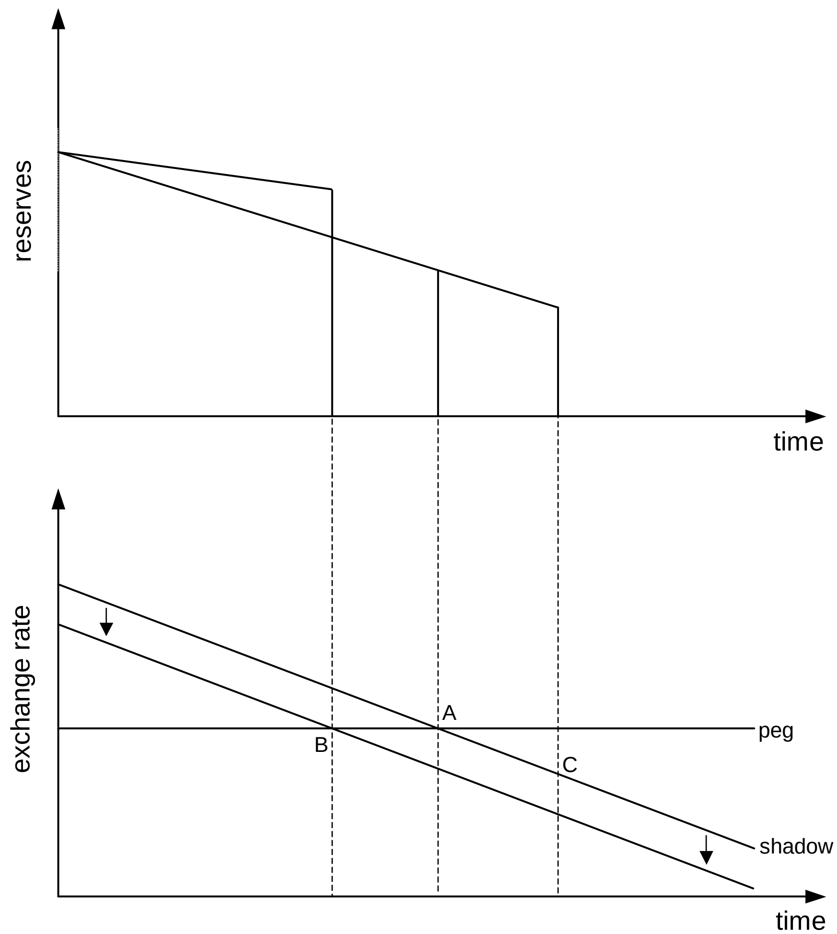

Figure 2: Central Bank reserves and exchanges rates in first-generation models.

equilibria might not be very robust, as adding even a small amount of noise to investors' perceptions about a country's fundamental eliminates the multiplicity of equilibria.[12]

Our model is also related to the social learning literature. For example, Caplin and Leahy (1994), Gul and Lundholm (1995), and Chamley (1998) present models with "informationally-driven" crises or clustering. Although these models do not deal with interest rate defenses, they provide the basic intuition for why small shocks can give rise to large crises in the presence of disaggregated information. However, they are not easy to interpret in the context of BOP crises, due to the "trade-offs" investors face in choosing their actions in those models. In the social learning

[12] Furman and Stiglitz (1998) argue that in models of multiple equilibria, since the gains from staying in the country if the peg survives is of order dt, while the costs if the peg does not survive is large, only unrealistically high interest rates would be effective in defending a peg.

literature, which is mostly concerned with industry dynamics or problems that give rise to similar "reduced-form" models, investors have an incentive to wait to observe other agents actions and face a cost of waiting. In episodes of BOP crises, on the other hand, investors have an incentive to move first (take their capital out of the country before the crisis takes place) and receive a flow benefit of waiting in the form of high returns on domestic assets. In addition, in episodes of BOP crises investors care about other investors' actions not only because they reveal their private information, but also because in case of a crisis those who leave first have a higher probability of doing so before the devaluation takes place.[13]

Two papers which also explore the role of private information during episodes of BOP crises are Drazen (1999) and Chari and Kehoe (2001). Drazen shows that, in a context where the government has private information about its objective function, high interest rates can serve as a signal of "toughness," providing a rational for interest rate defenses. Chari and Kehoe show that when investors have private information, they can infer other investors' private information by observing their actions, which leads to volatile capital flows and unpredictable crises (herd-like behavior). We view the model in this paper as complementary to Drazen's and Chari and Kehoe's. A crucial difference, however, is that they model BOP crises as a sequence of one-shot games, while we model BOP crises as a fully dynamic game. In Drazen's words, they make the "crucial assumption that a representative speculator can fully adjust his position in a given currency at the beginning of a period. Hence, risk-neutral speculators need only consider the probability of devaluation in the current period, and need not form expectations of the probability of devaluation in future periods, to derive their optimal positions." By treating crises as sequences of one shot games, these models do not capture the important trade-off investors face in such environments, namely, to earn high returns on domestic assets while attempting to pull out before the crisis takes place.[14] An important contribution of this paper is thus to characterize this trade-off and its interaction with interest rate policy.

[13] In social learning models that incorporate non-informational externalities, such as Chamley (1998), complementarities give agents an incentive to move *simultaneously*. These models do not capture the incentive to pull out first.

[14] A more technical issue is that, since in these kind of models the probability of devaluation in future periods does not affect current investors' actions, crises can be both *predictable* and associated with *discrete devaluations*. This is because it is not possible to rule out such cases by backward induction, as the one-stage games are only "connected" through the learning process.

3 The Model

The model is based on a linear first-generation-type framework. The main difference between our model and other models in the literature is the assumption that investors have private information regarding the level of the exchange rate in case the peg is abandoned.

Time is continuous and there are two kinds of players; a monetary authority, which attempts to keep a fixed exchange rate, and a set of investors, who at each point in time decide how much of their capital to invest in domestic assets. The state of the economy is summarized by a fundamental that deteriorates monotonically. While the peg lasts, investors receive a return on domestic assets which is higher than the international rate of return. If there is a speculative attack, investors who are able to convert their holdings of domestic currency into foreign currency before reserves are exhausted do not suffer capital losses, while other investors suffer losses equal to the size of the devaluation. The interplay between the incentive to pull out before others and the high nominal returns on domestic assets provides the main forces affecting the behavior of investors.

Monetary Authority

The monetary authority follows a simple rule: buy and sell foreign currency at the fixed exchange rate while reserves last.[15] Without loss of generality, the exchange rate is fixed at 1. Once reserves are exhausted, the currency is floated.

Investors

Investors are risk-neutral. They initially have some capital invested in the country, for which they receive a constant and exogenous return $r > 0$ in domestic currency. At each point in time, investors decide how much of their capital to invest in the country, and how much to invest abroad. The international rate of return is 0, and there are no transaction costs associated with capital movements. We also assume that investors have a maximum amount of capital (equal to their initial holdings for simplicity) and that there are no other investors who could invest in the country.[16]

[15] In Section (5) we analyze the optimal interest rate policy, allowing the monetary authority to set interest rates in order to delay, and possibly avoid, the crisis.

[16] This assumption can be justified by assuming that investors are capital-constrained "specialists." In section (6) we will argue that if a pool of uninformed investors existed who could bring their capital to take advantage of the high returns, the results would be stronger.

Investors are heterogeneous and have private information regarding their idiosyncratic characteristics. The specific dimension of heterogeneity is not crucial for the qualitative predictions of the model but, for concreteness, we assume that investors differ in the amount of domestic assets they would be able to liquidate easily if they wanted to take their capital out.[17] In the context of this model, liquid assets are assets which can be sold instantaneously and at a price which is fixed in local currency (e.g. short-term local-currency bank deposits), while illiquid assets cannot be sold at any price (or at a big loss, such as real estate or FDI).[18]

There are two groups of atomistic investors of mass 1 each. All investors within each group have the same amount of liquid assets or "type," denoted a_i for $i = 1, 2$.

Assumption 1. *Each investor knows his own type (and that of the rest of his group), but does not know the type of the other group. The a_i's are distributed with density function $g(\cdot)$ and support $[a_m, a_M]$. $g(\cdot)$ has no atoms and is common knowledge.*

The proportion of liquid assets invested in the country by investor $j \in [0, 1]$ of group $i \in \{1, 2\}$ at time t is denoted by $x(i, j, t)$.[19]

Environment

Time is continuous. Investors observe capital movements by all other investors.[20] The state of the economy at time t is summarized by a fundamental $f(t)$, which affects both the level of reserves and the value of the shadow exchange rate (i.e. the exchange rate if the government were to abandon the peg).[21]

Assumption 2. *Reserves at time t are given by*

$$R(t) = f(t) - \sum_{i=1,2} a_i \int_0^1 (1 - x(i, j, t))\, dj.$$

[17] In a setup in which the players are a monetary authority defending a peg and a set of *speculators* deciding when to attack the currency, identical results would be obtained if speculators had private information regarding the amount of resources they have available for the attack.

[18] At least in principle, it is possible to determine the amount of foreign investment by looking at capital flows. However, information regarding the types of investments and off-balance sheet transactions is scarce. See discussion in Garber (1998).

[19] Note the abuse of notation in that we do not explicitly include the history of past capital flows as an argument of $x(\cdot)$.

[20] We could alternatively assume that only net flows are observed without affecting the results.

[21] This would be the case if the fundamental corresponded to domestic credit, as in Krugman (1979) and Flood and Garber (1984).

In other words, reserves equal $f(t)$ minus the amount of capital investors have taken out.

Assumption 3. *The shadow exchange rate at time t is given by*

$$E_s(t) = 1 + f(t) + e_0 - a_1 - a_2 \tag{1}$$

where $e_0 \in (0, a_m)$ is a constant.

As a result, the size of the devaluation, which is given by $a_1 + a_2 - f(t) - e_0$, is increasing in the amount of liquid assets.[22]

The fundamental $f(t)$ deteriorates monotonically at speed μ and time is defined, without loss of generality, such that $f(0) = 2a_m - e_0$. We assume that the game starts at a time $\underline{t} < 0$ early enough such that there is an initial period when a devaluation cannot occur.

Assumption 4. *The fundamental $f(t)$ follows*

$$f(t) = (2a_m - e_0) - \mu t$$

In addition, $\underline{t} < -2\frac{(a_M - a_m)}{\mu}$ (i.e. $E_s(\underline{t}) > 1$ for all a_1 and a_2).

Since $f(t)$ falls monotonically at speed μ, the peg cannot last forever. Let $\bar{t}$ be the time at which the peg is abandoned, which is given by

$$\bar{t} = \sup \left\{ t : \forall \tau \in (\underline{t}, t)\, R(\tau) > 0 \right\}, \tag{2}$$

i.e. when reserves at the central bank reach zero.

The peg is thus abandoned when some investors decide to pull out and reserves are not enough to cover all desired outflows. In that case, reserves are allocated according to a sequential servicing constraint. The investors who initiated the attack are able to exchange their domestic currency before others, and reserves are assigned randomly if they are not sufficient to cover a group that moves simultaneously.

Investor i in group j chooses strategy $x(i, j, t)$ to maximize

[22] Equation (1) implies that the size of the devaluation is increasing in the amount of liquid assets that cannot be covered by existing reserves, which equals $a_1 + a_2 - f(t)$.

$$E\left[\int_{\underline{t}}^{\bar{t}} x(i,j,t)r\,dt - x^{+}(i,j,\bar{t})(1-E_s(\bar{t})) - \right.$$
$$\left.(x(i,j,\bar{t}) - x^{+}(i,j,\bar{t}))\left(\frac{A(\bar{t})-R(\bar{t})}{A(\bar{t})}\right)(1-E_s(\bar{t}))\right]$$

where $\bar{t}$ is given by equation (2), $x^{+}(i,j,t) \equiv \lim_{\tau\to t^{+}} x(i,j,\tau)$, and $A(t)$ is the amount of *desired outflows* at time t

$$A(t) = \sum_{i=1,2} a_i \int_0^1 (x(i,j,t) - x^{+}(i,j,t))\,dj.$$

The first term in the maximization problem accounts for the returns while the peg survives. The second term accounts for the devaluation losses from the capital that the investor did not attempt to take out at $\bar{t}$. The third term accounts for the devaluation losses from the capital that the investor attempted to take out, which incorporates the fact that this capital can be taken out before reserves are exhausted with probability $\frac{R(\bar{t})}{A(\bar{t})}$.

Four technical assumptions are needed to rule out some forms of unrealistic behavior. Some of these assumptions will only be used in the appendix, where a formal analysis of the game is presented.

Technical Assumption 1. *Strategies must be "well-behaved." For all flow histories,* $x^{-}(i,j,t) \equiv \lim_{\tau\to t^{-}} x(i,j,\tau)$ *exists,* $x^{+}(i,j,t) \equiv \lim_{\tau\to t^{+}} x(i,j,\tau)$ *exists, and* $x^{-}(i,j,t) = x(i,j,t)$.

Technical Assumption 2. *The game is the limit, as* $\epsilon \to 0$*, of the game in which the strategies* $x(i,j,t)$ *can be conditioned on flows only up to time* $t-\epsilon$.

Technical Assumption 3. *The model is the limit of a model with transaction costs as these costs tend to zero.*

Technical Assumption 4. *Investors within each group have access to a "correlating device" that allows them to follow "mixed-like" strategies. The two groups have independent correlating signals that cannot be observed by investors in the other group.*[23]

[23] This is analogous to assuming that investors can communicate with other investors within each group and agree (but not commit) to follow an action. None of the main results of the paper rest on this assumption, as it does not play any role in the main stage of the game. (It applies only after a failed probing attack.)

4 Analysis

As a benchmark, it is helpful to start by analyzing the model when there is no private information:

Proposition 1. *If a_1 and a_2 are common knowledge there is a unique Nash equilibrium. Investors leave their capital in the country until time*

$$\bar{t} = -\frac{(a_1 - a_m) + (a_2 - a_m)}{\mu},$$

which satisfies $E_s(\bar{t}) = 1$. At that point they all try to take their capital out, the peg is abandoned, and the size of the devaluation is zero.

Proof: It is trivial to show that the proposed solution is an equilibrium. To prove uniqueness note that, since $f(t)$ falls at speed μ, the peg must be abandoned, at the latest, when $f(t) = 0$. In pure strategies, investors cannot stay past $\bar{t}$ in equilibrium, since the crisis would involve a predictable depreciation. Mixed-strategy equilibria are not possible either, because they must involve randomizations over exit times up to the time when the crisis is inevitable. As a result, the "crisis hazard rate" would approach infinity at a point at which $E_s(t) < 1$, which cannot occur in equilibrium. □

This example shows that, in the model presented in this paper, the timing of crises is *independent of the interest rate r* when there is no private information.[24] The rest of this section studies the dynamics of crises and the effect of interest rates on their timing when private information is present.

For a formal analysis of the model, the reader should see the appendix. Here, we take the following proposition as a starting point, and present a more heuristic approach.

Proposition 2. *There is a unique and symmetric Nash equilibrium. The equilibrium is symmetric both between the two groups and between different investors in a single group. Symmetry within groups means that investors "move together," i.e. for all $j, j' \in [0,1]$ and $i \in \{0,1\}$, and for all histories of capital flows, $x(i,j,t) = x(i,j',t)$. In addition, investors always want to have either all their capital in the country, or all out, i.e. $x(i,j,t) \in \{0,1\}$.*

[24] This is due to the fact that our model ignores the effect of interest rates on the shadow exchange rate. See Calvo (1995), Flood and Jeanne (2000), and Lahiri and Végh (2000) for discussions on how interest rates affect the timing of crises through their effect on the shadow exchange rate.

Proof: See appendix.

The analysis is greatly simplified by two features of the model. First, since investors are atomistic, they do not act strategically, i.e. they take the actions of other investors as given, as opposed to only their strategies. Together with the absence of transaction costs, this implies that the investors' maximization problem can be solved *pointwise*.

The equilibrium of the game is composed of several "stages." In the first stage, investors' types are private information and their strategies can be summarized by a function $\bar{f}(a)$, which indicates at which value of the fundamental they would leave, conditional on their type. The first stage ends when a group of investors start taking their capital out, thereby revealing their type. Investors in the other group then either leave or stay, depending on their type. Unless their amount of investments in liquid assets is very low, investors in the second group leave, exhausting the government's reserves and ending the game; otherwise, they stay, the first group returns, and the second stage begins.

In the second stage, the type of the group that initiated the first attack (type 2 without loss of generality) is known, but the type of the other group is only known to be below some value $\underline{a}$, consistent with not having pulled out. The equilibrium in this stage is characterized by a function $\bar{f}_1(a_1; a_2)$, which indicates at which value of the fundamental investors in group 1 would leave as a function of their type and conditional on the type of group 2, and a hazard rate $\bar{h}(f; a_2, \underline{a})$, which indicates the probability density of group 2's leaving when the fundamental is f, as a function of their type and conditional on the maximum possible type for group 1. If investors in group 1 are the first to take their capital out in the second stage, investors in group 2 follow, reserves are depleted, and the game ends. If the attack is initiated by investors in group 2, the response of investors in group 1 again depends on their type. If their type is not very low, investors in group 1 also take their capital out, exhausting reserves and ending the game. Otherwise, investors in group 2 return and stage 3 begins. Each stage thereafter is identical to stage 2, and the same functions $\bar{f}_1(\cdot)$ and $\bar{h}(\cdot)$ apply.

Since investors can solve their maximization problem pointwise, each stage of the game can be solved independently. We concentrate on the first stage, where most of the insights become clear, and then briefly describe the subsequent stages.

With some abuse of notation, let us define $a(t) = \bar{f}^{-1}(f(t))$, where $\bar{f}^{-1}$ denotes the inverse of

$\bar{f}$.[25] The function $a(t)$ denotes then the "marginal type" at time t, i.e. the type such that an investor with that type would leave exactly at time t. For t such that $f(t) > \bar{f}(a_M)$, we define $a(t) = a_M$, since $\bar{f}^{-1}(f(t))$ is not defined. Similarly, for t such that $f(t) < \bar{f}(a_m)$, we define $a(t) = a_m$.

We can solve for $a(t)$ by noting that, when the crisis hazard rate is positive, the marginal type must be indifferent between investing in the country and abroad in equilibrium.

Proposition 3. *In the unique and symmetric Nash equilibrium, the first stage of the game is characterized by the "marginal type" function $a(t)$. Investors take all their capital out of the country when $a(t)$ reaches their type. The function $a(t)$ satisfies the differential equation*

$$r = \frac{g(a(t))}{G(a(t))}(-\dot{a}(t))\left(\frac{2a(t) - f(t)}{a(t)}\right)(2a(t) - f(t) - e_0) \tag{3}$$

and the boundary condition

$$a(0) = a_m.$$

Proof: Let us define $a_1(t)$ and $a_2(t)$ as the marginal type functions for groups 1 and 2 respectively.[26] In equilibrium, the marginal investor must be indifferent between staying or leaving. The returns outside the country are 0, while the returns inside the country consist of the sum of r and the expected losses from devaluation.

The expected losses from devaluation for an investor in group 1 arise because, when invested in the country, there is a positive hazard rate for group 2's pulling out, in which case the investor would suffer devaluation losses with positive probability.[27] The hazard rate for group 2's pulling out is given by

$$\gamma_2(t) = \frac{g(a_2(t))}{G(a_2(t))}(-\dot{a}_2(t))$$

[25] That such an inverse exists follows from the fact that if $\bar{f}(a)$ where not strictly increasing, the crisis probability density would have positive mass at some point in time, which cannot occur in equilibrium since returns are only of order dt.

[26] We will use the fact that the equilibrium is symmetric below.

[27] To make this step rigorous, TA2 is needed. Otherwise, there could be other equilibria in which a group leaves even though r is higher than the expected devaluation losses due to attacks initiated by the other group. This would be possible because if one group left at such a time, and the "reaction time" were zero, the other group could follow *immediately*. A more formal treatment of this point can be found in the appendix.

where $G(\cdot)$ is the cumulative distribution of $g(\cdot)$, and $\frac{g(a_2(t))}{G(a_2(t))}$ is the density of a_2 at $a_2(t)$, conditional on $a_2 \leq a_2(t)$. The probability of an investor in group 1 not being able to take his capital out conditional on group 2 pulling out is given by $\left(\frac{a_1(t)+a_2(t)-f(t)}{a_1(t)}\right)$, since after investors in group 2 take their capital out only an amount $f(t) - a_2(t)$ of reserves are left. Finally, the new exchange rate is given by equation (1).

Since an investor in group 2 faces a similar problem, $a_1(t)$ and $a_2(t)$ must satisfy[28]

$$\begin{aligned} r &= \frac{g(a_1(t))}{G(a_1(t))}(-\dot{a}_1(t))\left(\frac{a_1(t)+a_2(t)-f(t)}{a_2(t)}\right)(a_1(t)+a_2(t)-f(t)-e_0) \\ r &= \frac{g(a_2(t))}{G(a_2(t))}(-\dot{a}_2(t))\left(\frac{a_2(t)+a_1(t)-f(t)}{a_1(t)}\right)(a_2(t)+a_1(t)-f(t)-e_0)\,. \end{aligned}$$

Since the solution is symmetric, we can define $a_1(t) = a_2(t) \equiv a(t)$, and $\gamma_1(t) = \gamma_2(t) \equiv \gamma(t)$, obtaining equation (3).

Finally, let τ be such that $a(\tau) = a_m$. Then, since $\gamma(t) \to \infty$ as $t \to \tau$, it must be the case that $E_s(\tau) = 1$; otherwise, some investors could suffer predictable capital losses by staying too long or miss predictable capital gains by leaving too early. This is equivalent to $a(0) = a_m$.[29] $\square$

Figure 3 shows the marginal type function $a(t)$ for different interest rates r.[30] The figure shows that, for any values of a_1 and a_2, the first attack occurs later the higher r is. The intuition behind this result is that, although "learning" (which is closely related to $\dot{a}(t)$) is faster when interest rates are high, the moment at which this learning starts is determined by the terminal condition. As a result, when interest rates are high the learning process starts later, since faster learning implies that more of it can take place closer to $t = 0$.

[28]If transaction costs were zero these differential equations would need to be satisfied only if $\dot{a}_1(t) < 0$ and $\dot{a}_2(t) < 0$. As a result, there would be other equilibria in which there are alternating periods in which the crisis hazard rate is zero, then positive, then zero, etc. However, this equilibria are ruled out by TA3. In the appendix we show that if $\dot{a}_i(t) = 0$ for some t, then $a_i(t') = a_M$ for all $t' \leq t$.

[29]For equation (3) to be valid, the constraint $\left(\frac{2a(t)-f(t)}{a(t)}\right) \in (0,1)$ for all $t \in [\underline{t}, \bar{t}]$ needs to be satisfied. It is satisfied at $t = 0$ when $a_1 = a_2 = a_m$ iff $e_0 \in (0, a_m)$, which we assumed in A3. For earlier times, it is also satisfied under the stronger restriction $e_0 \in (0, a_m - (a_M - a_m))$. However, unless under extremely high interest rates, the weaker restriction is enough. Moreover, even if the constraint were not satisfied under the proposed solution, the qualitative behavior of the model would be the same. The only difference would be that the path of $a(t)$ in the true equilibrium would be flatter than in the proposed one, but the effects of interest rates and the information structure on the timing of the crisis would remain unchanged.

[30]Equation (3) can only be solved analytically for $r = 0$, in which case $a(t) = a_m - \frac{\mu}{2}t$. For $r > 0$, it can be shown that $a(t; r_1) < a(t; r_2)$ for all t if $r_1 < r_2$. In addition, $\dot{a}(0) = -\frac{\mu}{2} - \frac{r}{2}\frac{a_m}{e_0}$, which is useful for the numerical simulation.

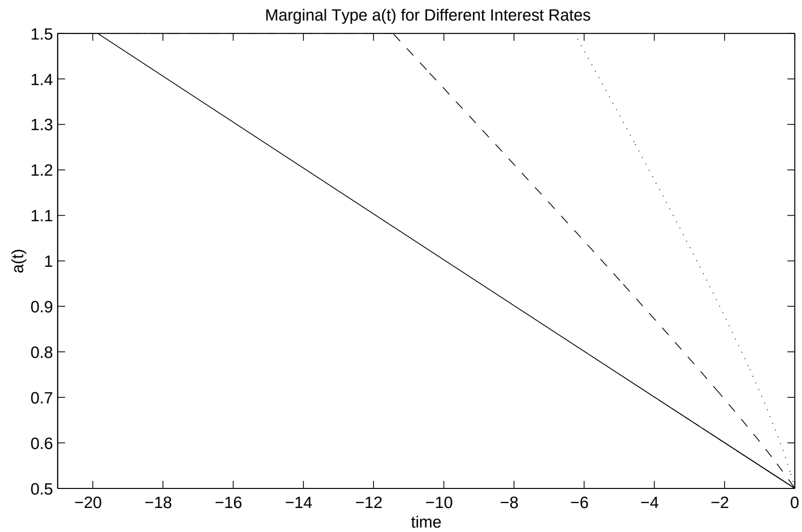

Figure 3: Marginal type a(t) for different interest rates. $a_m = 0.5$, $a_M = 1.5$, $g(a)$ is uniform, $\mu = 0.1$, and $e_0 = 0.2$. Solid line: $r = 0$. Dashed line: $r = 0.05$. Dotted line: $r = 0.2$.

Let t_1 be the time at which the first attack occurs and the first stage ends, i.e. $a(t_1) = \max\{a_1, a_2\}$. After the initial attack either the peg is abandoned or the second stage of the game begins. Without loss of generality, let us assume $a_2 > a_1$, so group 2 is the first to leave.

Proposition 4. *If $E_s(t_1) = 1 + f(t_1) + e_0 - a_1 - a(t_1) < 1$, investors in group 1 also leave and the peg is abandoned immediately. Otherwise, investors in group 2 return and the second stage begins. The equilibrium is characterized by a marginal type function $a^1(t)$, which denotes the type of investors in group 1 that would take their capital out at time t, and $h(t)$, which denotes the hazard rate of investors in group 2's pulling out. The function $a^1(t)$ satisfies the differential equation*

$$r = \frac{g(a^1(t))}{G(a^1(t))}(-\dot{a}^1(t))\left(\frac{a^1(t) + a_2 - f(t)}{a_2}\right)\left(a^1(t) + a_2 - f(t) - e_0\right) \tag{4}$$

and the boundary condition

$$a^1\left(-\frac{a_2 - a_m}{\mu}\right) = a_m. \tag{5}$$

The hazard rate $h(t)$ solves

$$r = h(t)\left(\frac{a^1(t) + a_2 - f(t)}{a^1(t)}\right)\left(a^1(t) + a_2 - f(t) - e_0\right) \tag{6}$$

for $a^1(t) \leq f(t_1) + e_0 - a(t_1)$ *and equals zero otherwise.*

Proof: See appendix.

To understand the second stage of the game, let us assume that r is very small, which makes it less likely that the first attack will be "successful." In this case, the equilibrium is characterized by investors pulling out in the first stage at a point when the expected devaluation losses are small, i.e. when t is such that $2a(t) - f(t) - e_0$ is close to zero.[31] As a result, there is a low probability that the group that did not initiate the attack has a type close enough to $a(t)$ such that the shadow exchange rate $E_s(t) < 1$, which implies that the currency will likely survive the first attack.

Figure (4) illustrates the case in which the first attack is not successful, assuming group 2 attacks first (i.e. $a_2 > a_1$). The solid line displays the learning process by investors in group 2, and shows the maximum possible type a_1. The dashed line is the marginal type $a(t)$ when no attack has taken place, and the dotted line is the marginal type $a^1(t)$ for group 1 in the second stage, conditional on a_2. The marginal type $a(t)$ falls until it reaches a_2 at some time t_1, prompting an attack by group 2. At that point, group 1 leaves and the peg is abandoned if a_1 is large enough such that the shadow exchange rate $E_s(t_1)$ is lower than 1. If group 1 stays, the second stage begins with an initial period when the crisis hazard rate is zero and no learning takes place. When the maximum type for group 1 consistent with the peg having survived intersects the marginal type function $a^1(t)$, investors start learning again. At some point, either $a^1(t)$ reaches a_1, which triggers a successful attack by group 1, or group 2 pulls out again. A sequence of "probing" attacks can ensue, until one takes place when $E_s(t) < 1$, in which case the attack is successful.

We now summarize a number of conclusions that can be drawn from the previous analysis regarding the effects of disaggregated information on the timing of BOP crises. First, once we account for the presence of disaggregated information, interest rates have a significant effect on the timing of crises, in contrast with the case presented at the beginning of the section. In particular, higher interest rates delay the onset of crises.

Second, for any interest rate r, the presence of private information delays the crisis, in the sense that the peg lasts longer for all realizations of investors' private information. The presence of private information has two main effects. On the one hand, it introduces "noise," so that investors

[31] When interest rate differentials are small, investors do not risk leaving their capital in the country if the expected devaluation losses are large.

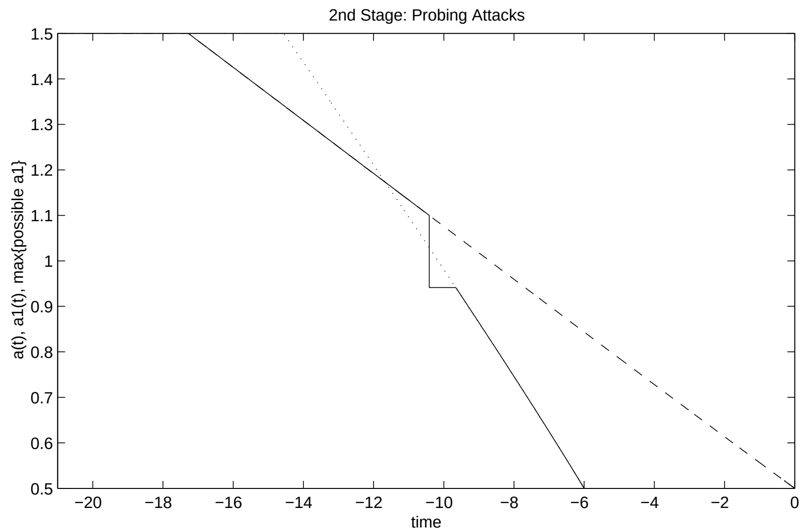

Figure 4: Second Stage: Probing Attacks. $a_m = 0.5$, $a_M = 1.5$, $a_1 = 1.1$, $a_2 < a_1$, $g(a)$ is uniform, $r = 0.005$, $\mu = 0.1$, and $e_0 = 0.2$. Dashed line: $a(t)$. Dotted line: $a^1(t)$. Solid line: $\max\{a_1 : a_1$ is consistent with group 1 not having left$\}$.

do not know precisely when the other investors are going to start leaving. As a result, investors can "coordinate" into staying in the country for a longer period of time and receiving the high returns.[32] This effect is illustrated in figure (3), as high interest rates "push back" the distribution of initial attack times.

On the other hand, disaggregated information makes investors "too optimistic" when the types are high (large amount of investments in liquid assets), and "too pessimistic" when the types are low (small amount of investments in liquid assets). The first (second) effect tends to make the first attack take place later (sooner) than under symmetric information. Although this seems to imply that the peg should last less under disaggregated information when types are low, this is not the case because there is an "asymmetric arbitrage." When one group leaves, the second group learns whether $E_s(t_1) < 1$, and would follow only if this would cause the exchange rate to devalue. Namely, the second group can "correct" the mistake by the first group if it leaves too early, but not if it leaves too late.

Third, in the context of BOP crises, private information gives rise to discontinuous drops in asset prices and, hence, complementarities in investors actions. The asymmetry in the movement

[32]The term coordination can be misleading, since investors are better off in the case with disaggregated information only if r is large relative to μ. However, if we take into account the effect of disaggregated information on interest rates (which we do in the next section), then investors are in fact better off in the case with disaggregated information.

of the exchange rate is due to the high returns inside the country, the fact that investors learn as the crisis progresses, and the existence of an agent (the monetary authority) which is willing to buy domestic currency even when a depreciation is expected.

Fourth, the model also sheds light onto the positive relationship between the rate of return and the speed at which learning takes place.[33] The reason is that the faster investors learn, the higher the crisis-hazard rate is and, thus, the higher the risk premium demanded by investors. However, in equilibrium an increase in the interest rate implies that the learning process starts *later*, as less time is needed to do the same amount of learning. As a result, it is the expectation of high interest rates in the future, with a correspondingly high learning speed, that makes investors stay now. Namely, high interest rates at a point in time push back the marginal type functions for all *earlier* times. This indicates the possibility of time inconsistency in the setting of interest rate policy.

5 Interest Rate Policy

In the previous section, we showed that domestic interest rates play an important role in the timing of BOP crisis. In this section, we study the optimal interest rate policy in such an environment.

We assume the monetary authority minimizes a loss function that incorporates the cost of abandoning the peg, the cost of raising interest rates to defend the currency, and the cost of reneging from a preannounced defense.

Assumption 5. *The monetary authority minimizes the loss function*

$$L = E\left[1_{[\text{peg is abandoned}]}\bar{D} + 1_{[\text{deviate}]}\Phi + \int_{\underline{t}}^{\bar{t}} c(r(t))\,dt\right]$$

where $\bar{D} > 0$ is the cost associated with abandoning the peg, $\Phi > 0$ is the cost associated with deviating from a preannounced interest rate policy, and $c(\cdot) \geq 0$ is the flow cost associated with raising interest rates. In addition, $c(0) = 0$, $c'(\cdot) > 0$, and $c''(\cdot) > 0$.

Without further changes, the optimal interest rate policy would be to set interest rates equal to zero. The reason is that the peg is abandoned regardless of interest rate policy and the monetary

[33] Stock (1987) finds that the business cycle evolves on an "economic time scale" rather than on a "calendar time scale." Interestingly, he finds that the most important determinant of the economic time scale is the short-term interest rate, which has an accelerating effect.

authority gains nothing by delaying the crisis. As a result, the monetary authority would set $r = 0$ and the peg would be abandoned as soon as the shadow exchange rate $E_s(t) = 1$.[34]

However, most crises have some liquidity component associated with them. For example, governments can implement policies to take a country out of an unsustainable path (such as increasing taxes or cutting government spending); the situation in international capital markets can improve and allow the country to find new sources of financing; or a positive terms-of-trade shock can take place. Usually, though, these developments take time and are not in the hands of the monetary authority. To capture such features of crises, we modify the model slightly to allow for the possibility of a turnaround in the economy.[35]

Assumption 6. *With hazard rate ρ, the "turnaround" hazard rate, the game ends, the peg survives, and the monetary authority is spared all further interest rate costs.*

To solve for the optimal interest rate policy, we first note that the equilibrium of the investors' game when the interest rate is variable is similar to the one found in the previous section under a constant interest rate. The only difference is that r is replaced by $r(t)$ in equation (3).[36]

Also, the monetary authority does not need to set a positive interest rate while the crisis hazard rate is zero. If

$$t_0 \equiv \sup \{t : a(t) = a_M\}$$

is the time at which the learning process starts, $r(t) = 0$ for $t < t_0$. For $t < t_0$ the loss function is then given by

$$L[t, \{r(s)\}] = \int_{t_0}^{0} \left[c(r(s)) + 2\frac{g(a(s))}{G(a(s))}(-\dot{a}(s))D(s, a(s)) \right] G(a(s))^2 e^{-\rho(s-t)} ds$$

where $D(s, a)$ equals the expected losses, as of time s, conditional on the first attack being initiated at time s by a group with type a. Note that the interest rate costs incurred before time s are not

[34]Note that we assume a zero discount rate. This is not an unreasonable assumption, since episodes of BOP crises usually only last a few months. However, the monetary authority might be interested in postponing the crisis for political reasons, which might imply a higher discount rate. In any case, the "turnaround" hazard introduced below can also account for a discount rate, without affecting the equilibrium of the game.

[35]The model presented in the previous section was one of solvency crises, since crises occurred with probability 1.

[36]The turnaround hazard rate does not affect investors' learning process. The reason is that if the turnaround takes place, investors get the same return (zero) on domestic and foreign assets.

included in $D(s,a)$.[37] The other terms in the expression are the hazard rate of having an initial attack at time s, conditional on not having had a previous attack or a turnaround before time s, which equals $2\gamma(s) = 2\frac{g(a(s))}{G(a(s))}(-\dot{a}(s))$; the interest rate flow cost at time s, conditional on the same event, $c(r(s))$; and the probability that neither a turnaround nor an attack take place before time s, which equals $G(a(s))^2 e^{-\rho(s-t)}$.[38]

In order to determine $D(s, a(s))$, one would need to solve a similar minimization problem. However, this introduces some additional difficulties because $D(s, a(s))$ is defined recursively. To keep the problem simple, we assume that $r = 0$ after an initial attack.[39] The function $D(s, a(s))$ is then given by

$$D(s,a(s)) = \bar{D}\left[\frac{G(a(s)) - G\left(f(s) + e_0 - a(s)\right)}{G(a(s))} + \int_s^{-\frac{a(s)-a_m}{\mu}} \mu \frac{g((f_m - \mu\tau) + e_0 - a(\tau))}{G(a(s))} e^{-\rho(\tau - s)} d\tau\right].$$

The first term is the probability that the second group has a type such that the shadow exchange rate $E_s(s) < 1$, i.e. the probability that the devaluation occurs immediately. The second term takes into account the fact that, if the second group has a type such that the devaluation occurs later, the probability that the turnaround takes place is higher. Note that, since we assume $r = 0$, there are no further interest rate costs.

To simplify notation, let

$$k(s,a(s)) \equiv \frac{G(a(s))}{g(a(s))}\left(\frac{a(s)}{2a(s) - f(s)}\right)\left(\frac{1}{2a(s) - f(s) - e_0}\right).$$

Then, from proposition (3), $a(s)$ is given by

[37] $D(s,a) = \bar{D}$ if the peg is abandoned at time s. If the type of the group that did not initiate the attack is low enough such that the peg is not immediately abandoned, $D(s,a) < \bar{D}$.

[38] Note that ρ enters the loss function $L[t, \{r(s)\}]$ in the same way a discount rate would. As a result, ρ can be taken as the sum of the turnaround hazard rate and the discount rate.

[39] This assumption does not affect the qualitative results of the model because the peg is abandoned after the first attack with high probability. The reason for this is that the optimal interest rate policy involves postponing the learning process and, as a result, after the first attack takes place the type of the group that did not initiate the attack is likely large enough such that $E_s < 1$. On the other hand, the "unconstrained" $D(s, a(s))$ is steeper than the one assumed here, so the incentives to postpone the learning would be slightly lower without this assumption.

$$
\begin{aligned}
\dot{a}(s) &= -r(s)k(s,a(s)) \qquad (7)\\
a(0) &= a_m
\end{aligned}
$$

We first ignore issues of time consistency by assuming $\Phi = \infty$. The monetary authority's problem is then to choose t_0 and $\{r(s)\}_{t=t_0}^{t=0}$ to minimize

$$
\begin{aligned}
L[t,\{r(s)\}] &= \int_{t_0}^{0} \left[c(r(s)) + 2\frac{g(a(s))}{G(a(s))} r(s)k(s,a(s))D(s,a(s)) \right] \times \\
&\quad G(a(s))^2 e^{-\rho(s-t)} ds
\end{aligned}
$$

subject to equation of motion (7), $a(t_0) = a_M$, and $a(0) = a_m$.

Proposition 5. *The solution to the monetary authority's problem when $\Phi = \infty$ is characterized by*

$$
\begin{aligned}
\dot{a}(s) &= -r(s)k(s,a(s)) \\
\dot{r}(s) &= \frac{1}{c''(r(s))} \Bigg\{ \left[\rho D(s,a(s)) - D_t(s,a(s))\right] 2\frac{g(a(s))}{G(a(s))} k(s,a(s)) + \\
&\quad c'(r(s)) \left[\rho + r(s) 2\frac{g(a(s))}{G(a(s))} k(s,a(s)) \right] - \\
&\quad c(r(s)) 2\frac{g(a(s))}{G(a(s))} k(s,a(s)) + c'(r(s)) \frac{k_t(s,a(s))}{k(s,a(s))} \Bigg\} \qquad (8)\\
r(t_0) &= 0 \\
a(t_0) &= a_M \\
a(0) &= a_m
\end{aligned}
$$

where a subscript t denotes the partial derivative with respect to time.

Proof: See appendix.

The solution is involved but intuitive. The $\dot{a}(s)$ equation has been described above. The equation for $\dot{r}(s)$ is composed of four terms and a scaling factor. The first term captures the fact that it is desirable to postpone the expected cost D if the turnaround hazard rate is positive or if D is expected to fall. The second term captures the fact that the monetary authority would like to postpone raising the interest rate to increase the probability that either the crisis or a turnaround take place before (saving the interest rate cost). The third term is due to the fact that, by postponing the increase in the interest rate, and thus the crisis distribution, it is more likely that the interest rate cost in the future will be incurred. The fourth term is associated with the fact that if the "effectiveness" of raising interest rates is increasing, the monetary authority has an incentive to postpone raising them. Finally, the larger the smoothing incentives (i.e. the higher the second derivative of the cost function) the less the previous four effects matter.[40] The condition that determines t_0 comes from the desire to smooth $r(s)$.

Figure (5) illustrates the optimal interest rate policy. The solid lines show the paths for $r(s)$, $a(s)$, and the probability density of time of first attack, corresponding to the solution when $\Phi = \infty$.[41] The paths of $r(s)$ and $a(s)$ are conditional on not having had an attack or turnaround prior to time s. The solution is characterized by a long initial period of "tranquility," in which $r = 0$ and the probability of a crisis is zero. Towards the end, however, interest rates are raised sharply at the same time that the crisis hazard rate increases. The figure highlights some of the results described in the previous section. First, the positive relationship between the interest rate and the speed of learning is *point by point*. Namely, along the optimal path of $r(s)$, times of high interest rates are times in which the probability of observing an attack is high. This is true for both the conditional probability (given approximately by the slope of $a(s)$) and the unconditional probability (given by the density function).

The optimal interest rate path is sharply increasing as fundamentals become very weak, as opposed to relatively constant at a low value. This is because high interest rates at a point in time push back the $a(s)$ schedule for all *earlier* times. In addition, the probability that the interest rate costs are incurred decreases with the time at which interest rates are supposed to be raised, since it is more likely that the game ends before that time. As a result, it is both more "efficient" and "cheaper" to raise interest rates later on.

[40] Note that partial time derivatives are present instead of total time derivatives. The reason for this is that the effect of $r(s)$ on the path of a exactly cancels out the term corresponding to the partial derivative with respect to a.

[41] To carry out the simulation, we use the fact that there are two initial conditions and one final condition for

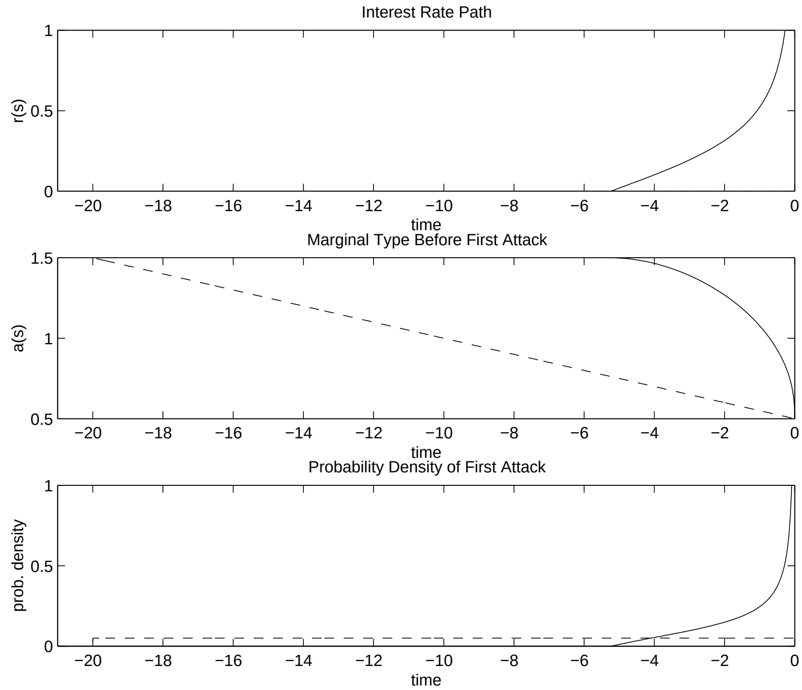

Figure 5: Optimal path for $r(s)$, corresponding marginal type $a(s)$, and density function of times of first attack. $a_m = 0.5$, $a_M = 1.5$, $g(a)$ is uniform, $\mu = 0.1$, $e_0 = 0.2$, $\rho = 0.1$, $\bar{D} = 1$, and $c(r) = 0.5r + r^2$. Solid line: with full commitment. Dashed line: with no commitment ($r = 0$).

However, the fact that the benefits from raising interest rates at a point in time is in postponing the crisis for *earlier* times suggests that a problem of time inconsistency might exist. As a result, we next consider the case in which $\Phi = 0$. To analyze this case, we need to make an assumption regarding the point at which the monetary authority sets interest rates for times close to $t = 0$.

Technical Assumption 5. *The model is the limit, as $\Delta t \to 0$, of a model in which interest rates are constant within $(-\Delta t, 0]$, $(-2\Delta t, -\Delta t]$, $(-3\Delta t, -2\Delta t]$, and so for. In addition the interest rate for $s \in (-(n+1)\Delta t, -n\Delta t]$ is set at time $-n\Delta t$.*

Proposition 6. *If $\Phi = 0$ and TA5 holds, the monetary authority cannot commit to* any *interest rate policy different from $r(s) \equiv 0$.*[42]

Proof: The proposition follows from a simple backward induction argument. Regardless of previous

system (8). We iterated over different t_0 until $a(0) = a_m$.

[42]It can be shown that if $c'(0) \geq \frac{\rho \bar{D}}{8} \frac{a_m}{\mu e_0}$ the proposition is true even if the interest rate for $s \in (-(n+1)\Delta t, -n\Delta t]$ is set at time $-(n+1)\Delta t$.

play, the monetary authority will set $r = 0$ for $s \in (-\Delta t, 0]$ at time 0. As a result, the peg must be abandoned at the latest at time $-\Delta t$ if $E_s(-\Delta t) < 1$. Assume that $r = 0$ for $s \in (-n\Delta t, 0]$ and that the peg must be abandoned at the latest at time $-n\Delta t$ if $E_s(-n\Delta t) < 1$. The monetary authority then does not have any incentive to set $r > 0$ for $s \in (-(n+1)\Delta t, -n\Delta t]$ at time $-n\Delta t$. By induction, $r(s) \equiv 0$. □

The dashed line in figure (5) shows paths for $r(s)$, $a(s)$, and the probability density of time of first attack, corresponding to the solution to the monetary authority's problem when $\Phi = 0$. The monetary authority cannot commit to raising interest rates and, as result, $r(s) \equiv 0$. The case of low interest rates was discussed in the previous section, and involves investors' leaving as soon as a devaluation is possible. This can be seen in the $a(s)$ schedule, which satisfies $f(s) + e_0 - 2\bar{a}(s) = 1$. The no-commitment case is then characterized by low interest rates, small devaluations, and vulnerable pegs.

In order to illustrate the problem of time inconsistency, figure (6) shows the incentives to deviate from the optimal full-commitment interest rate policy. The solid line corresponds to the expected *future* costs faced by the monetary authority as the crisis progresses, conditional on no previous attacks or turnaround. For early times the expected costs are an increasing function of time, since as time passes the probability that the turnaround takes place decreases. The expected costs eventually become larger than $\bar{D} = 1$, since they include both the likely devaluation and interest rate costs. As the interest rate costs become sunk, the expected future costs start decreasing. As $s \to 0$, the costs tend to $\bar{D} = 1$ since the crisis is imminent but no further interest rate costs need to be incurred. This is in sharp contrast with the behavior of the expected future costs if the monetary authority deviated from its pre-announced policy. The dashed line shows the expected costs, as of time s, assuming the monetary authority deviates at s and sets $r = 0$ thereafter.[43] The reputation cost Φ is not accounted for in the schedule. For early times, while $f(t) + e_0 - 2a_M > 1$, this path coincides with the expected costs under $\Phi = 0$, which are much higher than under full commitment. As the crisis progresses, though, the benefits from high future interest rates become sunk, and the two schedules start approaching each other. Eventually, the schedule becomes lower than that under commitment, and the monetary authority has an incentive to deviate.

[43]Once the monetary authority deviates, Φ is sunk and it becomes impossible to credibly announce any policy different from $r(s) \equiv 0$.

As a result, in order for the pre-announced full-commitment interest rate policy to be credible, Φ needs to be larger than the maximum distance between the two schedules in figure (6). For intermediate Φ, the monetary authority can only credibly commit to an interest rate defense which is less "aggressive."

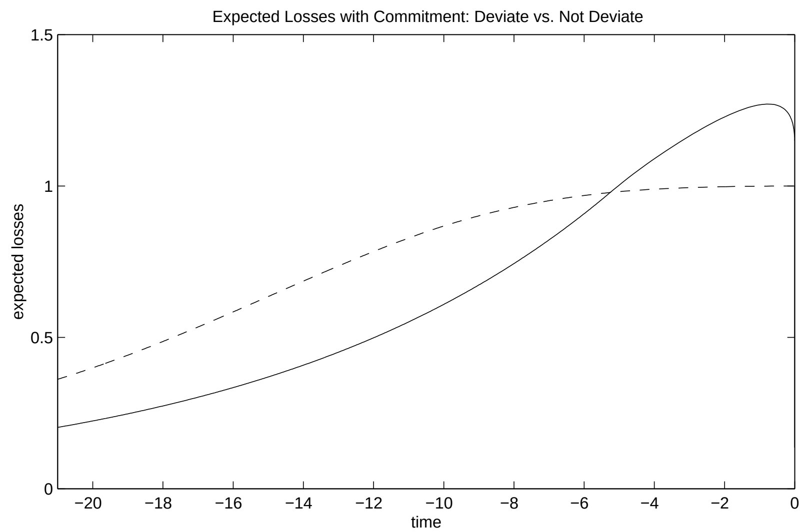

Figure 6: Solid line: $L(t)$, as of t, conditional on reaching t before the first attack. Dashed line: Idem, but assuming $r = 0$ thereafter.

In addition, note that in the absence of disaggregated information the peg would be abandoned, as in proposition (1), at a time such that the size of the devaluation is zero. Since this is the same condition that determines the timing of the crisis under the no-commitment solution, the monetary authority is better off in the presence of disaggregated information. This is not due to some form of transfer from investors, because with perfect information investors would get 0 ($r = 0$), whereas they get, in expectation, a positive return under imperfect information. In a sense, the monetary authority is willing to pay investors to stay longer in order to have a higher probability that the peg be saved and investors would like to "coordinate" into staying for longer and receive this payment. However, this coordination breaks down unless investors have private information.[44]

Finally, the results presented in this section depend crucially on the type of crisis the country is facing. When the probability of a turnaround is high, the monetary authority gains more by

[44]We cannot make strong claims as to the welfare implications of disaggregated information since this paper ignores important ingredients of BOP crises, such as moral hazard considerations. However, although this might imply that the monetary authority should require financial institutions to provide information regarding their activities, our results suggest that there is a case for not making this information public.

preannouncing a strong interest rate defense, not only because it is more likely that by delaying the crisis the peg could be saved, but also because it is less likely that the costs associated with the interest rate defense will be incurred. In other words, the peg is defended by committing to raising interest rates in the few cases in which the turnaround does not take place. As a result, it is optimal to have a more aggressive interest rate defense in cases of liquidity crises than in cases of solvency crises.

6 Robustness and Alternative Scenarios

Many of the ingredients in the model were introduced in reduced form. Apart from making the model more tractable, the reduced-form approach allows for a fairly general interpretation of the results. However, special attention needs to be paid to the question of robustness. This section explains which assumptions are essential for the results, and in which scenarios they are likely to be valid.

Policy instruments:

In this paper, we assume that the monetary authority controls domestic interest rates. This can be rationalized by assuming that domestic and foreign assets are imperfect substitutes. For example, in the model by Lahiri and Végh (2000) the monetary authority controls the return on a domestic bond which, as opposed to the foreign bond, provides liquidity services.

On the other hand, our model does not rely, in "spirit," on deviations from uncovered interest parity. Rather, the main point is that investors' actions depend not only on present interest rates but also on the path of expected future rates. When facing a "tough" monetary authority, investors expect that the monetary authority will raise interest rates sharply if fundamentals became very weak. Hence, investors are less likely to take their capital out for intermediate values of fundamentals. When, and if, fundamentals deteriorate, interest rates rise and the crisis becomes more likely, consistent with uncovered interest parity.

In a more general setting, the monetary authority could benefit from having reserves after the peg is abandoned. The problem of time inconsistency would likely be more severe in this case, as the monetary authority would have an incentive to abandon the peg before reserved are exhausted.

Constrained specialists and excess returns:

Investors receive excess returns from their investments in the domestic economy since they are indifferent between investing in the country and abroad only at the point at which they take their capital out. To avoid that private information be revealed by capital flows in the run-up to the crisis, we assume that investors are financially constrained and that the supply of capital is perfectly inelastic. However, even under a less than perfectly inelastic supply of capital, investors are probably not able to easily infer other investors' types by observing capital flows, as long as there is enough noise and uncertainty in the economy.[45]

Moreover, adding uninformed investors with a more elastic supply of capital actually makes our results stronger. For example, assume that uninformed investors bring an amount of capital $k(\bar{r}(t))$, where $\bar{r}(t)$ are excess returns which take into account expected devaluation losses for investors who do not know a_1 or a_2. In addition, assume that when a group pulls out, the uninformed investors have the same probability of taking their capital out before the devaluation as the group of investors that did not initiate the attack. The only difference this would make to the equilibrium comes from the ratio $\left(\frac{2a(t)-f(t)}{a(t)}\right)$ in equation (3), which would be replaced by $\left(\frac{2a(t)-f(t)}{a(t)+k(\bar{r}(t))}\right)$. This would imply that learning could take place even faster, and high interest rates would postpone the crisis even more than before.[46]

Sources of private information:

If we assumed that investors have private information about the post-devaluation exchange rate, without assuming that there is any relationship between "types" and holdings of liquid assets, the results of the model would not change. However, we prefer to assume that this information is due to some characteristic that also affects the size of the outflows because, otherwise, we would need to assume that reserves are enough to cover one group but not both. In the model presented in this paper, this constraint is satisfied without any special assumptions on initial reserves.

Other sources of private information that likely play important roles during BOP crises include: (i) risk characteristics of bank lending, since banks have better information about their own clients than about those of other banks; (ii) liquidation value of investments; (iii) outside opportunities of investors; (iv) margin calls investors would be forced to make if a crisis occurs; and (v) investors'

[45] In the context of herding in financial markets, Avery and Zemsky (1998) show that if there exists uncertainty in a large enough number of dimensions, it can take a long time for investors to learn even if they observe at which price other investors trade. In addition, off balance sheet transactions can obscure investors' positions.

[46] The existence of excess returns for informed investors does not mean that the free entry condition to become a specialist does not hold ex-ante, since this decision is made earlier on and might involve costs.

assessments about the prospects of the country.

Information structure:

The assumption of aggregate uncertainty is necessary for the results and, as a result, it is important that there be a small number of groups. However, similar results can be obtained with a unimodal distribution if two requirements are met: there exist "steep" edges or discontinuities, because if the distribution were smooth private information would be revealed slowly; in addition, if an investor with type a knows that the discontinuity in the distribution is to the left of $a + \epsilon$, he must assign a probability to the discontinuity being in $[a, a + \epsilon]$ that goes to zero as ϵ goes to zero. A distribution that satisfies these requirements is the one used in Chamley (1998): a rectangular distribution whose position is unknown, on top of a wider rectangular distribution. These two assumptions can be somewhat relaxed in the presence of observation "noise."

We chose to assume the existence of two groups for a number of reasons. First, this allows for the existence of "probing attacks," which are necessary to illustrate the one-sided arbitrage channel in which private information delays the crisis. Second, with the unimodal distribution we would need to make an ad-hoc assumption about the order in which investors that did not initiate the attack access foreign currency reserves, which might make the results suspect. Third, there are actually different types of investors and sometimes crises can be traced to the actions of one of them. For example, some observers argue that the behavior of hedge funds was important in the onset and spread of the Asian crisis, and versions exist about the Russian crisis being triggered by domestic investors refusing to roll over Russia's short-term debt which prompted a similar response from foreign investors.

Different scenarios:

There are a number of scenarios that can be represented by the model presented in this paper. Most literally, one could think that investors have their capital deposited in domestic currency at local banks, and decide whether to withdraw their deposits and exchange them for foreign currency. Or that they have to decide whether to attack the currency by borrowing in domestic currency at the prevailing interest rate. Another possible scenario is that of a government which is trying to roll over short-term debt, with investors deciding whether the spreads compensate for the risk of default. Even if government finances are in order, the private sector (especially domestic banks) might face similar liquidity needs. The fundamental could then represent domestic credit, as in first-generation models, the size of government's or banks' short-term liabilities, or the size of bad loans in the financial sector. In these cases, as in our model, investors receive high returns while the crisis does not occur, their decisions about whether to invest and receive these returns have an effect on the timing of the crisis, and there is an incentive to be the first to leave.

7 Concluding Remarks

This paper presents a framework for understanding the dynamics and timing of BOP crises. It shows that the presence of private information on the part of investors in a simple first-generation model can account for important features of BOP crises. First, crises can involve large drops in asset prices in the absence of large shocks even in a single equilibrium model. Second, even in countries whose fundamentals are known to be weak high interest rates can delay the onset of crises.

The paper shows that the effectiveness of interest rate defenses increases with the degree of private information. When interest rates are low *or* there is no private information, pegs are abandoned at a time such that the exchange rate is continuous. When interest rates are high *and* there is private information, pegs last longer and are abandoned at a point such that the exchange rate suffers a discrete devaluation.

In addition, the paper shows that the optimal interest rate policy in episodes of BOP crises is to sharply raise interest rates when fundamentals become very weak, rather than raising interest rates by a smaller amount for a longer period of time. However, a problem of time inconsistency arises. The monetary authority has an incentive to deviate and not to raise interest rates once fundamentals become weak enough. This emphasizes the importance of commitment devices such as currency boards or a role for international financial institutions such as the IMF.

The model also shows that crises are more likely when interest rates are high, even conditioning on the level of fundamentals. This has important implications for empirical studies on the effectiveness of interest rate defenses against BOP crises (such as Kraay (1999)). For example, an episode in which interest rates are raised but the monetary authority is nonetheless forced to abandon the peg could be taken as evidence that raising interest rates is not very useful in defending a currency under attack. However, in the model presented in this paper pegs are more likely to survive if interest rates are expected to be sharply raised in the future (i.e. strong defense) even though crises are more likely while interest rates are high.[47]

If the dimension along which investors have private information reflects some "intrinsic" characteristic, the model can be easily extended to account for the phenomenon of contagion. In such an extension, crises would only be transmitted to countries whose fundamentals are sufficiently weak.[48] In addition, an externality would exist between the setting of monetary policy in different countries since, by delaying the crisis in one country, monetary authorities delay the learning process in all other countries as well.

Finally, the framework presented in this paper can probably be applied to episodes of asset market bubbles and banking crises. The relationship between the rate of return on domestic assets and the probability of a currency crisis is analogous to the relationship between the high returns to holding an asset with a bubble and the probability that the bubble "bursts." Under the assumption that investors have private information regarding the fundamental value of the asset (i.e. its price when the bubble bursts), bubbles could probably exist even if investors know that the bubble cannot last forever. In episodes of banking crises, the relationship between the interest rate on deposits and the probability that some investors assign to the bank failing due to other investors' withdrawals poses a similar trade off on investors' actions.

[47] In the aftermath of the Brazilian devaluation in January 1999 the Argentine peso suffered very little pressure. This is likely because investors knew that interest rates would be sharply raised in case of a speculative attack due to the strong commitment to the currency board. After the Mexican devaluation in December 1994, when this commitment had not been previously tested, the pressure on the Argentine peso was much greater.

[48] See Tornell (1998) for evidence that crises are more likely to be transmitted to countries with weak fundamentals.

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A Analysis of Model with Constant r

Here we present a more formal analysis of the model. Since investors are atomistic and there are no transaction costs, investors' actions are taken so as to maximize expected returns pointwise. Thus, the model can be divided in different "stages," depending on the information investors have about each other. At the beginning of the first stage, types are private information. As the stage progresses, information is slowly revealed until a first "crisis" occurs, in which a group of investors pulls out, revealing its type. If the attack is not large enough to force the abandonment of the peg, a second stage begins, which is similar to the first, except that now the type of one of the groups is common knowledge. The "interaction" between the different stages is limited, in the sense that the different stages can be analyzed almost independently.

The first stage of the game lasts until some investors "move" for the first time. To be more precise, let

$$A_i(t) = \int_j (1 - x(i, j, t))\, dj$$

be the proportion of their capital taken out by investors in group i. The first stage of the game ends at time

$$t_1 = \sup \left\{ \tau : \forall \tau' \in (\underline{t}, \tau] \ A_1(\tau') = 0 \text{ and } A_2(\tau') = 0 \right\}$$

since that is the first time at which some investors observe a "movement" by investors in the other group.

First Stage:

Let

$$\begin{aligned} a_i(t, \nu) \quad = \quad & \left\{ a_i : \sup \left\{ \tau : \forall \tau' \in (\underline{t}, \tau] \ A_i(\tau') = 0 \right.\right. \\ & \left.\left. \text{if } \forall \tau' \in (\underline{t}, \tau] \ A_{-i}(\tau') = 0 \right\} \in (t - \nu, t + \nu) \right\} \end{aligned}$$

be the set of types a_i such that, conditional on not having observed any movement by investors in the other group, the earliest time a positive amount of capital from investors in group i leaves the country falls in the interval $(t-\nu, t+\nu)$.[49] Let

$$a_i(t) = \bigcap_{\nu>0} a_i(t,\nu)$$

be the set of types a_i that would start pulling out exactly at t, conditional on the other group not having pulled out before. Let

$$T = \{t : a_i(t) \neq \emptyset \text{ or } a_{-i}(t) \neq \emptyset\}$$

Proposition 7. *In equilibrium, $\exists t_0 \in \left(-\frac{2(a_M - a_m)}{\mu}, 0\right)$ such that, for $i \in \{1, 2\}$, $a_i(t)$ is a continuous strictly-decreasing function of t for $t \in [t_0, 0]$, $a_i(0) = a_m$, $a_i(t_0) = a_M$, and $A_i(t) = 0$ for $t < t_0$.*

Proof: The proof contains several intermediate steps:

(i) *For all t and ν, $a_1(t,\nu) = \emptyset$ iff $a_2(t,\nu) = \emptyset$. In addition, $\forall a_i \in a_i(t,\nu)$ $\exists a_{-i} \in a_{-i}(t,\nu)$ such that $a_i + a_{-i} - f(t+\nu) - e_0 > 0$.* This follows from the fact that investors can only condition their actions at t on flows up to $t-\epsilon$ (TA2). As a result, an investor in group i would not pull out at time $t' \in (t-\nu, t+\nu)$ if the probability of the other group pulling out is zero or if, even if the probability is positive, the crisis cannot bring about a positive devaluation.

(ii) *For all t, $a_i(t) = \emptyset$ iff $a_{-i}(t) = \emptyset$. In addition, $\forall a_i \in a_i(t)$ $\exists a_{-i} \in a_{-i}(t)$ such that $a_i + a_{-i} - f(t) - e_0 \geq 0$.* This follows from (i) and the definition of $a_i(t)$.

(iii) *If $a' \in a_i(t)$, $a'' \in a_i(t)$, $a'' > a'$, then $[a', a''] \subseteq a_i(t)$.* This depends on the form of the equilibrium in the second stage of the game, which will be analyzed below. For now, we just need that if a group starts pulling out and $a_i + a_{-i} - f(t) - e_0 > 0$ a crisis with positive devaluation takes place immediately with probability 1. As a result, if an investor of type a' finds it optimal to leave (or is indifferent between leaving and staying) then an investor with more liquid investments will strictly prefer to leave. In addition, if an investor of type a'' has not left before time t, then an

[49] It is not possible for investors to play mixed-like strategies in the first stage of the game in equilibrium. The reason is that if investors are indifferent between moving at two different times when they are of type a_i, they will strictly prefer to move at the earlier (later) time when their type is higher (lower) than a_i. As a result, since the probability of investors being of a certain type is zero (i.e. $g(a)$ has no atoms), the mass of investors who can play mixed-like strategies is zero. The situation is different in the following stages, since the type of one group of investors is common knowledge; hence, that group can play mixed-like strategies.

investor with less liquid assets would have strictly preferred to stay until t.

(iv) *For* $i = 1{,}2$ *and for all* t $a_i(t)$ *is either empty or a single point.* First, $a_i(t)$ cannot have positive measure. If it did, there would be a positive probability of group i reaching its "threshold value" at t. As a result, for all $a_i \in a_i(t)$ and $a_{-i} \in a_{-i}(t)$, $a_i + a_{-i} - f(t) - e_0 \leq 0$ since otherwise there would be a positive probability of crisis with positive devaluation at t which cannot occur in equilibrium. But then there would be $a_i' \in a_i(t)$ such that $a_i' + a_{-i} - f(t) - e_0 < 0$ for all $a_{-i} \in a_{-i}(t)$ which contradicts (ii). Since $a_{-i}(t)$ cannot be empty either due to (ii), we conclude that $a_i(t)$ cannot have positive measure. This, together with (iii) implies (iv).

(v) T *is dense in* $T' \equiv [\inf\{T\}, \sup\{T\}]$. That T is dense at $\inf\{T\}$ and $\sup\{T\}$ is obvious. Now assume $\exists\ \tau_1,\tau_2 \in T'$ such that $[\tau_1, \tau_2] \bigcap T = \emptyset$. Let $\tau_1' = \sup\{t \in T : t < \tau_1\}$. Now we use the assumption that takes the model to be the limit of a model with transaction costs as these costs tend to zero (TA3). For any positive transaction cost, and regardless of how short $[\tau_1, \tau_2]$ is, $\exists \tau_1'' \in T$ that is so close to τ_1' that the probability of having a crisis before τ_1 is low enough so that $a_i(\tau_1'')$ has to be empty.[50] This contradicts $\tau_1'' \in T$.

(vi) $T = T'$. Since $a_i(t)$ is either empty or a single point, and for every $a \in [a_m, a_M]$ $\exists t$ such that $a \in a_i(t)$, we can define a function $t_i(a) : [a_m, a_M] \to T'$ as the inverse of $a_i(t)$. $t_i(a)$ is decreasing and, from (v), its image is dense in T'. This implies $T = T'$.[51]

(vii) *For* $i = 1{,}2$, $a_i(t) : T \to [a_m, a_M]$ *are continuous strictly decreasing functions.* This follows from the fact that $a_i(t)$ is 1-to-1 and decreasing.

(viii) $T = [t_0, 0]$, *where* $t_0 \in \left(-\frac{2(a_M - a_m)}{\mu}, 0\right)$. If $t_i(a_m) < 0$, as $t \to t_i(a_m) < 0$ the expected devaluation losses would tend to infinite since the hazard rate of crisis tends to infinite while the size of devaluation does not tend to zero. In addition, in equilibrium an investor of type a_m would not leave at $t_i(a_m) > 0$ because the devaluation would be negative with probability 1, and he would prefer to stay longer. Finally, $t_0 < -\frac{a_M - a_m}{\mu}$ is impossible because investors would not leave if, even in the case where $a_1 = a_2 = a_M$, liquid investment are not large enough to exhaust all reserves. $\square$

[50] Investors would compare an arbitrarily small probability of crisis with the transaction costs associated with leaving at τ_1'' and coming back right after τ_1', or the losses associated with being out of the country at a time when a crisis cannot take place (which is at least $r(\tau_2 - \tau_1)$).

[51] This can be easily proved. Assume $\tau \in T'$ but $\tau \notin T$. Since T is dense, $\exists\{\tau_1, \ldots, \tau_n, \ldots\}$ such that $\tau_n \to \tau$, $\tau_1 < \cdots < \tau_n < \tau_{n+1} \cdots < \tau$, and $\forall n\ \tau_n \in T$. (For $\tau = \inf\{T\}$ a symmetric argument applies.) Let, for all n, $\alpha_n \equiv a_i(\tau_n)$. Then $\{\alpha_1, \ldots, \alpha_n, \ldots\}$ is a bounded decreasing sequence. Let α be its limit. If $t_i(\alpha) < \tau$, then τ_n is bounded away from τ and $\tau_n \not\to \tau$. If $t_i(\alpha) > \tau$, then it is impossible that $\tau_n < \tau\ \forall n$.

Proposition 8. *The first stage of the game has a unique and symmetric equilibrium. The equilibrium is characterized by a function $a(t)$, which denotes the type of investors that would take their capital out at time t. $a(t)$ is continuous, strictly-decreasing, differentiable, and is the unique solution to differential equation*

$$r = \frac{g(a(t))}{G(a(t))}(-\dot{a}(t))\left(\frac{2a(t) - f(t)}{a(t)}\right)(2a(t) - f(t) - e_0) \tag{9}$$

that satisfies the boundary condition

$$a(0) = a_m. \tag{10}$$

Note: The function $a(t)$ is only defined for $t \in [t_0, 0]$, where t_0 satisfies $a(t_0) = a_M$.

Proof: The proof contains several intermediate steps:

(i) *For $i = 1{,}2$, $a_i(t) : T \to [a_m, a_M]$ is differentiable.* Since $a_i(t)$ is monotone it must be differentiable almost everywhere. (See Kolmogorov and Fomin (1970) page 321.) Thus, if $a_i(t)$ is not differentiable at τ, $\exists \nu > 0$ such that $a_i(t)$ is differentiable at all points in $[\tau - \nu, \tau + \nu]$ except at τ. We want to show that $a_i(t)$ must be differentiable from the left and from the right at τ. Assume it is not differentiable from the left. This means that $\exists \epsilon_l$ such that $\forall \nu' > 0$, $[\max\{\dot{a}_i(t) : t \in (\tau - \nu', \tau)\} - \min\{\dot{a}_i(t) : t \in (\tau - \nu', \tau)\}] > \epsilon_l$. But this is not possible because, since $a_{-i}(t)$ is monotone, the hazard rate of group 1's reaching its threshold value (which equals $\mu(-\dot{a}_i(t))$) cannot decrease arbitrarily fast. As a result, the derivatives from the left and from the right must exist at τ. But they cannot be different because $a_{-i}(t)$ is continuous at τ, which implies $a_i(t)$ is differentiable at τ.

(ii) *For $i = 1{,}2$ all investors in group i move simultaneously and take all their capital out at $t_i(a_i)$.* This follows from the fact that, for all $t > t_i(a_i)$, $\exists a_i' < a_i$ such that investors would leave at $t = t_i(a_i')$ if their type were a_i'. But that means that for all $t > t_i(a_i)$ investors of type a_i are strictly worse off staying in the country than outside and, as a result, they would leave at $t_i(a_i)$.

(iii) *$a_1(t)$ and $a_2(t)$ are solutions to the system of differential equations*

$$
\begin{aligned}
r &= \frac{g(a_1(t))}{G(a_1(t))}(-\dot{a}_1(t))\left(\frac{a_1(t)+a_2(t)-f(t)}{a_2(t)}\right)(a_1(t)+a_2(t)-f(t)-e_0) \\
r &= \frac{g(a_2(t))}{G(a_2(t))}(-\dot{a}_2(t))\left(\frac{a_2(t)+a_1(t)-f(t)}{a_1(t)}\right)(a_2(t)+a_1(t)-f(t)-e_0)
\end{aligned}
$$

and satisfy

$$a_1(0) = a_2(0) = a_m$$

$$a_1(t_0) = a_2(t_0) = a_M$$

for some $t_0 \in \left(-\frac{a_M - a_m}{\mu}, 0\right)$. The boundary conditions were obtained in proposition 7. The form of the differential equations is derived in the main text.

(iv) $\forall t\ a_1(t) = a_2(t)$. The system of differential equations that determine $a_1(t)$ and $a_2(t)$ is symmetric and, in addition, $a_1(t)$ and $a_2(t)$ must satisfy the same initial condition. As a result, $a_1(\cdot) = a_2(\cdot)$. □

Later Stages:

Without loss of generality, we assume that group 2 is the one that started pulling out at the end of the first stage and, as a result, a_2 is common knowledge. In addition, agents in the second group know that $a_1 \in [a_m, a_2]$. We start the analysis of the second stage assuming that $f(t)$ is high enough when the first stage ends, so that a crisis cannot occur immediately, i.e.

$$f(t_1) > 2a_2 - e_0.$$

This is impossible in equilibrium, but it is easier to start with this case.

Let

$$
\begin{aligned}
a^1(t,\nu) = \ & \big\{a_1 : \sup\big\{\tau : \forall\tau' \in (t_1, \tau]\ A_1(\tau') = 0 \\
& \text{if } \forall\tau' \in (t_1, \tau]\ A_2(\tau') = 0\big\} \in (t-\nu, t+\nu)\big\}
\end{aligned}
$$

be the set of types a_1 such that, conditional on not having observed any movement by investors in group 2, the earliest time a positive amount of capital from investors in group 1 leaves the country falls in the interval $(t-\nu, t+\nu)$.[52] Let

$$a^1(t) = \bigcap_{\nu>0} a^1(t,\nu)$$

be the set of types a_1 that would start pulling out exactly at t, conditional on group 2 not having pulled out before.

The characterization of group 2's play is different from that of group 1, because a_2 is known and, as a result, investors in group 2 can play mixed-like strategies. Let

$$\begin{aligned} d(t,\nu) &= \Pr\Big[\sup\big\{\tau : \forall\tau' \in (t_1,\tau]\ A_2(\tau')=0 \\ &\qquad \text{if } \forall\tau' \in (t_1,\tau]\ A_1(\tau')=0\big\} \in (t-\nu,t+\nu)\Big] \end{aligned}$$

be the probability that, conditional on not having observed any movement by investors in group 1, the earliest time a positive amount of capital from investors in group 2 leaves the country falls in the interval $(t-\nu, t+\nu)$. Let

$$d(t) = \lim_{\nu\to 0} \frac{d(t,\nu)}{2\nu}$$

be the probability density of investors in group 2's starting to pull out exactly at t, conditional on group 1 not having pulled out before.[53] Let

$$T = \big\{t : a^1(t) \neq \emptyset \text{ or } d(t) > 0\big\}$$

and

$$t_m = -\frac{a_2 - a_m}{\mu}.$$

[52] As in the first stage, group 1 cannot play a mixed-like strategy in equilibrium because the are no points with positive mass in the distribution of types a_1.

[53] If $\lim_{\nu\to 0} d(t,\nu) > 0$, we can define $d(t)$ like a distribution with positive mass at t, i.e. a "delta function." But it is not necessary to worry much about this because in equilibrium, as will be shown below, the limit always exists.

Proposition 9. *In equilibrium,* $\exists t_0^1 \in \left(-\frac{2(a_2-a_m)}{\mu}, t_m\right)$ *such that* $a^1(t)$ *is a continuous strictly-decreasing function of* t *and* $d(t) > 0$ *for* $t \in (t_0^1, t_m)$*,* $a^1(t_m) = a_m$*,* $a^1(t_0^1) = a_2$*,* $\int_{t_0^1}^{t_m} d(t)\, dt = 1$*, and* $A_i(t) = 0$ *for* $t < t_0^1$*.*

Proof: It is not necessary to present it because it is very similar to the proof of proposition 7.

Proposition 10. *When* a_2 *is common knowledge,* $a_1 \in [a_m, a_2]$*, and time starts at* $t < -\frac{2(a_2-a_m)}{\mu}$*, the game has a unique equilibrium in which all investors in each group share the same strategies. The equilibrium is characterized by a function* $a^1(t)$*, which denotes the type of investors in group 1 that would take their capital out at time* t*, and* $d(t)$*, which denotes the probability density of investors in group 2's pulling out. The function* $a^1(t)$ *is continuous, strictly-decreasing, differentiable, and is the unique solution to differential equation*

$$r = \frac{g(a^1(t))}{G(a^1(t))}(-\dot{a}^1(t))\left(\frac{a^1(t) + a_2 - f(t)}{a_2}\right)\left(a^1(t) + a_2 - f(t) - e_0\right) \tag{11}$$

that satisfies the boundary condition

$$a^1(t_m) = a_m. \tag{12}$$

The function $d(t)$ *is given by*

$$d(t) = \frac{d}{dt}\left(\frac{e^{\int_{t_1}^{t} h(\tau)d\tau}}{e^{\int_{t_1}^{t_m} h(\tau)d\tau}}\right)$$

where $h(t)$ *is the hazard rate of investor 2's pulling out, which satisfies*

$$r = h(t)\left(\frac{a^1(t) + a_2 - f(t)}{a^1(t)}\right)\left(a^1(t) + a_2 - f(t) - e_0\right). \tag{13}$$

Note: The function $a^1(t)$ *is only defined for* $t \in [t_1, t_m]$*, where* t_1 *satisfies* $a^1(t_1) = a_M$*.*

Proof: It is not necessary to present it because it is very similar to the proof of proposition 8. The only difference is in step (ii) of the proof. In this case, it is not necessary that investors in group 2 move simultaneously if there is a positive response time. However, only the equilibrium in which they move simultaneously survives in the limit as the response time goes to zero (TA2). Also, note that although in the proofs we used $d(t)$, $h(t)$ is a more useful characterization of the equilibrium. $\square$

Now consider a case when

$$f(t_1) < 2a_2 - e_0.$$

In this case there are multiple equilibria; however, one of them strongly *dominates* the others. Let

$$\underline{a}(t; a_2) \equiv \max\{f(t) - a_2 + e_0, a_m\} \tag{14}$$

be the lowest a_1 such that $E_s(t_1) \leq 1$ (or a_m if no such a_1 exists). Also let

$$\bar{a}(t; a_2) \equiv \min\{a_2, a^1(t; a_2)\}.$$

Proposition 11. *When a_2 is common knowledge, $a_1 \in [a_m, a_2]$, and time starts at $t_1 > -\frac{2(a_2 - a_m)}{\mu}$, the game has multiple equilibria. The equilibria are characterized by $a^* \in [\underline{a}(t_1; a_2), \bar{a}(t_1; a_2)]$. Investors in group 2 try to take their capital out immediately. Investors in group 1 do the same if $a_1 \in (a^*, a_2]$. If $a_1 \in (a^*, a_2]$, reserves are exhausted and the peg is abandoned immediately. Otherwise, investors in group 2 learn that $a_1 \leq a^*$ and bring their capital back. After that point the game follows the unique equilibrium described in proposition (10). Namely, there is a period in which no attack can occur, which lasts until time $t_0^1(a^*)$ such that $a_1(t_0^1(a^*); a_2) = a^*$. After $t_0^1(a^*)$ the learning process starts, following $a^1(t; a_2)$ and $h(t, a_2)$.*

Proof: In the proposed equilibria no investor has an incentive to deviate. First, at the beginning of the game investors in group 2 have an incentive to leave because there is a positive probability that group 1's type is such that a crisis immediately follows. In addition, an investor in group 1 also has an incentive to leave immediately if $a_1 > a^*$ because he knows that all other investors will leave and, as a result, he would suffer devaluation losses if he stayed. After that point, the proposed strategies constitute a unique equilibrium as proved in proposition (10). That no other equilibria exist follows from the fact that, if $a_1 < f(t_1) - a_2 + e_0$, there would be a *revaluation* if the peg were abandoned immediately and, as a result, investors in group 1 would not leave. $\square$

Out of the continuum of possible equilibria, the one that corresponds to $a^* = \underline{a}(t_1; a_2)$ *dominates* the others. For example consider an equilibrium corresponding to $a^* > \underline{a}(t_1; a_2)$. Imagine, though, that an investor in group 1 thinks there is an arbitrarily small probability $\epsilon > 0$ that the equilibrium is actually the one corresponding to $a^{*\prime} \in [\underline{a}(t_1; a_2), a^*)$. Then if that investor had a type $a_1 \in [a^{*\prime}, a^*)$, he would think there is a *positive* probability ϵ that the devaluation takes place immediately. As a result, he would have an incentive to deviate and leave for a brief moment, to return only after observing that, in fact, the other investors in group 1 did not leave. The equilibrium corresponding to a^* should then not be expected to be played. Note that the equilibrium corresponding to a^* is dominated not only by the one corresponding to $\underline{a}(t_1; a_2)$, but also by all intermediate equilibria. In addition, investors would deviate even if they assign an arbitrarily small probability of deviation by other investors. The equilibrium corresponding to $a^* = \underline{a}(t_1; a_2)$ then strongly dominates all others.[54]

We can now give a full description of the equilibrium of the game when both types are private information.

Proposition 12. *When both a_1 and a_2 are private information, $a_1, a_2 \in [a_m, a_M]$, and time starts at $\underline{t} < -2\frac{(a_M - a_m}{\mu}$, the game has a unique and symmetric equilibrium, with a multi-stage structure. In the first stage, investors stay in the country until time t_1 such that $\max\{a_1, a_2\} = a(t_1)$, where $a(t)$ is determined by equations (9) and (10). Without loss of generality, we assume $a_2 > a_1$. At time t_1, investors in group 2 leave. If $a_1 > \underline{a}(t_1; a_2)$, where $\underline{a}(t_1; a_2)$ is defined in equation (14), investors in group 1 also leave, reserves are exhausted and the peg is abandoned. Otherwise, investors in group 2 return and the second stage begins. Investors stay in the country until investors in group 1 pull out when $a_1 = a^1(t)$, where $a^1(t)$ is determined by equations (11) and (12), or until investors in group 2 pull out, which occurs with hazard rate $h(t)$, where $h(t)$ is determined by equation (13). If the "attack" is initiated by investors in group 1, investors in group 2 follow, reserves are exhausted, and the peg is abandoned. If it is initiated by investors in group 2 and $a_1 > \underline{a}(t; a_2)$ investors in group 1 follow, reserves are exhausted, and the peg is abandoned. Otherwise, investors in group 2 return and stage 3 begins. The game then continues with all other stages being identical to stage 2.*

[54] This is an extreme form of *risk dominance*. See Harsanyi and Selten (1988) for an introduction to the concept of risk dominance.

B Government's Problem with Commitment

The government minimization problem involves finding the first order condition with respect to changes in the interest rate at every point in time. However, given the boundary conditions on $a(t)$, the interest rate at some other point in time needs to adjust accordingly. In particular, we choose t_0 as this "reference" point. For any functional F, let

$$\frac{\partial F[\{r(s)\}]}{\partial r(v)} = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left. \frac{dF[\{\bar{r}(s; v, r, \epsilon)\}]}{dr} \right|_{r=r(v)}$$

where

$$\bar{r}(s; v, r, \epsilon) = \begin{cases} 0 & \text{for } s < t_0 \\ r_0(v, r, \epsilon) & \text{for } s \in [t_0, t_0 + \epsilon) \\ r(s) & \text{for } s \in [t_0 + \epsilon, v - \frac{\epsilon}{2}) \\ r & \text{for } s \in [v - \frac{\epsilon}{2}, v + \frac{\epsilon}{2}] \\ r(s) & \text{for } s \in (v + \frac{\epsilon}{2}, 0] \end{cases}$$

and $r_0(v, r, \epsilon)$ is such that the boundary conditions are satisfied. This definition is just a way of formalizing the simple idea that we look at the effect of changes in the interest rate at times close to v, tracking their effect on r_0.

We start by determining how changes in $r(v)$ affect $a(\tau)$. It can be shown that

$$\frac{\partial a(\tau)}{\partial r(v)} = -\frac{\partial \dot{a}(v)}{\partial r(v)} \phi(\tau, v)$$

where

$$\phi(\tau, v) = \begin{cases} e^{-\int_\tau^v \frac{\partial \dot{a}(l)}{\partial a(l)} dl} & \text{for } \tau < v \\ \frac{1}{2} & \text{for } \tau = v \\ 0 & \text{for } \tau > v \end{cases}$$

The reason why $\frac{\partial a(\tau)}{\partial r(v)} = 0$ for $\tau > v$ is that $a(\tau)$ only depends on future interest rates, as the equilibrium is obtained through backward induction. For $\tau < v$, on the other hand, $r(v)$ affects $a(\tau)$ not only through its direct effect on $\dot{a}(v)$, but also through its indirect effect on $\dot{a}$ for all intermediate times.

We can then obtain the effect of changes in $r(v)$ on r_0.[55]

$$\frac{\partial r_0}{\partial r(v)} = -\frac{k(v,a(v))}{k(t_0,a(t_0))}\phi(t_0,v).$$

The effect of changes in $r(v)$ on $L[t,\{r(s)\}]$ is calculated by taking into account the direct effects on $c(r(v))$ and the indirect effects through the path $\{a(\tau)\}_{\tau=t}^{\tau=v}$. It is given by

$$\begin{aligned}
& e^{-\rho t}\frac{\partial L[t,\{r(s)\}]}{\partial r(v)} = \\
& \quad -e^{-\rho t_0}\frac{k(v,a(v))}{k(t_0,a(t_0))}\phi(t_0,v)\left[c'(r(t_0)) + 2g(a(t_0))D(t_0,a(t_0))k(t_0,a(t_0))\right] + \\
& \quad +e^{-\rho v}\left[G(a(v))^2 c'(r(v)) + 2g(a(v))G(a(v))D(v,a(v))k(v,a(v))\right] + \\
& \quad +\int_{t_0}^{v}\left[2g(a(\tau))G(a(\tau))c(r(\tau)) + 2g(a(\tau))^2 D(\tau,a(\tau))r(\tau)k(\tau,a(\tau)) + \right. \\
& \qquad 2g'(a(\tau))G(a(\tau))D(\tau,a(\tau))r(\tau)k(\tau,a(\tau)) + \\
& \qquad 2g(a(\tau))G(a(\tau))D(\tau,a(\tau))r(\tau)k_a(\tau,a(\tau)) + \\
& \qquad \left. 2g(a(\tau))G(a(\tau))D_a(\tau,a(\tau))r(\tau)k(\tau,a(\tau))\right]k(v,a(v))\phi(\tau,v)e^{-\rho\tau}d\tau
\end{aligned} \tag{15}$$

where the subscript "a" means partial derivative with respect to a.

We then set $\frac{\partial L[t,\{r(s)\}]}{\partial r(v)} = 0$, use $\phi(\tau,v) = \frac{\phi(t_0,v)}{\phi(t_0,\tau)}$ and $\frac{d}{dv}[k(v,a(v))\phi(t_0,v)] = k_t(v,a(v))\phi(t_0,v)$, and take the derivative with respect to v. After a few cancellations and rearrangements, we obtain

$$\begin{aligned}
\dot{r}(s) \;=\; & \frac{1}{c''(r(s))}\left\{[\rho D(s,a(s)) - D_t(s,a(s))]\,2\frac{g(a(s))}{G(a(s))}k(s,a(s)) + \right. \\
& c'(r(s))\left[\rho + r(s)2\frac{g(a(s))}{G(a(s))}k(s,a(s))\right] - \\
& \left. c(r(s))2\frac{g(a(s))}{G(a(s))}k(s,a(s)) + c'(r(s))\frac{k_t(s,a(s))}{k(s,a(s))}\right\}.
\end{aligned}$$

Apart from this first-order differential equation, we need a boundary condition to fully characterize $r(s)$. Assume $r(t_0) > 0$ and consider the alternative definition for the effect of $r(v)$ on the

[55] Note that $\frac{\partial \dot{a}(v)}{\partial r(v)} = -k(v,a(v))$.

functional $F[\{r(s)\}]$

$$\frac{\partial F[\{r(s)\}]}{\partial r(v)} = \lim_{\epsilon \to 0} \frac{1}{\epsilon} \left. \frac{dF[\{\tilde{r}(s; v, r, \epsilon)\}]}{dr} \right|_{r=r(v)}$$

where

$$\tilde{r}(s; v, r, \epsilon) = \begin{cases} 0 & \text{for } s < t_0(v, r, \epsilon) \\ r(t_0) & \text{for } s \in [t_0(v, r, \epsilon), t_0) \\ r(s) & \text{for } s \in [t_0, v - \frac{\epsilon}{2}) \\ r & \text{for } s \in [v - \frac{\epsilon}{2}, v + \frac{\epsilon}{2}] \\ r(s) & \text{for } s \in (v + \frac{\epsilon}{2}, 0] \end{cases}$$

and $t_0(v, r, \epsilon)$ is such that the boundary conditions are satisfied. Namely, t_0 is adjusted instead of r_0 as above.[56] It is easy to show that

$$\frac{\partial t_0}{\partial r(v)} = \frac{k(v, a(v))}{k(t_0, a(t_0))} \frac{\phi(t_0, v)}{r(t_)}.$$

and that, as a result, $\frac{\partial L[t, \{r(s)\}]}{\partial r(v)}$ only differs from the one above in its first term. Equalizing the first terms from the two expressions and cancelling common factors we obtain

$$c'(r(t_0) = \frac{c(r(t_0))}{r(t_0)}.$$

Since $c(0) = 0$ and $c(\cdot)$ is convex, this expression is satisfied (in the limit) if and only if $r(t_0) = 0$. This contradicts our assumption that $r(t_0) > 0$ and, as a result, the boundary condition is given by[57]

$$r(t_0) = 0.$$

[56] Note the abuse of notation in that t_0 is both a constant and a function.

[57] Basically, we have shown that as long as $r(t_0) > 0$ the loss function can be reduced by "smoothing out" $r(t_0)$, while preserving the boundary conditions on $a(s)$.