

Final Exam

Wednesday March 23

Question 1. Short answer questions (30 points)

1. (10 points) What is the main prediction of Tobin's q-theory of investment? How can it be tested?
2. (10 points) Durable consumption is more volatile than non-durable consumption. Therefore, if we would include durable expenditures in our measure for aggregate consumption, we would be less likely to find excess smoothness but more likely to find excess sensitivity.
3. (10 points) True, false or uncertain. Explain. It is not clear that a model with liquidity constraints can quantitatively match excess sensitivity and excess smoothness in the data, but at least it does better than the standard model without liquidity constraints in both dimensions.

Question 2. Consumption with endogenous labor supply (40 points) Consider a standard consumer who maximizes the ENPV of her flow utility over consumption subject to the usual initial condition on assets, no-Ponzi-game condition and budget constraint.

$$A_{t+1} = R(A_t + y_t - c_t)$$

However, assume that the consumer can affect her income by working harder at a given wage rate, so that $y_t = wh_t + e_t$, where w is the wage, h_t is hours worked and e_t is non-labor income, which is an i.i.d. random variable with $E_t e_{t+1} = e$. Asset returns $R = 1 + r$ and the wage w are non-stochastic and constant. The consumer likes leisure, so her flow utility is given by $u(c_t) - v(h_t)$, where $u' > 0$, $u'' < 0$, $v' > 0$ and $v'' > 0$.

1. (5 points) Write down the Bellman equation for this problem. *Hint:* Note that we cannot use cash-on-hand $X_t = A_t + wh_t + e_t$ as a state variable. Why?
(5 points) Derive the Euler equation for consumption.
2. (5 points) Derive a static first order condition that links hours worked to consumption in every period. Interpret this condition.
3. (15 points) Assume the utility function is quadratic over both consumption and hours worked, $u(c_t) = -\frac{1}{2}\theta(\phi - c_t)^2$ and $v(h_t) = \frac{1}{2}\eta h_t^2$. Find the optimal policy rule for c_t and h_t . For simplicity, you may assume that $\beta = 1/R$ and $\eta = \theta w$ with $w < 1$. (If you need more space, please continue on the back of the page)
4. (10 points) Can endogenous labor supply explain the empirical failures (excess sensitivity, excess smoothness, equity premium puzzle) of the standard consumption model? Why (not)?

Question 3. Equity premium puzzle (50 points) Consider a consumer, who invests in a portfolio of N assets with stochastic returns $R_{it} = 1 + r_{it}$. Let ω_{it} denote the fraction of total wealth invested in asset i , so that $\sum_{i=1}^N \omega_{it} = 1$. Labor income y_t is also stochastic. The consumer's budget constraint is given by,

$$A_{t+1} = \left(\sum_{i=1}^N \omega_{it} R_{it+1} \right) (A_t + y_t - c_t)$$

where A_t denotes beginning of period total asset holdings. As usual, the consumer chooses portfolio shares and total investment to maximize the expected net present value of her utility over consumption, discounting future periods with a discount factor β .

1. (10 points) Derive pricing kernels for all assets. *Hint:* Do not forget to impose the constraint that the asset shares have to add up to one.
2. (10 points) Derive an expression for the expected return of each asset in terms of the covariance of that return with the marginal discount factor M_{t+1} . Then linearize the stochastic discount factor around $c_{t+1} = c_t$ and derive an expression in terms of the covariance with consumption growth. For simplicity, you may assume expected consumption growth is zero. *Hint:* If you could not do part 1, assume that the pricing kernels are of the form $1 = E_t [M_{t+1} R_{it+1}]$, where $M_{t+1} = \beta u'(c_{t+1}) / u'(c_t)$ is the stochastic discount factor.
3. (10 points) How could we use the expression derived in part 2 to test the model using data on the cross-section of asset returns, in the spirit of Mehra and Prescott's (1985) famous equity premium puzzle paper. *Hint:* The difference with Mehra and Prescott is that there is no risk-free asset. If you could not do part 2, assume $E_t R_{it+1} = R + \theta \text{cov}_t(R_{it+1}, \Delta c_{t+1} / c_t)$, where unlike in Mehra and Prescott, R is not observable.
4. (10 points) Is the test in part 3 valid if the consumer engages in precautionary saving? If yes, explain why precautionary saving does not affect the result of the test. If not, explain how we could change the test so that it is valid also under precautionary saving.
5. (10 points) Now suppose that the consumer forms external habits over her consumption, i.e. she derives utility in part from consuming more than her neighbours \tilde{c}_t so that the period utility function is given by $u(c_t - \gamma \tilde{c}_t)$ for $0 \leq \gamma \leq 1$. How does this change the tests derived in parts 3 and/or 4? Can external habit formation explain the equity premium puzzle? Explain. *Hint:* The consumer cannot affect her neighbour's consumption choices.