Barcelona Graduate School of Economics, Universitat Pompeu Fabra
Advanced Macroeconomics I, Spring 2010, part II
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Problem set 5: Consumption CAPM
Optional practice problems

Problem 1. Short answer questions Your answer to the following questions should not exceed half a page each. If appropriate, you are encouraged to refer to papers or to include a graph, table or equation in your answer. Are the following statements true, false or uncertain? Explain.

1. One can argue that the equity premium puzzle is not really a puzzle. If over the next decade, the stock market crashes and the crash is large enough, then the puzzle would be 'solved'.
2. The standard consumption model with stochastic asset returns and stochastic labor income exhibits certainty equivalence if and only if the policy rule is linear.

Problem 2. Merton model of portfolio choice Consider a consumer who can choose to invest in stocks or bonds. The return on stocks Z_t follows a stochastic Markov process. The return on bonds is risk-free and equal to R. Denote the fraction of assets invested in stocks by ω_t. Labor income is zero so that the consumer's dynamic budget constraint is given by

$$A_{t+1} = (\omega_t Z_{t+1} + (1 - \omega_t) R)(A_t - c_t)$$

The consumer maximizes the expected net present value of utility over consumption subject to initial assets A_0 and a no-Ponzi-game condition.

1. Calculate the first order conditions for consumption c_t and the portfolio share in stocks ω_t, and the envelope condition for assets.
2. Use the conditions in part 1 to find the pricing kernels for stocks and bonds.
3. Let $M_{t+1} = \beta u'(c_{t+1}) / u'(c_t)$ be the stochastic discount factor. What is $E_t M_{t+1}$?
4. Derive an expression for the equity premium, defined as the average excess return of stocks over bonds.
5. Assume utility is of the CRRA form and stock returns are i.i.d. Derive the policy rules for consumption and portfolio choice.

Problem 3. Portfolio choice with many assets Consider a consumer with constant absolute risk aversion, who chooses a portfolio of N stocks with stochastic return Z_{it} (for stock i) and a riskless bond with return R. Stock returns are i.i.d. over time, but vary across stocks. Let ω_{it} denote the fraction of total wealth invested in asset i. Labor income is zero in all periods.

1. What is the consumer's dynamic budget constraint. Assume that uncertainty about stock returns is revealed after the consumption decision has to be made and let A_t denote beginning of period assets. Simplify by writing it in terms of excess returns $\hat{Z}_{it} = Z_{it} - R$.

2. Derive pricing kernels for all assets.

3. Solve for the optimal policy for consumption and portfolio shares. *Hint:* Try 'guessing' $c_t = aA_t + b$ and $\omega_{it} = [p_i A_t + q_i]^{-1}$. You can solve explicitly for all parameters of the policy rules up to N constants k_i, for which you can find implicit expressions.