Barcelona Graduate School of Economics, Universitat Pompeu Fabra
Advanced Macroeconomics II, Spring 2010
Thijs van Rens

Midterm Exam
Monday May 10, 2010

Question 1. Short answer questions (40 points)

1. (10 points) Consider a simple asset pricing model, where the current price P_t of a risky asset depends on current dividents d_t, which follow an i.i.d. stochastic process with mean d and variance σ^2, and the expected price next period $E_t P_{t+1}$, according to the following Bellman equation,

$$P_t = d_t + R^{-1} E_t P_{t+1}$$

where $R = 1 + r$ is the return on a risk-free bond. Find a parameter condition for this model to have a unique solution and derive this solution.

2. (10 points) True, false or uncertain. Explain. The failure of the RBC model to match the data is due in part to the specific assumptions we make on preferences. If we relax these assumptions, technology shocks may have intra-temporal effects on hours worked.

3. (10 points) In its battle against a persistently high unemployment rate, the government decides to start a job creation program. A government agency is made in charge of posting vacancies, recruiting workers and organizing the public production production process. Assume government jobs are equally productive as private sector jobs. In the standard (general equilibrium) search and matching model, what is the effect of this program on the unemployment rate and wages. Briefly explain your answer.

4. (10 points) Consider tables 1 and 3 below (last page of the exam), which are taken from Robert Shimer (2005), "The Cyclical Behavior of Equilibrium Unemployment and Vacancies", American Economic Review. Based on the results in these tables, can we conclude that the search and matching model lacks amplification, propagation, neither or both? How we can see this from the tables.

Question 2. TFP with endogenous capacity utilization and effort (40 points) Consider a production function that depends not only on the input of capital K_t and labor H_t, but also on the degree to which these inputs are utilized: capacity utilization z_t for capital and effort e_t for labor. The production function is Cobb-Douglas and can be written as

$$Y_t = A_t \left(z_t^{\phi} K_t \right)^{\alpha} \left(e_t^{\psi} H_t \right)^{1-\alpha}$$

where $0 < \phi < 1$ and $0 < \psi < 1$. This production technology is embedded in an otherwise standard RBC model. The period utility function of the representive worker is given by $\log c_t + v\,(1 - H_t - e_t)$, and capital accumulates according to $K_{t+1} = (1 - \delta z_t)\, K_t + Y_t - c_t$. Households discount the future with discount rate β. Total factor productivity A_t follows a stochastic Markov process and is the only shock in the model. Throughout this exercise, we will assume that all structural parameters of the model are known.

1. (5 points) The standard Solow residual is defined as $\varepsilon_t \equiv \log Y_t - \alpha \log K_t - (1 - \alpha) \log H_t$. Show that in this model, the Solow residual is not a good measure of total factor productivity $\log A_t$.

2. (10 points) Derive the equilibrium (or efficiency) conditions for this model. Make sure you have 5 (rational expectations difference) equations for the 5 endogenous variables c_t, K_t, z_t, H_t and e_t.

3. (10 points) Derive expressions for equilibrium effort e_t and capacity utilization z_t in terms of A_t, K_t and H_t.

4. (5 points) Derive a version of the Solow residual, corrected for endogenous effort and capacity utilization, which provides a correct measure for total factor productivity $\log A_t$ under this model.

5. (10 points) True, false or uncertain. Explain. If the model considered in this question is correct, then we should expect the standard Solow residual to be more volatile than total factor productivity.

Question 3. A search and matching model with two sectors (40 points) Consider an otherwise standard general equilibrium search and matching model, with two sectors (types of jobs). Employed workers earn wage w_1 if they are employed in sector 1 or wage w_2 if they work in sector 2, and unemployed workers receive unemployment benefit b. Search is random and the matching function is constant returns to scale, so that workers find jobs in sector $i \in \{1, 2\}$ with probability $p(\theta_i)$, where $\theta_i = v_i/u$, u is the unemployment rate and v_i are vacancies in sector i. There is free entry of vacancies in both sectors. Productivity y and the separation rate λ are the same for all jobs, but vacancy posting costs are higher in sector 2, $\kappa_1 < \kappa_2$. Wages in all jobs are set by (generalized) Nash bargaining, where ϕ denotes workers' bargaining power. Both workers and firms discount current and future payoffs with discount factor $1/(1+r)$.

1. (5 points) Set up the model, i.e. write down Bellman equations for the value of an employed worker in each sector, W_1, W_2, the value of an unemployed worker U, the value of a filled job in each sector, J_1, J_2 and any other equations you need to fully determine the equilibrium (NB and free entry).

2. (5 points) Show that total match surplus is the same in the two sectors, $S_1 = S_2 = S$.

3. (5 points) Show that profits (and wages) are the same in the two sectors.

4. (5 points) Does this model have an interior solution, i.e. in equilibrium, do firms post vacancies in both sectors, or only in sector 1, which has lower vacancy posting costs? Why? (no math required)

5. (5 points) Solve for total match surplus S in terms of model parameters (including productivity) and labor market tightness in both sectors.

6. (15 points) Calculate the steady state elasticity of the aggregate job finding probability, $p_{\text{agg}} \equiv p(\theta_1) + p(\theta_2)$, with respect to productivity. (If you cannot do this, I will give partial credit for the steady state elasticity of the job finding rate in sector 1.) Can this model help explain Shimer's (2005) unemployment volatility puzzle?

Tables (Shimer 2005)

TABLE 1—SUMMARY STATISTICS, QUARTERLY U.S. DATA, 1951–2003

		u	v	v/u	f	s	p
Standard deviation		0.190	0.202	0.382	0.118	0.075	0.020
Quarterly autocorrelation		0.936	0.940	0.941	0.908	0.733	0.878
Correlation matrix	u	1	−0.894	−0.971	−0.949	0.709	−0.408
	v	—	1	0.975	0.897	−0.684	0.364
	v/u	—	—	1	0.948	−0.715	0.396
	f	—	—	—	1	−0.574	0.396
	s	—	—	—	—	1	−0.524
	p	—	—	—	—	—	1

Notes: Seasonally adjusted unemployment u is constructed by the BLS from the Current Population Survey (CPS). The seasonally adjusted help-wanted advertising index v is constructed by the Conference Board. The job-finding rate f and separation rate s are constructed from seasonally adjusted employment, unemployment, and mean unemployment duration, all computed by the BLS from the CPS, as explained in equations (1) and (2). u, v, f, and s are quarterly averages of monthly series. Average labor productivity p is seasonally adjusted real average output per person in the non-farm business sector, constructed by the Bureau of Labor Statistics (BLS) from the National Income and Product Accounts and the Current Employment Statistics. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5.

TABLE 3—LABOR PRODUCTIVITY SHOCKS

		u	v	v/u	f	p
Standard deviation		0.009	0.027	0.035	0.010	0.020
		(0.001)	(0.004)	(0.005)	(0.001)	(0.003)
Quarterly autocorrelation		0.939	0.835	0.878	0.878	0.878
		(0.018)	(0.045)	(0.035)	(0.035)	(0.035)
Correlation matrix	u	1	−0.927	−0.958	−0.958	−0.958
			(0.020)	(0.012)	(0.012)	(0.012)
	v	—	1	0.996	0.996	0.995
				(0.001)	(0.001)	(0.001)
	v/u	—	—	1	1.000	0.999
					(0.000)	(0.001)
	f	—	—	—	1	0.999
						(0.001)
	p	—	—	—	—	1

Notes: Results from simulating the model with stochastic labor productivity. All variables are reported in logs as deviations from an HP trend with smoothing parameter 10^5. Bootstrapped standard errors—the standard deviation across 10,000 model simulations—are reported in parentheses. The text provides details on the stochastic process for productivity.