Modelling the Labor Market

- Competitive labor markets

\[ w_t - p_t = mrs_t \]

where \( mrs_t = \sigma c_t + \varphi n_t \)

- General labor market imperfections

\[ w_t - p_t = \mu^w_t + mrs_t \]

where \( \mu^w_t \): (log) wage markup.

*Example*: monopolistic union with isoelastic labor demand:

\[ \mu^w_t = \log \frac{\epsilon_w}{\epsilon_w - 1} \equiv \mu^w \]
Recall

\[ \pi_t^p = \beta E_t\{\pi_{t+1}^p\} - \lambda_p(\mu_t^p - \mu^p) \]

Assuming constant returns (for simplicity)

\[ \mu_t^p = p_t - (w_t - a_t) \]
\[ = a_t - \omega_t \]
\[ = a_t - (\mu_t^w + \sigma c_t + \varphi n_t) \]
\[ = (1 + \varphi)a_t - (\sigma + \varphi)y_t - \mu_t^w \]

In deviations from natural levels (assuming constant natural markups):

\[ \mu_t^p - \mu^p = -(\sigma + \varphi)\tilde{y}_t - (\mu_t^w - \mu^w) \]

Implied New Keynesian Phillips Curve:

\[ \pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \kappa p\tilde{y}_t + \lambda_p\hat{\mu}_t^w \]

\[ \implies \text{tradeoff between inflation and output gap stabilization} \]

**Question:** What determines the evolution of the wage markup?
A Model with Sticky Wages and Prices: Assumptions

- Price setting: as in basic NK model
- Differentiated labor services ("occupations") represented by unions
  \[\Rightarrow\] market power when setting wages
- Staggered nominal wage setting
- Cashless limit (focus on interest rate rules)
Households

- Representative household’s problem

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \{N_t(j)\}; Z_t)
\]

subject to

\[
P_tC_t + Q_tB_t \leq B_{t-1} + \int_0^1 W_t(j)N_t(j)\,dj + D_t
\]

where

\[
U(C_t, \{N_t(j)\}; Z_t) = \begin{cases} 
\left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \int_0^1 \frac{N_t(j)^{1+\varphi}}{1+\varphi} \,dj \right) Z_t & \text{for } \sigma \neq 1 \\
\left( \log C_t - \int_0^1 \frac{N_t(j)^{1+\varphi}}{1+\varphi} \,dj \right) Z_t & \text{for } \sigma = 1
\end{cases}
\]

with \( z_t \equiv \log Z_t \sim AR(1) \) and \( \{N_t(j)\} \) taken as given.
**Optimality condition**

\[ Q_t = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{Z_{t+1}}{Z_t} \right) \left( \frac{P_t}{P_{t+1}} \right) \right\} \]

or, in log-linearized form:

\[ c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}^p\} - \rho) + \frac{1}{\sigma} (1 - \rho_z) z_t \]
Wage Setting

- Fraction of occupations/unions adjusting nominal wage: $1 - \theta_w$
- Optimal wage setting

$$\max_{W_t^*} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \left( C_{t+k}^{-\sigma} \frac{W_{t+k}^*}{P_{t+k}} N_{t+k|t} - \frac{N_{t+k|t}}{1 + \varphi} \right) Z_{t+k}$$

subject to labor demand schedule

$$N_{t+k|t} = \left( \frac{W_{t+k}^*}{W_{t+k}} \right)^{-\varepsilon_w} \left( \int_0^1 N_{t+k}(i) di \right)$$
Wage Setting

- Optimality condition:

\[
\sum_{k=0}^{\infty} (\beta \theta_w)^k E_t \left\{ N_{t+k|t} Z_{t+k} C_{t+k}^{-\sigma} \left( \frac{W^*_t}{P_{t+k}} - \mathcal{M}_w MRS_{t+k|t} \right) \right\} = 0
\]

where \( MRS_{t+k|t} \equiv C^\sigma_{t+k} N^\varphi_{t+k|t} \) and \( \mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1} \).

- Log-linearized version:

\[
w^*_t = \mu^w + (1 - \beta \theta_w) \sum_{k=0}^{\infty} (\beta \theta_w)^k E_t \left\{ mrs_{t+k|t} + p_{t+k} \right\}
\]

- Equivalently:

\[
w^*_t = (1 - \beta \theta_w) \sum_{k=0}^{\infty} (\beta \theta_w)^k E_t \left\{ w_{t+k} - (1 + \epsilon_w \varphi)^{-1} \hat{\mu}^w_{t+k} \right\}
\]

where \( \mu^w_t \equiv (w_t - p_t) - mrs_t \) and \( mrs_t = \sigma c_t + \varphi n_t \).
Aggregate wage dynamics

\[ w_t = \theta_w w_{t-1} + (1 - \theta_w) w_t^* \]

Wage inflation equation

\[ \pi^w_t = \beta E_t \{ \pi^w_{t+1} \} - \lambda_w (\mu^w_t - \mu^w) \]

where \( \lambda_w \equiv \frac{(1 - \beta \theta_w)(1 - \theta_w)}{\theta_w (1 + \phi \epsilon_w)} \)
Firms

- Final goods: same as in the basic NK model
- Intermediate goods: technology given by

\[
Y_t(i) = A_t N_t(i)^{1-\alpha}
\]

where \(N_t(i) \equiv \left( \int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w-1}}\) and \(a_t \equiv \log A_t \sim AR(1)\)

Cost minimization:

\[
N_t(i,j) = \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i)
\]

where \(W_t \equiv \left( \int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}}\)

Implication:

\[
\int_0^1 W_t(j) N_t(i,j) dj = W_t N_t(i)
\]
Firms

- Price setting: same as in the basic NK model

\[ \pi_t^p = \beta E_t \{ \pi_{t+1}^p \} - \lambda_p (\mu_t^p - \mu^p) \]

where \( \lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p} \)
Equilibrium

- **Goods market clearing:**

  \[ Y_t(i) = C_t(i) \text{ all } i \in [0, 1] \Rightarrow Y_t = C_t \]

  where \( Y_t \equiv \left( \int_0^1 Y_t(i)^{1 - \frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \)

- **Aggregate employment and output**

  \[
  N_t \equiv \int_0^1 \int_0^1 N_t(i, j) dj di = \Delta_{w,t} \int_0^1 N_t(i) di \\
  = \Delta_{w,t} \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{Y_t(i)}{Y_t} \right)^{\frac{1}{1-\alpha}} di = \Delta_{w,t} \Delta_{p,t} \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}}
  \]

  where \( \Delta_{w,t} \equiv \int_0^1 \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} dj \) and \( \Delta_{p,t} \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon_p}{1-\alpha}} di. \)

  Up to a first order approximation:

  \[ (1 - \alpha) n_t = y_t - a_t \]
Equilibrium

The Wage Gap

\[ \tilde{\omega}_t \equiv \omega_t - \omega^n_t \]

where \( \omega_t \equiv w_t - p_t \) and \( \omega^n_t \) is the natural real wage:

\[ \omega^n_t = \psi_\omega + \psi_{\omega a} a_t - \mu^p \]

where \( \psi_{\omega a} \equiv \frac{\sigma + \varphi}{\sigma(1-\alpha) + \varphi + \alpha} \) and \( \psi_\omega \equiv -\frac{\alpha \log(1-\alpha)}{\sigma(1-\alpha) + \varphi + \alpha} \).

Identity:

\[ \tilde{\omega}_t \equiv \tilde{\omega}_{t-1} + \pi^w_t - \pi^p_t - \Delta \omega^n_t \]
Equilibrium

- **Price markup gap**

\[
\mu^p_t = \log(1 - \alpha) + (a_t - \alpha n_t) - \omega_t
\]

\[
\mu^p = \log(1 - \alpha) + (a_t - \alpha n_t) - \omega^n_t
\]

\[
\Rightarrow \quad \hat{\mu}_t^p = -\frac{\alpha}{1 - \alpha} \tilde{y}_t - \tilde{\omega}_t
\]

- **Implied price inflation equation:**

\[
\pi_t^p = \beta E_t \{ \pi_{t+1}^p \} + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t
\]

where \( \kappa_p \equiv \frac{\alpha \lambda_p}{1 - \alpha} \).
Equilibrium

- **Wage markup gap:**

  \[
  \mu_t^W = \omega_t - (\sigma y_t + \varphi n_t)
  \]

  \[
  \mu^n_w = \omega^n_t - (\sigma y^n_t + \varphi n^n_t)
  \]

  \[
  \Rightarrow \hat{\mu}_t^W = \tilde{\omega}_t - \left(\sigma + \frac{\varphi}{1 - \alpha}\right) \tilde{y}_t
  \]

- **Implied wage inflation equation:**

  \[
  \pi_t^W = \beta E_t \{ \pi_{t+1}^W \} + \nu_w \tilde{y}_t - \lambda_w \tilde{\omega}_t
  \]

  where \( \nu_w \equiv \lambda_w \left(\sigma + \frac{\varphi}{1 - \alpha}\right) \)
**Equilibrium**

- *Dynamic IS equation*

\[
\tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t \{ \pi_{t+1}^p \} - r_t^n) + E_t \{ \tilde{y}_{t+1} \}
\]

where \( r_t^n \equiv \rho - \sigma \psi_y (1 - \rho_a) a_t + (1 - \rho_z) z_t \)

- *Interest Rate Rule:*

\[
i_t = \rho + \phi_p \pi_t^p + \phi_w \pi_t^w + \phi_y \tilde{y}_t + \nu_t
\]
**Equilibrium**

- *Dynamical system:*

\[
A^w_0 x_t = A^w_1 E_t \{x_{t+1}\} + B^w_0 u_t
\]

where \( x_t \equiv [\tilde{y}_t, \pi_t^p, \pi_t^w, \tilde{\omega}_{t-1}]' \), \( u_t \equiv [\tilde{r}_t^n - \nu_t - \phi_y \tilde{y}_t^n, \Delta \omega_t^n]' \),

\[
A^w_0 \equiv \begin{bmatrix}
\sigma + \phi_y & \phi_p & \phi_w & 0 \\
-\kappa_p & 1 & 0 & 0 \\
-\kappa_w & 0 & 1 & 0 \\
0 & -1 & 1 & 1 \\
\end{bmatrix}
\]

\[
A^w_1 \equiv \begin{bmatrix}
\sigma & 1 & 0 & 0 \\
0 & \beta & 0 & \lambda_p \\
0 & 0 & \beta & -\lambda_w \\
0 & 0 & 0 & 1 \\
\end{bmatrix}
\]\n
; \quad B^w_0 \equiv \begin{bmatrix}
1 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 1 \\
\end{bmatrix}
Equilibrium

- **Local uniqueness condition**

\[
\phi_p + \phi_w + \phi_y \left( \frac{1 - \beta}{\sigma + \frac{\alpha + \varphi}{1 - \alpha}} \right) \left( \frac{1}{\lambda_p} + \frac{1}{\lambda_w} \right) > 1
\]

Particular case \((\phi_y = 0)\):

\[
\phi_p + \phi_w > 1
\]
Figure 6.1 Determinacy and Indeterminacy Regions
Calibration and Simulations

- **Interest rate rule**: \( \phi_p = 1.5 \), \( \phi_y = \phi_w = 0 \), \( \rho_v = 0.5 \)
- **New parameter**: \( \epsilon_w = 4.5 \)
- **Three calibrations of price and wage rigidities:**
  1. **Baseline**: \( \theta_p = 3/4 \), \( \theta_w = 3/4 \)
  2. **Flexible wages**: \( \theta_p = 3/4 \), \( \theta_w = 0 \)
  3. **Flexible prices**: \( \theta_p = 0 \), \( \theta_w = 3/4 \)
- Remaining parameters as in baseline model
- **Simulations**: *Dynamic responses to monetary policy shock*
Figure 6.2 Dynamic Responses to a Monetary Policy Shock
The Social Planner’s Problem

\[ \max U(C_t, \{N_t(j)\}; Z_t) \]

subject to:

\[ C_t = A_t \left[ \int_0^1 \left( \int_0^1 N_t(i, j)^{1-\frac{1}{\epsilon_w}} \, dj \right)^{\frac{\epsilon_w(1-\alpha)}{\epsilon_w-1}} \left(1 - \frac{1}{\epsilon_p}\right) \right]^{\frac{\epsilon_p}{\epsilon_p-1}} \]

\[ N_t(j) = \int_0^1 N_t(i, j) \, di \]

- Optimality conditions:

  \[ C_t(i) = C_t, \quad \text{all } i \in [0, 1] \]

  \[ N_t(i, j) = N_t(j) = N_t(i) = N_t, \quad \text{all } i, j \in [0, 1] \]

  \[ -\frac{U_{n,t}}{U_{c,t}} = MPN_t \]

  where \( MPN_t = (1 - \alpha) A_t N_t^{-\alpha} \)
In the decentralized economy with flexible prices and wages:

\[ P_t = M_p \frac{(1 - \tau) W_t}{MPN_t} \]

\[ \frac{W_t}{P_t} = - \frac{U_{n,t}}{U_{c,t}} M_w \]

for all goods and occupations, where \( M_p \equiv \frac{\epsilon_p}{\epsilon_p - 1} \) and \( M_w \equiv \frac{\epsilon_w}{\epsilon_w - 1} \).

Letting \( M \equiv M_p M_w \)

\[ - \frac{U_{n,t}}{U_{c,t}} = \frac{1}{M(1 - \tau)} MPN_t \]

Condition for efficiency of the natural equilibrium: \( M(1 - \tau) = 1 \)

**Remark**: natural equilibrium generally not attainable with sticky prices and wages (proof)
Optimal Monetary Policy

\[
\min \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left( \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}^2_t + \frac{\varepsilon_p}{\lambda_p} (\pi_t^p)^2 + \frac{\varepsilon_w (1 - \alpha)}{\lambda_w} (\pi_t^w)^2 \right)
\]

subject to:

\[
\pi_t^p = \beta E_t \{ \pi_{t+1}^p \} + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t
\]

\[
\pi_t^w = \beta E_t \{ \pi_{t+1}^w \} + \kappa_w \tilde{y}_t - \lambda_w \tilde{\omega}_t
\]

\[
\tilde{\omega}_{t-1} \equiv \tilde{\omega}_t - \pi_t^w + \pi_t^p + \Delta \omega_t^n
\]
Optimal Monetary Policy

- **Optimality conditions:**

\[
\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right) \tilde{y}_t + \kappa_p \zeta_{1,t} + \kappa_w \zeta_{2,t} = 0
\]

\[
\frac{\epsilon_p}{\lambda_p} \pi^p_t - \Delta \zeta_{1,t} + \zeta_{3,t} = 0
\]

\[
\frac{\epsilon_w (1 - \alpha)}{\lambda_w} \pi^w_t - \Delta \zeta_{2,t} - \zeta_{3,t} = 0
\]

\[
\lambda_p \zeta_{1,t} - \lambda_w \zeta_{2,t} + \zeta_{3,t} - \beta E_t \{ \zeta_{3,t+1} \} = 0
\]

for \( t = 0, 1, 2, \ldots \) given \( \zeta_{1,-1} = \zeta_{2,-1} = 0 \) and given \( \tilde{\omega}_{-1} \).

- **Equilibrium under the optimal policy:**

\[
A^*_0 x_t = A^*_1 E_t \{ x_{t+1} \} + B^*_0 \Delta a_t
\]

where \( x_t \equiv [\tilde{y}_t, \pi^p_t, \pi^w_t, \tilde{\omega}_{t-1}, \zeta_{1,t-1}, \zeta_{2,t-1}, \zeta_{3,t}]' \).
Optimal Monetary Policy

- **Optimal Policy in Response to Demand Shocks**

\[ A_0^* x_t = A_1^* E_t \{ x_{t+1} \} \]

Under the assumption of \( \omega_{t-1} = 0 \),

\[ x_t = 0 \]

for all \( t \)

**Implementation:**

\[ i_t = r_t^n + \phi_p \pi_t \]

where \( r_t^n = \rho + (1 - \rho_z) z_t \), and \( \phi_p > 1 \)

- **Optimal Policy in Response to Technology Shocks**
Figure 6.3  Dynamic Responses to a Technology Shock under the Optimal Monetary Policy

- Output gap
- Price inflation
- Wage inflation
- Real wage

Legend:
- Blue circles: baseline
- Red diamonds: flexible wages
- Black squares: flexible prices
Combining the wage and price inflation equations:

\[ \pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa \tilde{y}_t \]

where \( \kappa \equiv \frac{\lambda_w \lambda_p}{\lambda_p + \lambda_w} \frac{\alpha}{1-\alpha} \) and

\[ \pi_t \equiv \frac{\lambda_w}{\lambda_p + \lambda_w} \pi_t^p + \frac{\lambda_p}{\lambda_p + \lambda_w} \pi_t^w \]

\[ \Rightarrow \text{ no tradeoff} \]
Assumptions

\[ \pi_p = \pi_w \equiv \pi \]
\[ \epsilon_p = \epsilon_w (1 - \alpha) \equiv \epsilon. \]

Implied optimality conditions:

\[ \pi_t = -\frac{1}{\epsilon} \Delta \tilde{y}_t \]

for \( t = 1, 2, 3, \ldots \) as well as

\[ \pi_0 = -\frac{1}{\epsilon} \tilde{y}_0 \]

Optimal policy:

\[ \pi_t = \tilde{y}_t = 0 \]

for all \( t \)
Strict Targeting Rules:

\[ \pi^i_t = 0 \]

Flexible Targeting Rules:

\[ i_t = 0.01 + 1.5\pi^i_t \]

Evaluation
### Table 6.1  Evaluation of Simple Rules

<table>
<thead>
<tr>
<th></th>
<th>Optimal</th>
<th>Strict Targeting</th>
<th>Flexible Targeting</th>
</tr>
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<tr>
<td></td>
<td>Price</td>
<td>Wage</td>
<td>Composite</td>
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<td><strong>Technology shocks</strong></td>
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<td>$\sigma(\pi^p)$</td>
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<td>$\sigma(\tilde{y})$</td>
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</tr>
<tr>
<td>$L$</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>
Figure 6.4: Welfare losses under alternative policies with sticky wages and prices.

Woodford (2003)
Households: An Alternative Formulation

- Representative household with a continuum of members, indexed by 
  \((j, s) \in [0, 1] \times [0, 1]\)
- Continuum of differentiated labor services, indexed by \(j \in [0, 1]\)
- Indivisible labor
- Disutility from working: \(\chi s^\phi\), for \(s \in [0, 1]\), where \(\phi \geq 0\)
- Full consumption risk sharing within the household
- Household utility:
  \[ E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \{N_t(j)\}; Z_t) \]

\[
U(C_t, \{N_t(j)\}; Z_t) \equiv \left( \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \chi_t \int_0^1 \int_0^{N_t(j)} s^\phi dsdj \right) Z_t \\
= \left( \frac{C_t^{1-\sigma} - 1}{1 - \sigma} - \chi_t \int_0^1 \frac{N_t(j)^{1+\phi}}{1 + \phi} dj \right) Z_t
\]

where \(\chi_t\) is a labor disutility shifter
Participation

- Participation condition for an individual \((j, s)\):
  \[
  \frac{W_t(j)}{P_t} \geq \chi_t C_t^{\sigma} s^{\varphi}
  \]

- Marginal participant, \(L_t(j)\), defined by:
  \[
  \frac{W_t(j)}{P_t} = \chi_t C_t^{\sigma} L_t(j)^{\varphi}
  \]

- Taking logs and integrating over \(i\),
  \[
  w_t - p_t = \sigma c_t + \varphi l_t + \xi_t
  \]

where \(w_t \sim \int_0^1 w_t(j) \, dj\) and \(l_t \equiv \int_0^1 l_t(j) \, dj\) is the (log) labor force.
Unemployment

Introducing Unemployment

- Unemployment rate

\[ u_t \equiv l_t - n_t \]

- Average wage markup and unemployment

\[ \mu^w_t = (w_t - p_t) - (\sigma c_t + \varphi n_t + \zeta_t) \]
\[ = \varphi u_t \]
Figure 7.1  The Wage Markup and the Unemployment Rate

$w_t - p_t$

$\mu_t^w$

$u_t$

$\mu_t$

$\mu_t$

$w_t - p_t$

$\mu_t^w$

$\mu_t$

$\mu_t$

$w_t - p_t$

$\mu_t^w$

$\mu_t$

$\mu_t$

$w_t - p_t$

$\mu_t^w$

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$w_t - p_t$

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Unemployment

Introducing Unemployment

- Unemployment rate
  \[ u_t \equiv l_t - n_t \]

- Average wage markup and unemployment
  \[ \mu_t^w = (w_t - p_t) - (\sigma c_t + \varphi n_t + \zeta_t) = \varphi u_t \]

- Under flexible wages:
  \[ \mu^w = \varphi u^n \]
  \[ \Rightarrow u^n: \textit{natural} \text{ rate of unemployment} \]

- Combined with wage inflation equation:
  \[ \pi_t^w = \beta E_t \{ \pi_{t+1}^w \} - \lambda_w \varphi (u_t - u^n) \]
Figure 7.2  Response of Labor Market Variables to a Monetary Policy Shock

- **Unemployment Rate**: The graph shows a significant decrease in unemployment rate following a monetary policy shock, reaching a steady state around the 10-period mark.

- **Employment**: Employment increases gradually after the initial shock, stabilizing around the 14-period mark.

- **Labor Force**: There is a slight decrease in the labor force, stabilizing around the 12-period mark.

- **Real Wage**: The real wage experiences a steady increase over the 16 periods following the shock.
Extensions (I): Indexation

- Indexation rule
  \[ w_{t+k|t} = w_{t+k-1|t} + \gamma \pi^p_{t-1} \]

- Implied wage inflation equation
  \[
  \tilde{\pi}_t^w = \beta E_t \{ \tilde{\pi}_{t+1}^w \} - \lambda w (\mu_t^w - \mu^w) \\
  = \beta E_t \{ \tilde{\pi}_{t+1}^w \} - \lambda w \varphi (u_t - u^n)
  \]
  where \( \tilde{\pi}_t^w \equiv \pi_t^w - \gamma \pi^p_{t-1} \)

- Solving forward and assuming \( \{u_t\} \sim AR(1) \),
  \[
  \pi_t^w = \gamma \pi^p_{t-1} - \frac{\lambda w \varphi}{1 - \beta \rho_u} u_t
  \]
Extensions (II): Parameterizing Wealth Effects

- Disutility from working: $\chi \Theta_t s^\phi$, where $\Theta_t \equiv Z_t / \overline{C}_t$ and $Z_t = Z_{t-1}^{1-\nu} \overline{C}_t$. Log utility of consumption (consistent with BGP).

- Participation condition for an individual $(j, s)$:

  \[
  \frac{W_t(j)}{P_t} \geq \chi \Theta_t C_t s^\phi
  \]

- Marginal participant, $L_t(j)$, defined by:

  \[
  \frac{W_t(j)}{P_t} = \chi \Theta_t C_t L_t(j)^\phi
  \]

- Taking logs, symmetric equilibrium, and integrating over $j$,

  \[w_t - p_t = z_t + \varphi l_t + \zeta\]

  where $w_t \approx \int_0^1 w_t(j) dj$ and $l_t \equiv \int_0^1 l_t(j) dj$ is the (log) labor force.

- Limiting cases: KPR ($\nu = 1$) and GHH ($\nu = 0$).
Figure 6. Monetary Policy Shocks and the Role of Wealth Effects
Wage inflation equation (standard)

\[ \pi^w_t = \beta E_t \{ \pi^w_{t+1} \} - \lambda w (\mu^w_{t} - \mu^n_{w,t}) \]

\[ = \beta E_t \{ \pi^w_{t+1} \} - \lambda w (\omega_t - \sigma c_t - \varphi n_t) - \lambda w \xi_t + \lambda w \mu^n_{w,t} \]

Wage inflation equation (alternative)

\[ \pi^w_t = \beta E_t \{ \pi^w_{t+1} \} - \lambda w (\mu^w_{t} - \mu^n_{w,t}) \]

\[ = \beta E_t \{ \pi^w_{t+1} \} - \lambda w \varphi u_t + \lambda w \mu^n_{w,t} \]

with labor supply shock identified from

\[ \omega_t - p_t = \sigma c_t + \varphi l_t + \xi_t \]
Figure 10. The Natural Rate of Unemployment
Figure 11. Sources of Unemployment Rate Fluctuations

Unemployment Rate decomposition

- Historical UR
- Supply shocks
- Demand shocks

Unemployment Rate decomposition

- Historical UR
- Labor supply
- Wage mark-up