Monetary Policy and the Open Economy

by

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Motivation

- The basic new Keynesian model for the closed economy
  - equilibrium dynamics: simple three-equation representation
  - ability to match much of the evidence on the effects of monetary policy and technology shocks
  - monetary policy: optimality of inflation targeting

- How does the introduction of open economy elements affect that analysis and prescriptions?

- What role should the exchange rate play in the design of policy? What is the optimal degree of exchange rate volatility?

- Issue: many possible combinations of assumptions on size, market completeness, pass-through, etc.
A Baseline New Keynesian Model of the Small Open Economy (Galí 2015, GM 2005)

Households
Representative household maximizing

\[
E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t; Z_t)
\]

where

\[
C_t \equiv \left( (1 - \nu)^{\frac{1}{\eta}} C_{H,t}^{1-\frac{1}{\eta}} + \nu^{\frac{1}{\eta}} C_{F,t}^{1-\frac{1}{\eta}} \right)^{\frac{\eta}{\eta-1}}
\]

\[
C_{H,t} \equiv \left( \int_0^1 C_{H,t}(i)^{\frac{\varepsilon-1}{\varepsilon}} di \right)^{\frac{\varepsilon}{\varepsilon-1}}
\]

subject to

\[
\int_0^1 P_{H,t}(i) C_{H,t}(i) di + P_{F,t} C_{F,t} + E_t \{ Q_{t,t+1} D_{t+1} \} \leq D_t + W_t N_t
\]

Assumption:

\[
U(C_t, N_t; Z_t) = \begin{cases} 
\left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \frac{N_t^{1+\varphi}}{1+\varphi} \right) Z_t & \text{for } \sigma \neq 1 \\
\log C_t - \frac{N_t^{1+\varphi}}{1+\varphi} Z_t & \text{for } \sigma = 1
\end{cases}
\]

with \( z_t \equiv \log Z_t = \rho_z z_{t-1} + \varepsilon_t \)
• Optimal allocation of expenditures

(i) Domestic goods:

\[ C_{H,t}(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} C_{H,t} \]

for all \( i \in [0, 1] \), where \( P_{H,t} \equiv \left( \int_0^1 P_{H,t}(i)^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}} \)

\[ \Rightarrow \int_0^1 P_{H,t}(i) C_{H,t}(i) \, di = P_{H,t} C_{H,t} \]

(ii) Domestic vs. Foreign

\[ C_{H,t} = (1 - \nu) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t \quad ; \quad C_{F,t} = \nu \left( \frac{P_{F,t}}{P_t} \right)^{-\eta} C_t \]

where \( P_t \equiv \left[ (1 - \nu)(P_{H,t})^{1-\eta} + \nu(P_{F,t})^{1-\eta} \right]^{\frac{1}{1-\eta}} \)

\[ \Rightarrow P_{H,t} C_{H,t} + P_{F,t} C_{F,t} = P_t C_t \]

• Other optimality conditions

\[ Q_{t,t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{Z_{t+1}}{Z_t} \right) \left( \frac{P_t}{P_{t+1}} \right) \]

\[ Q_t = E_t \{ Q_{t,t+1} \} = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{Z_{t+1}}{Z_t} \right) \left( \frac{P_t}{P_{t+1}} \right) \right\} \]

\[ C_t^{\sigma} N_t^{\phi} = \frac{W_t}{P_t} \]
• Some definitions and identities

Terms of trade:

\[ s_t \equiv \log S_t = p_{F,t} - p_{H,t} \]

CPI:

\[ p_t = (1 - \nu)p_{H,t} + \nu p_{F,t} \]
\[ = p_{H,t} + \nu s_t \]

CPI vs. Domestic inflation:

\[ \pi_t = \pi_{H,t} + \nu \Delta s_t \]

where \( \pi_{H,t} \equiv p_{H,t} - p_{H,t-1} \)

• Law of one price

\[ p_{F,t} = e_t + p_t^* \]

• Real exchange rate

\[ q_t \equiv p_{F,t} - p_t \]
\[ = s_t + p_{H,t} - p_t \]
\[ = (1 - \nu)s_t \]
International risk sharing

\[
\beta \left( \frac{C_{t+1}^*}{C_t^*} \right)^{-\sigma} \left( \frac{P_t^*}{P_{t+1}^*} \right) \left( \frac{\mathcal{E}_t}{\mathcal{E}_{t+1}} \right) = Q_{t,t+1}
\]

\[
\Rightarrow C_t = \eta C_t^* Z_t^\frac{1}{\sigma} Q_t^\frac{1}{\sigma}
\]

\[
c_t = y_t^* + \frac{1}{\sigma} \left( z_t + q_t \right)
\]

\[
= y_t^* + \frac{1}{\sigma} z_t + \left( \frac{1 - \nu}{\sigma} \right) s_t
\]

given \( c_t^* = y_t^* \)
Firms

- Technology

\[ Y_t(i) = A_t N_t(i)^{1-\alpha} \]

where \( a_t \equiv \log A_t = \rho_a a_{t-1} + \varepsilon_t^\alpha \).

- Optimal price setting

\[ \bar{p}_{H,t} = \mu + (1 - \beta \theta) \sum_{k=0}^{\infty} (\beta \theta)^k E_t\{\psi_{t+k}|t\} \]

- Domestic inflation dynamics

\[ \pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} - \lambda \hat{\mu}_t \]

where \( \hat{\mu}_t \equiv P_{H,t} - \psi_{t+k} - \mu \) and \( \lambda \equiv \frac{(1-\theta)(1-\beta \theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha \varepsilon} \)

Exports

\[ X_t(i) = \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\epsilon} X_t \]

where \( X_t \equiv \left( \int_0^1 X_t(i)^{\frac{\epsilon-1}{\epsilon}} \ di \right)^{\frac{\epsilon}{\epsilon-1}} \) given by

\[ X_t = v \left( \frac{P_{H,t}}{\varepsilon_t P_t} \right)^{-\eta} Y_t^* \]

\[ = v S^n Y_t^* \]
Equilibrium

- Goods market clearing

\[
Y_t(i) = C_{H,t}(i) + X_t(i) \\
= \left( \frac{P_{H,t}(i)}{P_{H,t}} \right)^{-\varepsilon} \left[ (1 - \nu) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \nu S^\eta Y_t^* \right]
\]

for all \( i \in [0, 1] \) and all \( t \). Combined with \( Y_t \equiv \left( \int_0^1 Y_t(i)^{1-\varepsilon} di \right)^{\frac{\varepsilon}{\varepsilon - 1}} \)

\[
Y_t = (1 - \nu) \left( \frac{P_{H,t}}{P_t} \right)^{-\eta} C_t + \nu S^\eta Y_t^*
\]

Log-linearized version:

\[
y_t = (1 - \nu)c_t + \nu(2 - \nu)\eta s_t + \nu y_t^*
\]

Euler equation (in terms of \( \pi_{H,t+1} \)):

\[
c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi_{H,t+1}\} - \rho) + \frac{\nu}{\sigma} E_t\{\Delta s_{t+1}\} + \frac{1}{\sigma}(1 - \rho z)z_t
\]

Terms of trade (using IRS condition):

\[
s_t = \sigma_v(y_t - y_t^*) - (1 - \nu)\Phi z_t
\]

where \( \sigma_v \equiv \sigma \Phi > 0 \) with \( \Phi \equiv \frac{1}{1 + v(\varpi - 1)} > 0 \) and \( \varpi \equiv \sigma \eta + (1 - \nu)(\sigma \eta - 1) \).
Combining them all

\[ \tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma_v}(i_t - E_t\{\pi_{H,t+1}\} - r^n_t) \]

where

\[ r^n_t = \rho + \sigma_v E_t\{\Delta y^n_{t+1}\} + \sigma_v \nu(\varpi - 1)E_t\{\Delta y^*_{t+1}\} + \Phi(1 - \nu)(1 - \rho_z)z_t \]

- Labor market clearing

\[ N_t \equiv \int_0^1 N_t(i)di = \left(\frac{Y_t}{A_t}\right)^{\frac{1}{1-\alpha}} \int_0^1 \left(\frac{P_{H,t}(i)}{P_{H,t}}\right)^{-\frac{\nu}{1-\alpha}} \, di \]

Up to a first-order approximation:

\[ y_t = a_t + (1 - \alpha)n_t \]

- Price markups and the output gap

\[ \mu_t = p_{H,t} - (w_t - a_t + \alpha n_t) \]
\[ = -(w_t - p_t) - (p_t - p_{H,t}) + a_t - \alpha n_t \]
\[ = -(\sigma c_t + \varphi n_t) - \nu s_t + a_t - \alpha n_t \]
\[ = -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)y_t + \nu(\varpi - 1)s_t + \left(1 + \frac{\varphi + \alpha}{1 - \alpha}\right)a_t - \nu z_t \]

Accordingly,

\[ \hat{\mu}_t = -\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)\tilde{y}_t + \nu(\varpi - 1)\tilde{s}_t \]
and

\[ y^n_t = \Gamma_a a_t + \Gamma z z_t + \Gamma_* y^*_t \]
\[ s^n_t = \sigma v (y^n_t - y^*_t) - (1 - \nu) \Phi z_t \]

with \( \Gamma_a \equiv \frac{1+\varphi}{\sigma_r(1-\alpha)+\varphi+\alpha} > 0 \), \( \Gamma_* \equiv -\frac{\nu(\omega-1)\sigma_r(1-\alpha)}{\sigma_r(1-\alpha)+\varphi+\alpha} \) and \( \Gamma_z \equiv -\frac{\nu \omega \Phi (1-\alpha)}{\sigma_r(1-\alpha)+\varphi+\alpha} \).

Using the fact that \( \tilde{s}_t = \sigma v \tilde{y}_t \),

\[ \tilde{\mu}_t = -\left( \sigma v + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t \]
\[ \pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \kappa v \tilde{y}_t \]

where \( \kappa_v \equiv \lambda \left( \sigma_v + \frac{\varphi + \alpha}{1 - \alpha} \right) \). Also

\[ r^n_t \equiv \rho - \sigma v \Gamma_a (1 - \rho_a) a_t + \Psi_* E_t \{ \Delta y^*_{t+1} \} + \Psi_z (1 - \rho_z) z_t \]

with \( \Psi_* \equiv \sigma_v (\nu (\omega - 1) + \Gamma_*) \) and \( \Psi_z \equiv (1 - \nu) \Phi - \sigma_v \Gamma_z \). Note that \( \lim_{\nu \to 0} \Psi_* = 0 \) and \( \lim_{\nu \to 0} \Psi_z = 1 \).
Equilibrium Dynamics under a Taylor-type Rule

\[ \pi_{H,t} = \beta E_t\{\pi_{H,t+1}\} + \kappa_0 \tilde{y}_t \]

\[ \tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma_v}(i_t - E_t\{\pi_{H,t+1}\} - r^n_t) \]

\[ i_t = \rho + \phi_\pi \pi_{H,t} + \phi_y \tilde{y}_t + v_t \]

where \( v_t = \rho v v_{t-1} + \varepsilon_v^n \) and \( r^n_t \equiv \rho - \sigma_v \Gamma_a (1 - \rho_a) a_t + \Psi_p E_t\{\Delta y_{t+1}^*\} + \Psi_z (1 - \rho_z) z_t \)

Equivalently

\[
\begin{bmatrix}
\tilde{y}_t \\
\pi_{H,t}
\end{bmatrix} = A_v \begin{bmatrix}
E_t\{\tilde{y}_{t+1}\} \\
E_t\{\pi_{H,t+1}\}
\end{bmatrix} + B_v u_t
\]

where

\[ u_t \equiv \tilde{r}_t^n - \phi_y \tilde{y}_t^n - v_t \]

\[ A_v \equiv \Omega_v \begin{bmatrix}
\sigma_v & 1 - \beta \phi_\pi \\
\sigma_v \kappa_v & \kappa_v + \beta (\sigma_v + \phi_y)
\end{bmatrix} ; \quad B_v \equiv \Omega_v \begin{bmatrix}
1 \\
\kappa_v
\end{bmatrix} \]

with \( \Omega_v \equiv \frac{1}{\sigma_v + \phi_y + \kappa_v \phi_\pi} \).

- **Uniqueness condition**

\[ \kappa_v (\phi_\pi - 1) + (1 - \beta) \phi_y > 0 \]

- **Effects of a monetary policy shock**
Dynamic responses to a monetary policy shock

- Output gap
- Employment
- Domestic inflation
- CPI inflation
- Domestic price level
- Consumer price level
- Terms of trade
- Nominal exchange rate
- Nominal rate
- Real rate
Optimal Monetary Policy: A Special Case

- **Assumptions:**
  \[ Z_t = 1 \text{ all } t \]
  \[ \sigma = \eta = 1 \]

- **Social Planner’s Problem**
  \[
  \max U(C_t, N_t; 1)
  \]
  subject to the consumption/output possibilities set
  \[ C_t = Y_t^{1-v}(Y_t^*)^v \]

Optimal allocation
\[
- \frac{U_n(C_t, N_t)}{U_c(C_t, N_t)} = (1 - v)(1 - \alpha) \frac{C_t}{N_t}
\]

Under the assumed preferences:
\[ C_t N_t^\varphi = (1 - v)(1 - \alpha) \frac{C_t}{N_t} \]
\[ \Rightarrow N = [(1 - v)(1 - \alpha)]^{\frac{1}{1+\varphi}} \]

- **Flexible price equilibrium**
  \[
  \mathcal{M} = \frac{P_{H,t}}{W_t/M P N_t} = \frac{(1 - \alpha)(Y_t/N_t)}{(1 - \tau)C_t N_t^\varphi S_t^\psi} = \frac{(1 - \alpha)}{(1 - \tau)N_t^{1+\varphi}}
  \]

Optimal subsidy:
\[ (1 - \tau)(1 - v)\mathcal{M} = 1 \]
• Implementation

\[ \tilde{y}_t = \pi_{H,t} = 0 \]
\[ i_t = r^n_t \]

Optimal interest rate rule:

\[ i_t = r^n_t + \phi_\pi \pi_{H,t} \]

• Macroeconomic implications of domestic inflation targeting

\[ s^n_t = \sigma_v (y^n_t - y^*_t) - (1 - v)\Phi z_t \]
\[ = \sigma_v \Gamma a_t + \sigma_v (\Gamma_ - 1)y^*_t + [\sigma_v \Gamma_z - (1 - v)\Phi] z_t \]
\[ e_t^{\text{DIT}} = s^n_t - p^*_t \]
\[ p_t^{\text{DIT}} = vs^n_t \]
\[ = v(e_t^{\text{DIT}} + p^*_t) \]
Evaluation of Alternative Policies

• Welfare losses (under special case)

\[ W = \frac{(1 - v)}{2} \sum_{t=0}^{\infty} \beta^t \left( \left( \frac{1 + \varphi}{1 - \alpha} \right) \tilde{y}_t^2 + \frac{\epsilon}{\lambda} \pi_{H,t}^2 \right) \]

\[ V = \frac{(1 - v)}{2} \left[ \left( \frac{1 + \varphi}{1 - \alpha} \right) var(\tilde{y}_t) + \frac{\epsilon}{\lambda} var(\pi_{H,t}) \right] \]

• Four suboptimal rules

\[ \pi_t = 0 \]
\[ \epsilon_t = 0 \]
\[ i_t = 0.01 + 1.5\pi_{H,t} + 0.125\tilde{y}_t \]
\[ i_t = 0.01 + 1.5\pi_t + 0.125\tilde{y}_t \]
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