The New Keynesian Model with Sticky Wages and Prices

by

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Alternative Labor Market Specifications

- Competitive labor markets
  \[ w_t - p_t = mrs_t \]
  where \( mrs_t = \sigma c_t + \varphi n_t \).

- General labor market imperfections
  \[ w_t - p_t = \mu_t^w + mrs_t \]
  where \( \mu_t^w \): (log) wage markup.

Example: monopolistic union with isoelastic labor demand:
\[ \mu_t^w = \log \frac{\epsilon_w}{\epsilon_w - 1} \equiv \mu^w \]
Sticky Wages: Implications for Inflation Dynamics

Recall

\[ \pi_t^p = \beta E_t \{ \pi_{t+1}^p \} - \lambda_p (\mu_t^p - \mu^p) \]

Assuming constant returns (for simplicity)

\[ \mu_t^p = p_t - (w_t - a_t) \]
\[ = a_t - \omega_t \]
\[ = a_t - (\mu_t^w + \sigma c_t + \varphi n_t) \]
\[ = (1 + \varphi) a_t - (\sigma + \varphi) y_t - \mu_t^w \]

In deviations from natural levels (assuming constant natural markups):

\[ \mu_t^p - \mu^p = -(\sigma + \varphi) \tilde{y}_t - (\mu_t^w - \mu^w) \]

Implied New Keynesian Phillips Curve:

\[ \pi_t^p = \beta E_t \{ \pi_{t+1}^p \} + \kappa_p \tilde{y}_t + \lambda_p \hat{\mu}_t^w \]

\[ \implies \text{tradeoff between inflation and output gap stabilization} \]

**Question:** What determines the evolution of the wage markup?
An Economy With Sticky Wages and Prices (EHL (2000))

- **Firms**

  \[ Y_t(i) = A_t N_t(i)^{1-\alpha} \]

  where \( N_t(i) \equiv \left( \int_0^1 N_t(i, j)^{1-1/\epsilon_w} dj \right)^{\epsilon_w/(\epsilon_w-1)} \) and \( a_t \equiv \log A_t \sim AR(1) \)

  **Cost minimization:**

  \[ N_t(i, j) = \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i) \]

  where \( W_t \equiv \left( \int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{1-\epsilon_w} \)

  **Implication:**

  \[ \int_0^1 W_t(j) N_t(i, j) dj = W_t N_t(i) \]
**Price setting (as in basic NK model):**

Fraction of firms adjusting price each period: \(1 - \theta_p\)

Firm’s problem:

\[
\max_{P^*_t} \sum_{k=0}^{\infty} \theta_p^k E_t \left\{ \Lambda_{t,t+k}(1/P_{t+k}) \left( P^*_t Y_{t+k|t} - C_{t+k}(Y_{t+k|t}) \right) \right\}
\]

subject to:

\[
C_{t+k}(Y_{t+k|t}) = W_{t+k} \left( Y_{t+k|t}/A_{t+k} \right)^{1/(1-\alpha)}
\]

\[
Y_{t+k|t} = (P^*_t/P_{t+k})^{-\epsilon_p} C_{t+k}
\]

Implied price setting rule (log-linearized):

\[
p^*_t = \mu^p + (1 - \beta \theta_p) \sum_{k=0}^{\infty} (\beta \theta_p)^k E_t \{\psi_{t+k|t}\}
\]

where \(\psi_{t+k|t} \equiv \log \Psi_{t+k|t}\) and \(\mu^p \equiv \log \frac{\epsilon_p}{\epsilon_p-1}\)

**Price dynamics:**

\[
p_t = \theta_p p_{t-1} + (1 - \theta_p)p^*_t
\]

**Price inflation equation:**

\[
\pi^p_t = \beta E_t\{\pi^p_{t+1}\} - \lambda_p(\mu^p_t - \mu^p)
\]

where \(\lambda_p \equiv \frac{(1-\theta_p)(1-\beta \theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha + \alpha \epsilon_p} \).
Households

\[
\max E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, \{N_t(j)\}; Z_t)
\]

subject to:

\[
\int_0^1 P_t(i)C_t(i)di + Q_tB_t \leq B_{t-1} + \int_0^1 W_t(j)N_t(j)dj + D_t
\]

where \(C_t \equiv \left( \int_0^1 C_t(i)^{1-1/\epsilon_p} di \right)^{\epsilon_p/(\epsilon_p-1)} \) and

\[
U(C_t, \{N_t(j)\}; Z_t) = \begin{cases} 
\left( \frac{C_t^{1-\sigma}-1}{1-\sigma} - \int_0^1 \frac{N_t(j)^{1+\varphi}}{1+\varphi} dj \right) Z_t & \text{for } \sigma \neq 1 \\
\left( \log C_t - \int_0^1 \frac{N_t(j)^{1+\varphi}}{1+\varphi} dj \right) Z_t & \text{for } \sigma = 1
\end{cases}
\]

with \(z_t \equiv \log Z_t \sim AR(1)\).

**Optimality conditions**

\[
C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon_p} C_t
\]

\[
Q_t = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{Z_{t+1}}{Z_t} \right) \left( \frac{P_t}{P_{t+1}} \right) \right\}
\]

or, in log-linearized form:

\[
c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(i_t - E_t\{\pi^p_{t+1}\} - \rho) + \frac{1}{\sigma}(1 - \rho_z)z_t
\]
Wage Setting

Fraction of occupations/unions adjusting nominal wage: \( 1 - \theta_w \)

Optimal wage setting:

\[
\max_{\tilde{W}_t^*} E_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \left( C_{t+k}^{-\sigma} \frac{W_t^*}{P_{t+k}} N_{t+k|t} - \frac{N_{t+k|t}^{1+\varphi}}{1 + \varphi} \right) Z_{t+k}
\]

subject to

\[
N_{t+k|t} = \left( \frac{W_t^*}{\tilde{W}_{t+k}} \right)^{-\epsilon_w} \left( \int_0^1 N_{t+k}(i) di \right)
\]

Optimality condition:

\[
\sum_{k=0}^{\infty} (\beta \theta_w)^k E_t \left\{ N_{t+k|t} Z_{t+k} C_{t+k}^{-\sigma} \left( \frac{W_t^*}{P_{t+k}} - \mathcal{M}_w M R S_{t+k|t} \right) \right\} = 0
\]

where \( M R S_{t+k|t} = C_{t+k}^\sigma N_{t+k|t}^{\varphi} \) and \( \mathcal{M}_w = \frac{\epsilon_w}{\epsilon_w - 1} \).

Log-linearized version:

\[
w_t^* = \mu^w + (1 - \beta \theta_w) \sum_{k=0}^{\infty} (\beta \theta_w)^k E_t \left\{ m r s_{t+k|t} + p_{t+k} \right\}
\]
Equivalently:

\[ w_t^* = (1 - \beta \theta_w) \sum_{k=0}^{\infty} (\beta \theta_w)^k E_t \{ w_{t+k} - (1 + \epsilon_w \varphi)^{-1} \hat{\mu}_{t+k} \} \]

where \( \mu_t^w \equiv (w_t - p_t) - mrs_t \) and \( mrs_t = \sigma c_t + \varphi n_t \).

**Aggregate wage dynamics**

\[ w_t = \theta_w w_{t-1} + (1 - \theta_w) w_t^* \]

**Wage inflation equation**

\[ \pi_t^w = \beta E_t \{ \pi_{t+1}^w \} - \lambda_w (\mu_t^w - \mu_t^w) \]

where \( \lambda_w = \frac{(1-\beta \theta_w)(1-\theta_w)}{\theta_w (1+\varphi \epsilon_w)} \)
Equilibrium

Goods market clearing:

\[ Y_t(i) = C_t(i) \text{ all } i \in [0, 1] \Rightarrow Y_t = C_t \]

where \( Y_t \equiv \left( \int_0^1 Y_t(i)^{1 - \frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}. \)

Aggregate employment

\[
N_t \equiv \int_0^1 \int_0^1 N_t(i, j) dj di = \int_0^1 N_t(i) \int_0^1 \frac{N_t(i, j)}{N_t(i)} dj di = \Delta_{w,t} \int_0^1 N_t(i) di
\]

\[
= \Delta_{w,t} \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \int_0^1 \left( \frac{Y_t(i)}{Y_t} \right)^{\frac{1}{1-\alpha}} di = \Delta_{w,t} \Delta_{p,t} \left( \frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}}
\]

where \( \Delta_{w,t} \equiv \int_0^1 \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} dj \) and \( \Delta_{p,t} \equiv \int_0^1 \left( \frac{P_t(i)}{P_t} \right)^{-\frac{\epsilon_p}{1-\alpha}} di. \)

Up to a first order approximation:

\[(1 - \alpha)n_t = y_t - a_t\]
Wage gap:
\[ \tilde{\omega}_t \equiv \omega_t - \omega^n_t \]
where \( \omega_t \equiv \omega_t - p_t \) and where \( \omega^n_t \) is the natural real wage:
\[ \omega^n_t = \log(1 - \alpha) + (a_t - \alpha n^n_t) - \mu^p \]
\[ = \log(1 - \alpha) + \psi_{wa} a_t - \mu^p \]
where \( \psi_{wa} \equiv \frac{1 - \alpha \psi_{ya}}{1 - \alpha} > 0 \) and \( \psi_{ya} \equiv \frac{1 + \phi}{\sigma(1 - \alpha) + \phi + \alpha} \).

Price markup gap:
\[ \mu^p_t = \log(1 - \alpha) + (a_t - \alpha n^n_t) - \omega_t \]
\[ \mu^p = \log(1 - \alpha) + (a_t - \alpha n^n_t) - \omega^n_t \]
\[ \Rightarrow \tilde{\mu}^p_t = -\frac{\alpha}{1 - \alpha} \tilde{y}_t - \tilde{\omega}_t \]

Hence:
\[ \pi^p_t = \beta E_t \{ \pi^p_{t+1} \} + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t \]
where \( \kappa_p \equiv \frac{\alpha \lambda_p}{1 - \alpha} \).
Wage markup gap:

\[
\begin{align*}
\mu_t^w &= \omega_t - (\sigma y_t + \varphi n_t) \\
\mu^w &= \omega_t^n - (\sigma y^n_t + \varphi n^n_t) \\
\Rightarrow \tilde{\mu}_t &= \tilde{\omega}_t - \left(\sigma + \frac{\varphi}{1 - \alpha}\right) \tilde{y}_t
\end{align*}
\]

Hence:

\[
\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \kappa \tilde{y}_t - \lambda \tilde{\omega}_t
\]

where \( \kappa \equiv \lambda_w \left(\sigma + \frac{\varphi}{1 - \alpha}\right) \).

In addition:

\[
\tilde{\omega}_t \equiv \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta \omega_t^n
\]

Dynamic IS equation

\[
\tilde{y}_t = -\frac{1}{\sigma} (i_t - E_t\{\pi_{t+1}^p\} - r_t^n) + E_t\{\tilde{y}_{t+1}\}
\]

where \( r_t^n \equiv \rho - \sigma(1 - \rho_y)\psi_y a_t + (1 - \rho_z) z_t \)

Interest Rate Rule:

\[
i_t = \rho + \phi_p \pi_t^p + \phi_w \pi_t^w + \phi_y \tilde{y}_t + v_t
\]
Dynamical system:

\[
A^w_0 x_t = A^w_1 E_t \{ x_{t+1} \} + B^w_0 u_t
\]

where \( x_t \equiv [\tilde{y}_t, \pi^p_t, \pi^w_t, \tilde{\omega}_{t-1}]' \), \( u_t \equiv [\hat{r}^n_t - v_t - \phi_y \tilde{y}^n_t, \Delta \omega^*_t]' \),

\[
A^w_0 \equiv \begin{bmatrix}
\sigma + \phi_y & \phi_p & \phi_w & 0 \\
-\xi_p & 1 & 0 & 0 \\
-\xi_w & 0 & 1 & 0 \\
0 & -1 & 1 & 1 \\
\end{bmatrix}
\]

\[
A^w_1 \equiv \begin{bmatrix}
\sigma & 1 & 0 & 0 \\
0 & \beta & 0 & \lambda_p \\
0 & 0 & \beta & -\lambda_w \\
0 & 0 & 0 & 1 \\
\end{bmatrix}, \quad B^w_0 \equiv \begin{bmatrix}
1 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 1 \\
\end{bmatrix}
\]

Conditions for uniqueness of the equilibrium\(^1\)

\[
\phi_p + \phi_w + \phi_y \left( \frac{1 - \beta}{\sigma + \frac{\alpha + \zeta}{1 - \alpha}} \right) \left( \frac{1}{\lambda_p} + \frac{1}{\lambda_w} \right) > 1
\]

Particular case (\( \phi_y = 0 \)):

\[
\phi_p + \phi_w > 1
\]

\(^1\)Flaschel-Franke (2008), Blasselle-Poissonier (2013)
Figure 6.1 Determinacy and Indeterminacy Regions

The diagram shows a graph with two axes: $\phi_y$ on the x-axis and $\phi_p + \phi_w$ on the y-axis. The region above the line $\phi_p + \phi_w = \phi_y$ is labeled as Determinacy, while the region below it is labeled as Indeterminacy.
Dynamic Responses to a Monetary Policy Shock

*Interest rate rule:* \( \phi_p = 1.5, \phi_y = \phi_w = 0, \rho_v = 0.5 \)

*New parameter:* \( \epsilon_w = 4.5 \)

*Three calibrations:*

**Baseline:** \( \theta_p = 3/4, \theta_w = 3/4 \)

**Flexible wages:** \( \theta_p = 3/4, \theta_w = 0 \)

**Flexible price:** \( \theta_p = 0, \theta_w = 3/4 \)

*Simulations*
Figure 6.2 Dynamic Responses to a Monetary Policy Shock

- **Output Gap**: Baseline, Flexible Wages, Flexible Prices
- **Price Inflation**: Baseline, Flexible Wages, Flexible Prices
- **Wage Inflation**: Baseline, Flexible Wages, Flexible Prices
- **Real Wage**: Baseline, Flexible Wages, Flexible Prices
Monetary Policy Design: The Social Planner’s Problem

\[
\max U(C_t, \{N_t(j)\}; Z_t)
\]

subject to:

\[
C_t(i) = A_t N_t(i)^{1-\alpha}
\]

\[
N_t(j) = \int_0^1 N_t(i, j) di
\]

where \(C_t \equiv \left( \int_0^1 C_t(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p-1}} \) and \(N_t(i) \equiv \left( \int_0^1 N_t(i, j)^{1-\frac{1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w-1}} \).

Optimality conditions:

\[
C_t(i) = C_t, \text{ all } i \in [0, 1]
\]

\[
N_t(i, j) = N_t(j) = N_t(i) = N_t, \text{ all } i, j \in [0, 1]
\]

\[
\frac{-U_{n,t}}{U_{c,t}} = MP N_t
\]

where \(MP N_t = (1 - \alpha) A_t N_t^{-\alpha}\)
Efficiency of the Natural Equilibrium

In the decentralized economy with flexible prices and wages:

\[ P_t = \mathcal{M}_p \frac{(1 - \tau)W_t}{MPN_t} \]

\[ \frac{W_t}{P_t} = -\frac{U_{n,t}}{U_{c,t}} \mathcal{M}_w \]

for all goods and occupations, where \( \mathcal{M}_p \equiv \frac{\epsilon_p}{\epsilon_p - 1} \) and \( \mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1} \). Thus,

\[ -\frac{U_{n,t}}{U_{c,t}} = \frac{1}{\mathcal{M}(1 - \tau)} MPN_t \]

where \( \mathcal{M} \equiv \mathcal{M}_p \mathcal{M}_w \).

Condition for efficiency of the natural equilibrium: \( \mathcal{M}(1 - \tau) = 1 \)

*Remark*: natural equilibrium generally not attainable with sticky prices and wages (proof)
Optimal Monetary Policy Problem

\[
\min \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \left( \left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t^2 + \frac{e_p}{\lambda_p} (\pi_t^p)^2 + \frac{e_w(1 - \alpha)}{\lambda_w} (\pi_t^w)^2 \right)
\]

subject to:

\[
\pi_t^p = \beta E_t \{ \pi_{t+1}^p \} + \kappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t
\]

\[
\pi_t^w = \beta E_t \{ \pi_{t+1}^w \} + \kappa_w \tilde{y}_t - \lambda_w \tilde{\omega}_t
\]

\[
\tilde{\omega}_{t-1} \equiv \tilde{\omega}_t - \pi_t^w + \pi_t^p + \Delta \omega_t^n
\]
Optimality conditions:

\[
\left( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_t + \pi_{p,t} \xi_{1,t} + \pi_{w,t} \xi_{2,t} = 0
\]

\[
\frac{\varepsilon_p}{\lambda_p} \pi_{p,t}^p - \Delta \xi_{1,t} + \xi_{3,t} = 0
\]

\[
\frac{\varepsilon_w (1 - \alpha)}{\lambda_w} \pi_{w,t}^w - \Delta \xi_{2,t} - \xi_{3,t} = 0
\]

\[
\lambda_p \xi_{1,t} - \lambda_w \xi_{2,t} + \xi_{3,t} - \beta E_t \{ \xi_{3,t+1} \} = 0
\]

for \( t = 0, 1, 2, \ldots \) given \( \xi_{1,-1} = \xi_{2,-1} = 0 \) and given \( \tilde{\omega}_{-1} \).

Equilibrium under the optimal policy:

\[
A_0^* x_t = A_1^* E_t \{ x_{t+1} \} + B_0^* \Delta a_t
\]

where \( x_t \equiv [\tilde{y}_t, \pi_{p,t}^p, \pi_{w,t}^w, \tilde{\omega}_{t-1}, \xi_{1,t-1}, \xi_{2,t-1}, \xi_{3,t}]' \).
**Optimal Policy in Response to Demand Shocks**

\[ A^*_0 x_t = A^*_1 E_t \{ x_{t+1} \} \]

Under the assumption of \( \tilde{\omega}_{t-1} = 0 \),

\[ x_t = 0 \]

for all \( t \)

**Implementation:**

\[ i_t = r^n_t + \phi_p \pi_t \]

where \( r^n_t = \rho + (1 - \rho_z) z_t \), and \( \phi_p > 1 \)

**Optimal Policy in Response to Technology Shocks**

Figure 6.3
Figure 6.3  Dynamic Responses to a Technology Shock under the Optimal Monetary Policy

- Output gap
- Price inflation
- Wage inflation
- Real wage

- Baseline
- Flexible wages
- Flexible prices
A New Keynesian Phillips Curve for Composite Inflation

$$\pi_t = \beta E_t \{ \pi_{t+1} \} + \kappa \bar{y}_t$$

where $$\kappa \equiv \frac{\lambda_w \lambda_p}{\lambda_p + \lambda_w} \frac{\alpha}{1-\alpha}$$

$$\pi_t \equiv \frac{\lambda_w}{\lambda_p + \lambda_w} \pi^p_t + \frac{\lambda_p}{\lambda_p + \lambda_w} \pi^w_t$$

$$\Rightarrow$$ no tradeoff
A Special Case

\[ \pi_p = \pi_w \equiv \pi \]
\[ \epsilon_p = \epsilon_w (1 - \alpha) \equiv \epsilon. \]

Implied optimality conditions:

\[ \pi_t = -\frac{1}{\epsilon} \Delta \tilde{y}_t \quad (1) \]

for \( t = 1, 2, 3, \ldots \) as well as

\[ \pi_0 = -\frac{1}{\epsilon} \tilde{y}_0 \quad (2) \]

*Optimal policy:*

\[ \pi_t = \tilde{y}_t = 0 \]

for all \( t \)
Figure 6.4: Welfare losses under alternative policies with sticky wages and prices.

Woodford (2003)
Evaluation of Simple Interest Rate Rules

Strict Targeting Rules:

\[ \pi_t^i = 0 \]

Flexible Targeting Rules:

\[ i_t = 0.01 + 1.5\pi_t^i \]

Table 6.1
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