Validating DSGE Models from SVARs with Short-Run Restrictions: Information Sets Matter!

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Abstract

This paper explores the quantitative implications of restricting the information sets conditional on which decisions are taken in DSGE models. We show that restricting the information set is of no substantial consequences in DSGE models with weak internal propagation mechanisms. However, in a DSGE model featuring a large number of real frictions that enhance its persistence properties, the Impulse Response Functions of certain variables can be modified in a marked way, i.e. their sign, amplitude and persistence can drammatically change. This shows that implementing short-run restrictions in DSGE models is not as innocuous as might seem at first glance.

Keywords: Short-run restrictions, SVAR, DSGE models.

JEL Class.: E24, E32

Introduction

The econometrics of Dynamic Stochastic General Equilibrium (DSGE) models has witnessed substantial advances over the recent years. It is nowadays more and more common to bring DSGE models to the data using a variety of formal statistical techniques, including Maximum Likelihood estimation (Altug, 1989, Ireland, 2004), Generalized Method of Moments (Christiano and Eichenbaum, 1992, Burnside, Eichenbaum and Rebelo, 1993), Bayesian techniques (Schorfheide, 2000, Smets and Wouters, 2003), Minimum Distance Estimation (Rotemberg and Woodford, 1997, Christiano, Eichenbaum and Evans, 2005). The present paper is concerned with the principles on which the latter econometric technique is based.

The Minimum Distance Estimation (MDE) technique consists in estimating the structural parameters of DSGE models so as to minimize a weighted distance between theoretical impulse responses of key macroeconomic variables and those derived from a Structural Vector Autoregression (SVAR). The attractive feature of this method is that it allows researchers to bring structural and empirical approaches to closer conformity. Another key advantage is that it is not necessary to fully specify the whole stochastic structure of the DSGE model. Moreover, it allows researchers to focus attention only on those shocks that are relevant for the question studied.[1]

This method requires that an auxiliary SVAR model be estimated prior to estimating the DSGE parameters. In doing so, a researchers has access to at least two broad types of identifying restrictions. Blanchard and Quah (1989) and Galí (1999) have proposed to identify shocks based on long-run restrictions through which some variables are asymptotically forbidden to react to certain shocks. For example, Galí (1999) identifies technology shocks as the only shocks that can impact the level of average labor productivity in the long-run. The attractive feature of this approach is that this type of restrictions holds in a large class of competing DSGE models, thus allowing researchers to assess the empirical relevance of one theory against the other using the Impulse Response Functions' (IRF) behavior in the short-

[1]For example, Rotemberg and Woodford (1997) and Christiano, Eichenbaum and Evans (2005) study the dynamic effects of monetary policy shocks.

run.[2] However, Erceg, Guerrieri, and Gust (2004), and Chari, Kehoe, and McGrattan (2005) have recently questioned the ability of long-run restrictions in SVAR to properly recover the shocks. An alternative consists in imposing short-run restrictions through which some variables are forbidden to react to some shocks on impact (Sims, 1980, Christiano, Eichenbaum, and Evans, 2005). Christiano, Eichenbaum and Vigfusson (2005) show that this latter technique performs remarkably well compared to long-run restrictions. In particular, short-run restrictions are able to correctly and precisely pin down the shocks. This suggests that SVAR with short-run restrictions can be a useful assessment device when constructing and evaluating a DSGE model.

Applying the MDE approach requires that DSGE and SVAR share the same restrictions, be it short-run or long-run. With short-run restrictions, this implies that some form of recursiveness be imposed in the DSGE to make it compatible with the SVAR. A researcher is thus conduced to restrict some variables not to respond to certain shocks on impact in the DSGE model. To do so requires that the information set in the DSGE be manipulated, i.e. certain variables must are restricted to be decided prior to observing the current period shocks, e.g. Rotemberg and Woodford (1997), Christiano, Eichenbaum and Evans (2005), Christiano, Eichenbaum and Vigfusson (2005), Boivin and Gianonni (2003), Gianonni and Woodford (2004).

Is this approach innocuous? No, if the DSGE dynamic properties are unaffected by the timing of decisions in the model. For example, Christiano, Eichenbaum and Evans (2005) show that manipulating the information set has negligible effects on the dynamic properties of their model, in response to a monetary policy shock identified with short-run restrictions. Similarly, Christiano, Eichenbaum and Vigfusson (2005) show that in a standard Real Business Cycle (RBC) model, the response of hours to a permanent technology shock is not dependent upon the timing of decisions regarding labor supply.

The purpose of the present paper is to warn researchers that manipulating the information sets conditional on which decisions are taken in the DSGE model is far from being innocuous.

[2] Galí (1999) uses the information conveyed by the short-run response of hours to a technology shock as a way of discriminating between flexible-price and sticky-price theories of the business cycle.

Obviously, information sets matter when it comes to the comovements between restricted and unrestricted variables. In many cases, restricting the information set has proved a fruitful research strategy, as exemplified by Burnside, Eichenbaum and Rebelo (1993) for example. Surprisingly, proponents of the MDE approach abstract from the quantitative implications of a change in the information sets. This should be of no importance if the overall pattern of impulse response functions in the DSGE model are left unaffected when the informations sets vary. To the contrary, the aim of this paper is to show using simple examples that information sets can matter for the dynamic properties of DSGE models.

This is first shown within the simple framework of a univariate dynamic rational-expectation model that encompasses a large class of small DSGE models. More precisely, it turns out that information sets do not matter as long as the model's dynamics is dominated by its forward-looking dimension. In other terms, information sets do not alter the dynamic properties of models with weak internal propagation mechanisms. In contrast, when these mechanism are important, manipulating the information sets dramatically affect the response of the economy to the shock under study. So as to illustrate these ideas, we consider a simple DSGE model with habit formation and no capital accumulation, which maps into the above framework. In this model, the response of hours to a permanent technology shock is highly modified when labor supply is decided prior to observing the current period technology shock, as opposed to what happens when this decision is taken after observing the shock. More precisely, in the latter case, (referred to as the full information case), hours respond negatively to a technology shock whereas they respond positively when current-period shocks are not observed (referred to as the incomplete information case). Thus the IRFs sign and amplification differ markedly. However, the persistence of hours conditional on technology shocks is not affected by the information set.

In a second step, we extend this basic framework to multivariate economies. More precisely, we consider an RBC model with habit formation and investment adjustment costs in the line of Boldrin, Christiano and Fisher (2001) and Christiano, Eichenbaum and Evans (2005). When habit and adjustment costs parameters are set to zero, i.e. when the forward-looking dimension of the model is prominent, information sets do not matter for the dynamics of hours. This is this version of our model that CEV (2005) use in order to assess the validity of SVARs with short-run restrictions. To the contrary, when habit and adjustment costs parameters are set to positive values, i.e. when the backward looking dimension of the model plays a significant role, we obtain the opposite result: information sets matter. More precisely, the impulse response functions of hours after a technology shock differ in terms of sign, amplitude and persistence. This draws an important warning to researchers willing to implement short-run restrictions in DSGE models. To the contrary, the long-run responses of variables in a DSGE are not dependent upon the timing of decisions. Unfortunately, SVAR seem to perform poorly when such long-run restrictions are used for the purpose of identifying shocks.

The paper is organized as follows. Section 1 discusses the properties of a canonical, univariate, rational-expectations model that allows us to identify a set of conditions under which information sets matter. These results are illustrated in a simple DSGE model with habit formation that maps into this framework. In section 2, we consider a more refined DSGE model with real frictions that are source of strong internal propagation properties. The last section briefly concludes.

1 Univariate DSGE Models

In this section, we use univariate DSGE models in order to highlight analytically the dynamic consequences of restricting the timing of certain decisions. We start by doing so in a rather general framework which we then specialize to the study of the dynamic response of hours to a permanent technology shock.

1.1 An Introductory Example

For presentation purpose, we first consider a simple rational expectations model in which a single endogenous variable y_t is a linear function of its expectation in the next period, its once-lagged value and an exogenous variable x_t. The model is deliberately kept at the simplest level, so as to highlight the main insight. The model takes the form

$$\mathrm{E}_t^\star y_{t+1} - (\lambda_1 + \lambda_2) y_t + (\lambda_1 \times \lambda_2) y_{t-1} + \mathrm{E}_t^\star x_t = 0, \tag{1}$$

where $\mathrm{E}_t^\star$ is a expectation operator conditional on an information set to be defined below. Implicitly, and without loss of generality, the coefficient in front of $\mathrm{E}_t^\star x_t$ is normalized to unity. In what follows, we assume $|\lambda_1| < 1$ and $|\lambda_2| > 1$. As is common practice in the business cycle literature, the exogenous variable x_t follows an autoregressive process of order one

$$x_t = \rho x_{t-1} + \sigma \varepsilon_t, \tag{2}$$

where $|\rho| < 1$, $\sigma > 0$ and ε_t is a white noise process with zero mean and unit variance. Without loss of generality, we omit a constant term both in (1) and (2) since the endogenous and exogenous variables can be considered as deviations from their average value. Equation (1) can define either a linear or a log-linear representation of the equilibrium conditions of a dynamic model. Moreover, equation (1) can represent an approximation to the equilibrium conditions of a non-linear model, as would result from the loglinearization of a DSGE model for example.

Despite its simplicity, this simple linear representation embodies several model economies: the Cagan model, the monetary model of exchange rate determination, asset pricing models, dynamic factor demand models, the basic real business cycle model, monetary models, and so on. We consider two versions of this model, depending on the information set available at the time agents take their decisions. In the first case (denoted model $\mathcal{M}_1$), we assume that the information set includes all the relevant variables in period t. More precisely, the shock of the period ε_t enters the information set conditional on which decisions are taken. In a second version of the model (denoted model $\mathcal{M}_2$), we assume that decisions about y_t are taken prior to observing the current period shock. In this case, ε_t is excluded from the information set on which expectations are conditioned.

Assuming that $\lambda_1 \neq \rho$, and using equations (1) and (2), it is straightforward to show that the solution to each model takes the form

$$y_t = \lambda_1 y_{t-1} + \frac{1}{\lambda_2 - \rho} x_t \qquad \text{[Model } \mathcal{M}_1\text{]}, \tag{3}$$

$$y_t = \lambda_1 y_{t-1} + \frac{\rho}{\lambda_2 - \rho} x_{t-1} \qquad \text{[Model } \mathcal{M}_2\text{]}. \tag{4}$$

Since $|\lambda_1| < 1$ and $|\rho| < 1$, the two reduced forms (3) and (4) admit the following infinite moving average representation

$$y_t = \frac{1}{\lambda_2 - \rho} \sum_{i=0}^{\infty} \left(\frac{\rho^{i+1} - \lambda_1^{i+1}}{\rho - \lambda_1} \right) \varepsilon_{t-i} \qquad \text{[Model } \mathcal{M}_1\text{]}, \tag{5}$$

$$y_t = \frac{\rho}{\lambda_2 - \rho} \sum_{i=0}^{\infty} \left(\frac{\rho^{i+1} - \lambda_1^{i+1}}{\rho - \lambda_1} \right) \varepsilon_{t-i-1} \qquad \text{[Model } \mathcal{M}_2\text{]}, \tag{6}$$

from which we can directly deduce the IRFs of y_t to an unexpected innovation ε_t to the exogenous variable x_t.

When comparing the impulse response functions of each model, it is important to notice that there is no point in comparing the impact response. The reason why is simple: in model $\mathcal{M}_2$, the impact response is restricted to be zero, since y_t is decided prior to observing the current period shock; in contrast, model $\mathcal{M}_1$ allows y_t to instantaneously react to an innovation to x_t. In the sequel, we exclusively focus on the models' IRFs at horizon $h \geq 1$. Both models can be thought to generate similar dynamics (conditional on x_t), if their IRFs share the same sign and patterns of amplification and propagation.

Let $I_i(h)$ denote the IRF of y_t in model i, $i = 1, 2$. From eqs. (5)-(6), we directly deduce

$$I_1(h) = \frac{1}{\lambda_2 - \rho} \left(\frac{\rho^{h+1} - \lambda_1^{h+1}}{\rho - \lambda_1} \right) \qquad \text{[Model } \mathcal{M}_1\text{]},$$

$$I_2(h) = \frac{\rho}{\lambda_2 - \rho} \left(\frac{\rho^{h} - \lambda_1^{h}}{\rho - \lambda_1} \right) \qquad \text{[Model } \mathcal{M}_2\text{]}.$$

The first salient result is as follows. When both models have no backward-looking dimension (i.e. when $\lambda_1 = 0$), $I_1(h)$ and $I_2(h)$ are strictly equal for $h \geq 1$

$$I_1(h) = I_2(h) = \frac{\rho^h}{\lambda_2 - \rho}.$$

Thus, for simple models without endogenous propagation mechanisms, restricting the information set available to the agents at the time decisions are taken is completely innocuous from the point of view of MDE. Unfortunately, these models stand in sharp contrast with the current research agenda in quantitative macroeconomics. Indeed, it is more and more common to include a variety of (real or nominal) frictions that generate strong internal propagation channels that are necessary in order to satisfyingly match the IRFs derived from the SVAR to which the DSGE is compared. Within the framework of the above models, this would suggest that λ_1 be relatively large.

Now, let us consider the case $\lambda_1, \lambda_2 > 0$, and we let ρ vary in the compact $[-1, 1]$.[3] In this case, we obtain a first notable result: for $h = 1$, the IRFs of models $\mathcal{M}_1$ and $\mathcal{M}_2$ do not necessarily share the same sign, provided that $\rho < 0$ and $|\rho| < \lambda_1$, as is the case with models the internal propagation mechanisms are richer than their exogenous sources of persistence. This is a sufficient feature to show that restricting the information set can deeply distort the IRFs of apparently similar models. As indicated before, a second element of models comparison relies on their amplification properties. This can be once again studied by inspecting the IRFs, notably for short horizons when one is interested in business cycle analysis. For example, in the case $h = 1$, we obtain IRFs

$$|I_1(1)| = \left|\frac{\rho + \lambda_1}{\lambda_2 - \rho}\right| \quad \text{and} \quad |I_2(1)| = \left|\frac{\rho}{\lambda_2 - \rho}\right|.$$

Here, we focus on absolute values in order to control for the possible sign difference. It is clear that for $\lambda_1 > 0$, both IRFs differ. Moreover, provided that $\rho + \lambda_1 > 0$, $|I_1(1)| - |I_2(1)|$ increases with λ_1. Thus, the stronger the persistence channels, the more different the amplification mechanisms of $\mathcal{M}_1$ and $\mathcal{M}_2$.

A final inspection of the models dynamics in response to x_t relates to their persistence mechanisms. However, IRFs are not necessarily easy to interpret when one is interested in

[3] Considering negative λ_1 is not particularly relavant for business cycle analysis, since the strong persistence of aggregate variables is a well-known stylized fact and requires a positive λ_1 to be matched. Restricting our attention to positive λ_2 is motivated by the fact that many business cycle models imply that λ_1 and λ_2 share the same sign, as is the case for example with the standard RBC model.

persistence. For example, notice that as soon as an IRF crosses the axis line, the half-life of convergence ceases to be informative. This is why the autocorrelation function (ACF) conditional on x_t should be considered as the proper measure of persistence.[4] However, in the present case, it is obvious from eqs. (3)-(4) that both models share the exact same autocorrelation function $\varrho(h)$

$$\varrho(h) = (\rho + \lambda_1)\,\varrho(h-1) - \rho\lambda_1\varrho(h-2)\,, \quad h = 1, 2$$

with $\varrho(0) = 1$ and $\varrho(-h) = \varrho(h)$.

This simple introductory example has shown that in DSGE models similar to equations (1)-(2), restricting the information set conditional on which decisions are taken can have substantial effects on the sign and amplitude of the responses to the shocks considered. Next, we illustrate this point more formally in the context of a business cycle model without capital where we study the response of hours to a technology shock. However, altering the persistence properties requires a more complicated model, with multiple endogenous state variables, the presentation of which is deferred until section 2.

1.2 A Simple Model with Habit Persistence

We consider a simple DSGE model with habit formation in consumption and a permanent technology shock. It can be seen as a flexible price version of the model studied by Galí and Rabanal (2004). We assume that intertemporal consumption choices are not time separable and that the service flows from consumption are a linear function of current and once-lagged consumptions choices (see e.g. Braun, Constandinines and Ferson, 1993). More precisely, the intertemporal expected utility function of the representative household is given by

$$\mathrm{E}_t^{\star} \sum_{i=0}^{\infty} \beta^i \left\{ \log(C_{t+i} - bC_{t+i-1}) + \bar{\chi}\,(1 - N_t) \right\} \qquad \bar{\chi} > 0, \tag{7}$$

[4]We hasten to insist that the relevant statistic is the correlation conditional on a particular shock. By contrast, in a multi-shock model, the autocorrelation function can be a complicated mix of all the shocks and it is difficult a priori to tell their influence apart.

subject to the per period budget constraint

$$C_t \leq W_t N_t + \Pi_t. \tag{8}$$

The parameter $\beta \in (0,1)$ is the subjective discount factor and $b \in (0,1)$ governs the evolution of consumption habits. $\mathrm{E}_t^\star$ is the expectation operator conditional on the information set available as of time t. The quantity of good consumed in period t is C_t. The variable N_t denotes hours worked, W_t is the real wage, and Π_t represents the profit that the household receives from the firm. The utility function is separable, logarithmic in consumption, and following Hansen (1985), linear in leisure, implying an infinite labor supply elasticity.[5] Without loss of generality, the time endowment is set to unity. The first order conditions of the household's problem (7)–(8) yield

$$\bar{\chi} = W_t \ \ \mathrm{E}_t^\star \left\{ \frac{1}{C_t - bC_{t-1}} - \beta b \frac{1}{C_{t+1} - bC_t} \right\}.$$

The representative firm produces a homogenous good with a constant returns to scale technology

$$Y_t = Z_t N_t,$$

The variable Z_t is the aggregate technology, the growth rate of which $\gamma_{z,t} \equiv \log(Z_t/Z_{t-1})$ is assumed to evolve according to

$$\gamma_{z,t} = \rho \gamma_{z,t-1} + (1-\rho)\bar{\gamma} + \sigma \varepsilon_{z,t} \qquad |\rho| < 1 \ , \ \sigma > 0$$

where $\varepsilon_{z,t}$ is *iid* with zero mean and unit variance. The first order condition of the firm is $W_t = Y_t/N_t$. Using the first order conditions of households and firms together with markets equilibrium $Y_t = C_t = Z_t N_t$, the equilibrium of the economy is given by the following expression

$$\bar{\chi} = Z_t \ \ \mathrm{E}_t^\star \left\{ \frac{1}{Z_t N_t - bZ_{t-1} N_{t-1}} - \beta b \frac{1}{Z_{t+1} N_{t+1} - bZ_t N_t} \right\}.$$

As above, we consider two versions of this model, each of them depending on the timing decisions of labor supply. In the first version (denoted $\mathcal{M}_1$), we follow the standard approach

[5] This modelling choice is of no substantial consequence on the mechanisms which we describe below.

and assume that all decisions (consumption and labor supply) are taken after the technology shock at t is observed. In this case, decision are made conditional on the information set generated by $\{\varepsilon_{z,t}, \varepsilon_{z,t-1}, ...\}$. Following Christiano, Eichenbaum and Vigfusson (2005), we consider a second version (denoted $\mathcal{M}_2$) in which labor supply decisions are taken before observing the technology shock. Labor supply decisions are now conditional on the information set generated by $\{\varepsilon_{z,t-1}, \varepsilon_{z,t-2}, ...\}$. Conversely, the decisions about consumption are taken after the observations of the this shock. We first apply a stationary–inducing transformation since output and consumption follow a stochastic trend. The approximate solution of the model is computed from a log–linearization of the stationary equilibrium conditions around the deterministic steady state. For model $\mathcal{M}_1$, equilibrium employment is given by

$$\widehat{n}_t = b\widehat{n}_{t-1} - b\left(\frac{1+\beta\rho(1-2b)}{1-\beta b\rho}\right)\widehat{\gamma}_{z,t}, \tag{9}$$

where $\widehat{\gamma}_{z,t} = \gamma_{z,t} - \bar{\gamma}$. Since $(1+\beta\rho(1-2b))/(1-\beta b\rho) > 0$ for $(\beta, b) \in (0,1)$ and $|\rho| < 1$, hours respond negatively and persistently to a technology shock when $b > 0$. For example, when the growth rate $\widehat{\gamma}_{z,t}$ is *iid*, the response of hours is always negative after a technology shock since

$$\widehat{n}_t = -b\sum_{i=0}^{\infty} b^i \varepsilon_{z,t-i}$$

For model $\mathcal{M}_2$, equilibrium employment is given

$$\widehat{n}_t = b\widehat{n}_{t-1} - b\rho\left(\frac{1+\beta\rho(1-2b)}{1-\beta b\rho}\right)\gamma_{z,t-1} \tag{10}$$

Notice that when the growth rate of technology is *iid*, hours never react to a technology shock. This simple example shows that a slight change in the information set may have dramatic effects on the dynamics of hours following a random-walk technology shock.

From (9) and (10), we can deduce IRFs to a technology shock for models $\mathcal{M}_1$ and $\mathcal{M}_2$

$$I_1(h) = -b\left(\frac{1+\beta\rho(1-2b)}{1-\beta b\rho}\right)\left(\frac{\rho^{h+1}-b^{h+1}}{\rho-b}\right) \qquad \text{[Model } \mathcal{M}_1\text{]},$$

$$I_2(h) = -b\rho\left(\frac{1+\beta\rho(1-2b)}{1-\beta b\rho}\right)\left(\frac{\rho^{h}-b^{h}}{\rho-b}\right) \qquad \text{[Model } \mathcal{M}_2\text{]}.$$

As in the introductory example, the response of hours in models $\mathcal{M}_1$ and $\mathcal{M}_2$.will critically depend on the process of the technology shock. Following Prescott (1986) and Hansen (1997), we consider that the growth rate of the technology is negatively serially correlated ($\rho = -0.2$). When $b + \rho > 0$, the two IRFs $I_1(h)$ and $I_2(h)$ differ sharply. First, model $\mathcal{M}_1$ implies a negative response at any horizon, whereas the response of hours is always positive in model $\mathcal{M}_2$. In model $\mathcal{M}_1$, the negative response originates from habit formation. With habit persistence, consumption does not change much following an increase in the labor income (i.e. an increase in Z_t), because agents are reluctant to adjust consumption rapidly. When labor supply decisions are taken after the realization of the technology shock at period t (model $\mathcal{M}_1$), households spend their new income in leisure (see Francis and Ramey, 2004 for a similar mechanism). When habit persistence is large enough (i.e. the habit persistence parameter satisfies $b > -\rho \equiv 0.2$), the response of hours is persistently negative. Notice that the negative response of hours is amplified when b increases (see equation (9)). Conversely, when labor supply decisions are taken before the realization of the technology shocks, hours do not respond on impact, but consumption will persistently increase. Moreover, households expect a decrease in the growth rate of the real wage. Hence, they must increase their labor supply in all subsequent periods if they are to sustain their consumption plans. Consequently, hours respond positively to a technology shock in model $\mathcal{M}_2$.

Figure 1 illustrates these mechanisms. In this figure, IRFs of model $\mathcal{M}_1$ (full information) and $\mathcal{M}_2$ (incomplete information) are obtained for $\beta = 0.99$, $b = 0.65$ and $\rho = -0.2$. Figure 1 highlights that the sign of the IRFs differs according to the information set. Moreover, the amplification of the technology shock are not the same. The response of hours in model $\mathcal{M}_1$ displays a larger magnitude than in model $\mathcal{M}_2$. The information set thus matters for the quantitative properties of the model, especially so if we are concerned with the sign and amplitude of these responses.[6] Notice however that the persistence property of hours conditional on the technology shock is the same for both models, as in the introductory model. We now explore the case of multivariate models for which persistence properties of

[6]Notice however that the long-run responses of output and consumption do not depend on the timing of decisions.

full and incomplete information models potentially differ.

2 Multivariate DSGE Models

In this section, we extend our previous analysis to larger models with multiple channels of internal propagation. We highlight the impact of restricting the information sets on the persistence properties of these models.

2.1 The Model

We consider an RBC model with habit formation and investment adjustment costs in the line of Boldrin, Christiano and Fisher (2001) and Christiano, Eichenbaum and Evans (2005). The model includes a random walk productivity shock denoted Z_t. As in the previous section, we assume that intertemporal consumption choices are not time separable and that the service flows from consumption are a linear function of current and once-lagged consumptions choices. More precisely, the intertemporal expected utility function of the representative household is given by eq. (7). In what follows, we will consider again two versions of this model, differentiated according to the timing decision of hours.

The representative firm uses capital K_t and labor N_t to produce the homogeneous final good Y_t. The technology is represented by the following constant returns–to–scale Cobb–Douglas production function

$$Y_t = K_t^{\alpha} \left(Z_t N_t\right)^{1-\alpha},$$

where $\alpha \in (0,1)$. Z_t is assumed to follow an exogenous process of the form

$$\log(Z_t) = \gamma_z + \log(Z_{t-1}) + \sigma_z \varepsilon_{z,t},$$

where $\sigma_z > 0$ and $\varepsilon_{z,t}$ is *iid* with zero mean and unit variance. The constant γ_z is a drift term in the random walk process of Z_t. The capital stock evolves according to the law of motion

$$K_{t+1} = (1-\delta) K_t + \mathcal{F}\left(I_t, I_{t-1}\right)$$

where $\delta \in (0,1)$ is the constant depreciation rate. The function $\mathcal{F}(\cdot,\cdot)$ represent a technology that allows to transform current and past investment into capital for the next period. This investment adjustment costs function is given by

$$\mathcal{F}(I_t, I_{t-1}) = \left[1 - \mathcal{S}\left(\frac{I_t}{I_{t-1}}\right)\right] I_t$$

As in Christiano, Eichenbaum and Evans (2005), we assume that the function $\mathcal{S}(\cdot)$ satisfies the following properties $\mathcal{S}(\gamma_z) = \mathcal{S}'(\gamma_z) = 0$ and $\xi = \mathcal{S}''(\gamma_z)\gamma_z^2 > 0$. Given the solution procedure adopted here, it follows that the steady state of the model does not depend on the parameter ξ while its dynamic properties do. This particular form of adjustment costs has proved useful for enhancing the internal propagation properties of DSGE models. Finally, the final good can be either consumed or invested

$$Y_t = C_t + I_t.$$

Once again, we consider two versions of this model, each of them depending on the timing decisions of labor supply. In the first version (denoted again $\mathcal{M}_1$), we follow the standard approach and assume that all decisions (consumption, labor supply, investment) are taken after the realization of the shock at t. Following Christiano, Eichenbaum and Vigfusson (2005), we consider a second version (model $\mathcal{M}_2$) in which labor supply decisions are taken before observing the technology shock. Conversely, the decisions about consumption and investment are taken after the observations of this shock.

We first apply a stationary–inducing transformation for variables that follow a stochastic trend. Output, consumption and investment are divided by Z_t, and the capital stock is divided by Z_{t-1}. The approximate solution of the model is computed from a log–linearization of the stationary equilibrium conditions around the deterministic steady state using the approach described in Christiano (2004).

2.2 Quantitative Results

We split the model parameters in two subsets $\{\theta_1, \theta_2\}$. The first θ_1 regroups the parameters $\{\alpha, \beta, \delta, \gamma_z\}$. These parameters are fixed in all our experimentations. We set the capital share α to 0.36.The depreciation rate δ is set to 0.025, as is customary in the literature. The growth factor γ_z is set to 1.005, to mimic the average growth rate of US output over the period 1955(1)-2002(4). Finally, we set the discount factor β to 0.99.

The second group θ_2 is composed of the habit persistence parameter b and the adjustment costs parameter in investment ξ. In a first case, we set these two parameters to zero and the model is a standard frictionless RBC model, similar to that considered by Chari, Kehoe and McGrattan (2005) and Christiano, Eichenbaum and Vigfusson (2005) in their assessment of SVARs. In a second case, we select $b = 0.8$ and $\xi = 2$ as in Eichenbaum and Fisher (2005) (see also Boldrin, Christiano and Fisher, 2001 and Christiano, Eichenbaum and Evans, 2005).

Let us first examine the effect of a modification of the information set in the standard RBC model. Figures 2 and 3 reports the IRFs and the ACFs of hours following a technology shock. The first thing to notice is that in both cases, IRFs look very similar at any horizon. The sign and amplification essentially share features. Additionally, the ACFs of hours, either in level or in first difference, are virtually indistinguishable. This result is consistent with our analytical findings of section 1. Indeed, as is well known, the dynamics of the capital stock in this model reduces to an equation identical to eq. (1). These figures demonstrate that in the case of the simple RBC model, a slight modification of the information set is of no substantial consequence on the models' dynamics. This result is of particular interest for the quantitative validation of a DSGE model. Indeed, CEV (2005) show that SVARs with short-run restrictions are able to properly recover the true (i.e. model-based) IRFs. Since imposing the same kind of short-run restriction in the standard RBC model is innocuous from a dynamic point of view, SVARs with short-run restrictions are a useful guide for model-building purposes.

Notice however that this model, as is well known in the literature (see Cogley and Nason, 1995), possesses extremely weak internal propagation mechanisms, as illustrated by the ACFs of hours in first difference (bottom panel of figure 3). Following the results obtained in the introductory model, this result is not completely surprising. Indeed, this simple model shows that absent a strong backward-looking dimension, the information set does not matter when comparing IRFs. Consequently, it is also of interest to consider models with stronger internal propagation mechanisms. When habit formation and dynamic investment adjustment costs are taken into account, the model gives us this opportunity.

Figures 4 and 5 reports the IRFs and the ACFs of hours following a technology shock when $b = 0.8$ and $\xi = 2$. In this case, the IRFs sharply differ as soon as hours are restricted not to react to current period shocks. In the model with full information, the short-run response of hours is negative and governed by the same mechanisms as those present in the simple model with habit formation and without capital accumulation. Since consumption adjusts only gradually and increasing investment is costly, the labor supply drops sharply due to a strong income effect. However, after a few quarters, the intertemporal substitution effect dominates, and thus hours increase. By way of contrast, model $\mathcal{M}_2$ delivers a very different picture. In this case, hours cannot respond on impact (by construction), but more importantly, they increase in all subsequent periods. The reason why is simple. Households increase consumption and investment on impact. Due to habit formation and dynamic investment adjustment costs, agents have a tendency to maintain higher consumption and investment levels in the subsequent periods. This leaves no other choice but to increase labor supply in order to sustain these plans. In equilibrium, hours persistently increase after a permanent technology shock. This dramatic modification of the models' behavior casts doubts on the innocuous nature of the information set. This is all the more striking when one considers the ACFs of hours (see figure 5). The model with incomplete information possesses stronger internal propagation mechanisms than the model with full information. This especially true when one considers the growth rate of hours, which exhibits a very strong and positive serial correlation in $\mathcal{M}_2$. In contrast, in $\mathcal{M}_1$, hours in first difference display a slightly negative serial correlation. Thus, the information set affects the dynamics of the model not only in terms of IRFs sign and amplification but also in terms of persistence. This draws an important warning to researchers willing to implement short-run restrictions in DSGE models. To the contrary, notice that restricting the timing of decisions has no long-run effects on the model's variables.

3 Concluding remarks

This paper investigates the quantitative implications of restricting the information sets conditional on which decisions are taken in DSGE models. Our results first show that restricting the information set is of no substantial consequences if one is dealing with a DSGE model with weak internal propagation mechanisms, such as the basic Real Business Cycle model. However, in a DSGE model featuring a large number of real frictions that enhance its persistence properties, we obtain a very different picture. More precisely, we showed by way of example that the Impulse Response Functions of certain variables can be modified in a marked way, i.e. their sign, amplitude and persistence can dramatically change. This shows that implementing short-run restrictions in DSGE models is not as innocuous as might seem at first glance.

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Figure 1: The Response of Hours to a Technology Schock

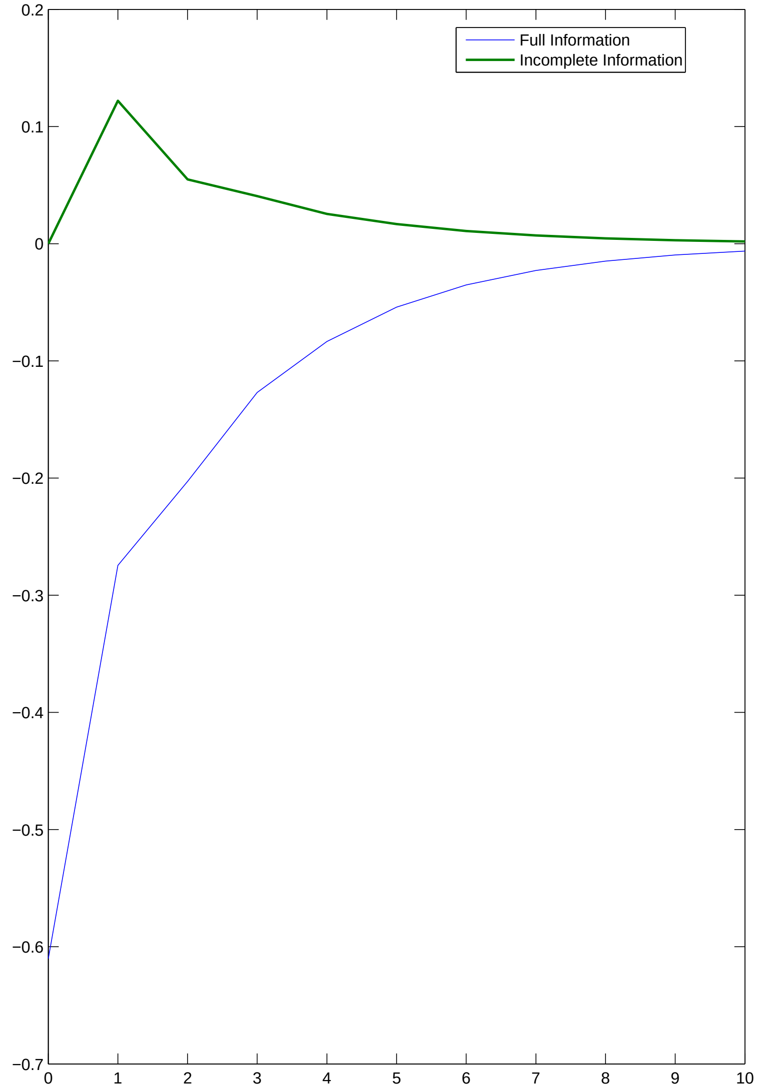

Figure 2: The Response of Hours to a Technology Schock

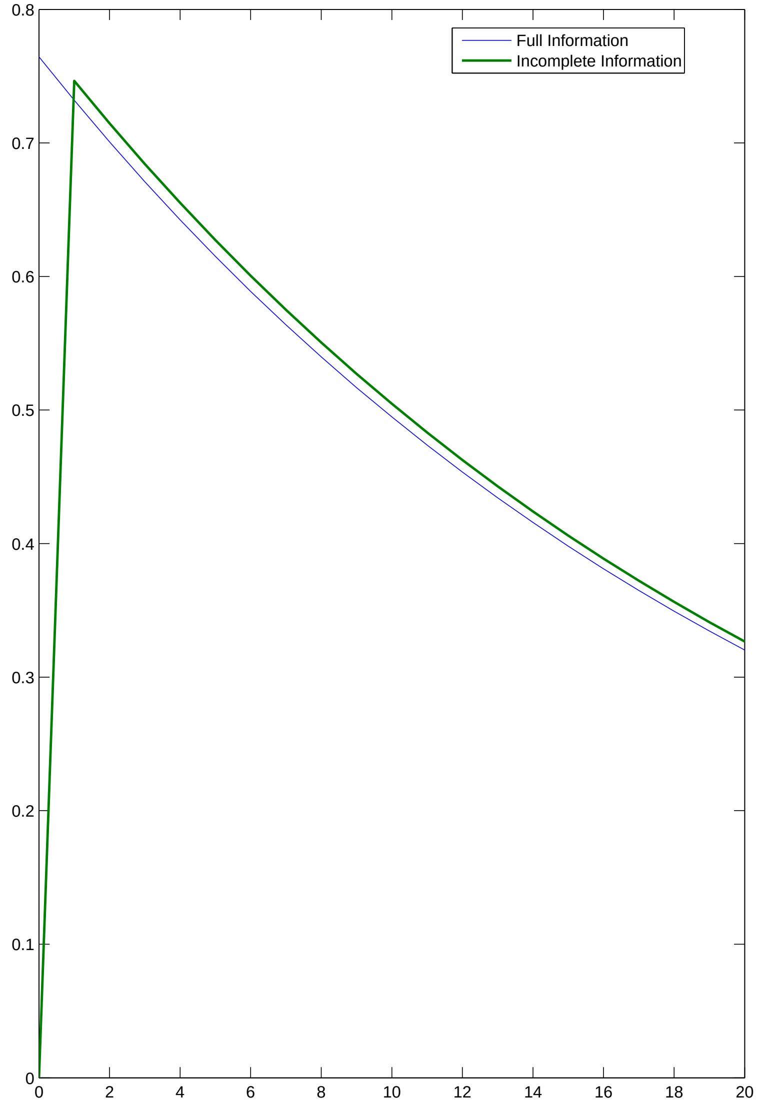

Figure 3: Autocorrelation Function of Hours

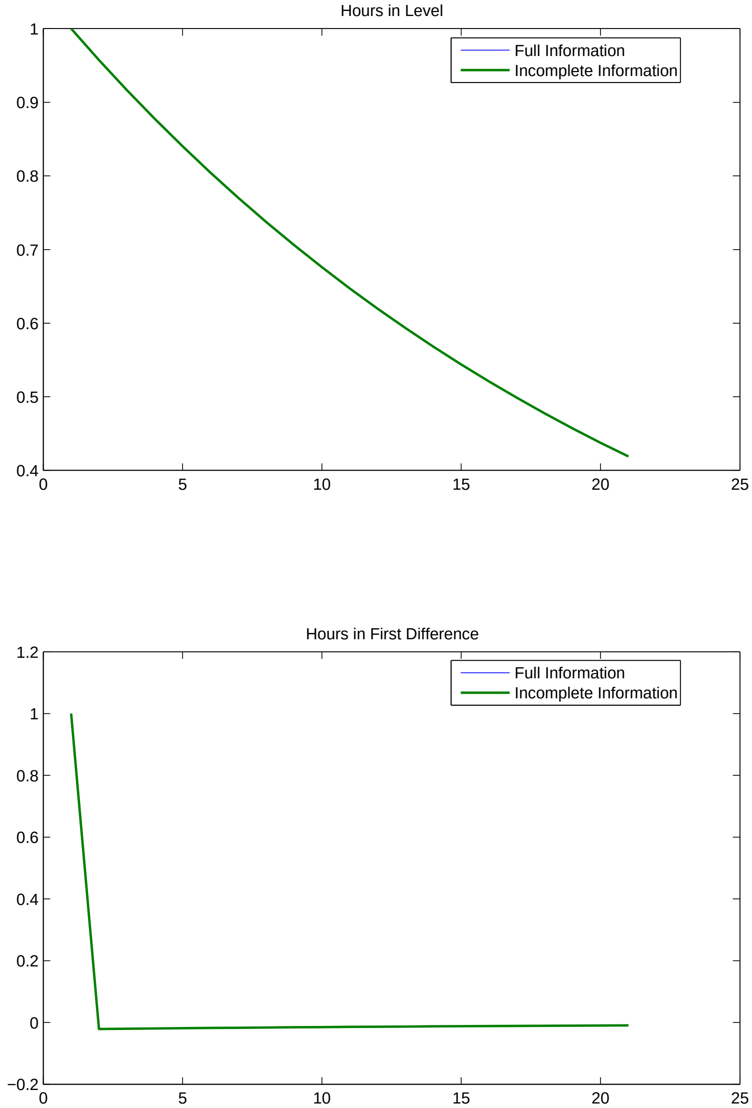

Figure 4: The Response of Hours to a Technology Schock

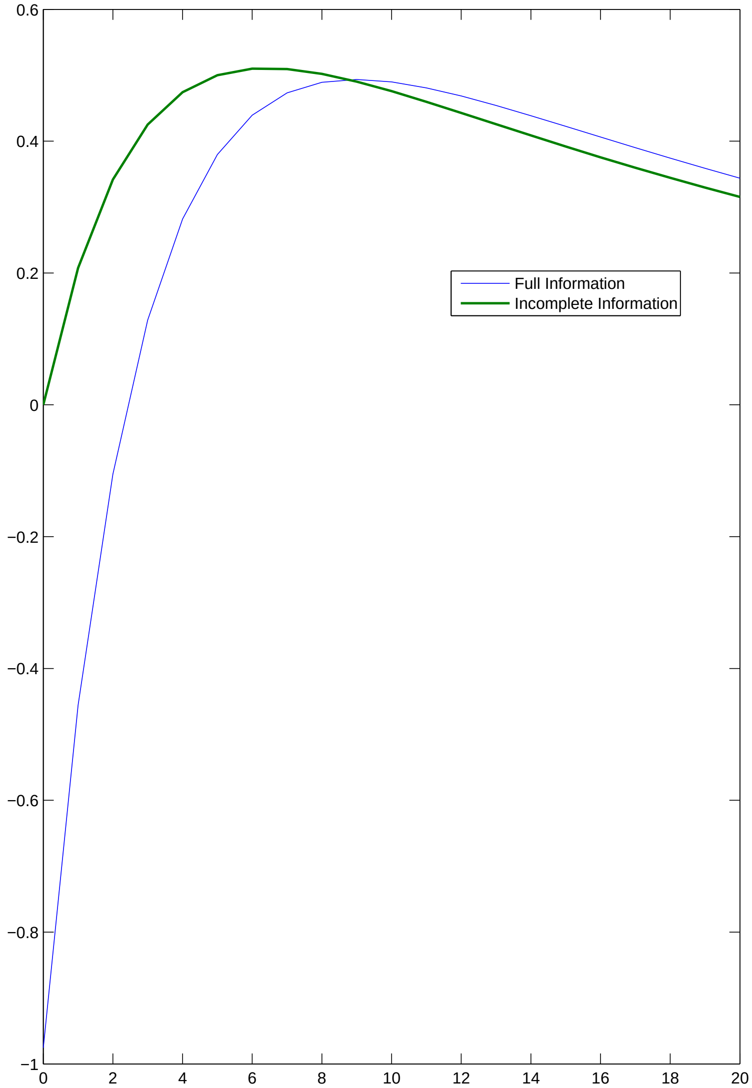

Figure 5: Autocorrelation Function of Hours