The choice of exchange rate regime and speculative attacks*

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Abstract

We develop a framework for studying the choice of exchange rate regime in an open economy where the local currency is vulnerable to speculative attacks. The optimal regime is determined by a policymaker who trades off the loss from nominal exchange rate uncertainty, against the cost of maintaining a given regime. This cost is affected in turn by the likelihood of a speculative attack. Searching for the optimal regime within the class of exchange rate bands, we show that the optimal regime is either a peg (a zero-width band), a free float (an infinite-width band), or a non degenerate finite width band. The paper shows that lower costs of moving across currencies induce policymakers to set more flexible exchange rate systems. This lowers, ceteris paribus, the likelihood of financial crises. One implication is that a Tobin tax does not necessarily reduce the likelihood of currency crises. The paper also investigates the effects of country size, openess, variability in fundamentals and reputation on the choice of exchange rate regime. The role of abrupt changes in reputation in triggerering crises (even with unchanged fundamentals) as an alternative to "sunspots" is discussed and illustrated.

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1 Introduction

The literature on speculative attacks and currency crises can be broadly classified into first-generation models (e.g., Krugman, 1979 and Flood and Garber, 1984) and second-generation models (e.g., Obstfeld, 1994 and 1996; Velasco, 1997; and Morris and Shin, 1998). Recent surveys by Flood and Marion (1999) and Jeanne (2000) suggest that the main difference between the two generations of models is that in first-generation models, the policies that ultimately lead to the collapse of fixed exchange rate regimes are specified exogenously, while in second-generation models, policymakers play an active role in deciding whether or not to defend the currency against a speculative attack. In other words, second-generation models endogenize the policymakers' response to a speculative attack. As Jeanne (2000, p. 5-6) points out, this evolution of the literature is similar to "the general evolution of thought in macroeconomics, in which government policy also evolved from being included as an exogenous variable in macroeconomic models to being explicitly modeled."

Although second-generation models explicitly model the policymakers' (ex post) response to speculative attacks, the initial (ex ante) choice of the exchange rate regime (typically a peg), is treated in this literature as exogenous. As a result, the interdependence between possible ex post currency attacks and the ex ante choice of exchange rate regime is ignored in this literature. A different line of literature that focuses on optimal exchange rate regimes (e.g., Helpman and Razin, 1982, and Devereux and Engel, 1999) also ignores this effect by abstracting from the possibility of speculative attacks.

In this paper, we make a first step towards filling this gap. We analyze a model in which both the ex ante exchange rate regime and the probability of ex post currency attacks are determined endogenously. The model has three stages: In the first stage, prior to the realization of a stochastic shock to the freely floating exchange rate (the "fundamental" in the the model), the policymaker chooses the exchange rate regime so as to maximize his objective function. In the second stage, after the realization of fundamentals, speculators decide whether or not to attack the exchange rate regime. Finally, in the third stage, the policymaker decides whether to defend the regime or abandon it. Thus, relative to second-generation models, our model explicitly examines the ex ante choice of the exchange rate regime. This makes it possible to analyze, for the first time, the strategic interaction between the ex ante choice of regime and the probability of ex post currency attacks.[1]

In order to model speculative attacks, we use the framework recently developed by Morris and Shin (1998) where each speculator observes a slightly noisy signal about the fundamentals of the economy, so that the fundamentals are not common knowledge among speculators. We use this framework not only because of its descriptive realism, but also because it yields a unique equilibrium. As Carlsson and van Damme (1993) first showed, strategic interactions that generate multiple equilibria under common knowledge, give rise to a unique equilibrium if the common knowledge assumption is relaxed.[2] In our context, this result implies that the fundamentals of the economy uniquely determine whether a currency attack will occur or not. This allows us, in turn, to determine the probability of a currency attack (which cannot be determined unambiguously in a model with multiple equilibria) and relate it to the choice of exchange rate regime.

In general, characterizing the best exchange rate regime in the face of a possible speculative attack is an extremely hard problem since the best regime may have an infinite number of arbitrary features. The difficulty is compounded by the fact that the exchange rate regime affects, in turn, the strategic behavior of speculators vis-a-vis the policymaker and vis-a-vis each other. We therefore limit the search for the "best" regime to the class of explicit exchange rate bands. This class of regimes is characterized by two parameters: the upper bound of the band and the lower bound of the band. The policymaker allows the exchange rate to move freely within these bounds but commits to intervene in the market and prevent the exchange rate from moving outside the band. The degree of flexibility of regimes within this class is directly related to the width of the band. Although the class of bands does not exhaust all the possible varieties of exchange rate regimes that human imagination might conceive it is, nonetheless, rather broad and includes as special cases the two most commonly analyzed regimes, namely pegs (zero-width bands) and free floats (infinitely wide bands).[3,4] Apart from making it possible to characterize the best regime in

[1]To focus attention on this strategic interaction, we analyze a three-stage model instead of a full-fledged dynamic framework. In that, we follow Obstfeld (1996) and Morris and Shin (1998), who analyze two-stage models.

[2]Recently, the Morris-Shin framework has been applied to study optimal transparency (Heinemann and Metz (2002)) and interest rate policy (Angeletos, Hellwig and Pavan (2002)). A similar framework has also been applied in other contexts (e.g., Goldstein and Pauzner (2000). For an excellent survey, see Morris and Shin (2001).

[3]Interestingly, Garber and Svensson (1995) note that "...fixed exchange rate regimes in the real world typically have explicit finite bands within which exchange rate are allowed to fluctuate".

[4]Intermediate regimes (bands of positive but finite width) have been adopted during the nineties by a good number of countries, including Brazil, Chile, Colombia, Ecuador, Finland, Hungary, Israel, Mexico, Norway, Poland, Russia, Sweden, The Czech Republic, The Slovak Republic, Venezuela and a number of emerging Asian countries.

a convenient way within a larger class of regimes than hitherto considered this approach extends the traditional currency crisis analysis, which focusses on speculative attacks only against pegs, to speculation against more flexible and more realistic regimes.

In order to focus on the strategic interaction between the choice of exchange rate regime and speculators we follow Obstfeld (1996) and Morris and Shin (1998) and model some of the underlying macroeconomic structure in a reduced form. A basic premise of our framework is that exporters and importers, as well as borrowers and lenders in foreign currency denominated financial assets dislike uncertainty about the level of the nominal exchange rate and that policymakers internalize at least part of this aversion. This premise is consistent with recent empirical findings by Calvo and Reinhart (2002). To reduce uncertainty in the nominal exchange rate, the policymaker may commit to an exchange rate band or even to a peg. Such commitment is, however, costly since maintainance of the currency within the band occasionally requires the policymaker to use up foreign exchange reserves or deviate from the interest rate level that is consistent with other domestic objectives. The cost of either option rises if the exchange rate comes under a speculative attack. If the policymaker decides to exit the band and avoid the costs of defending it, he losses credibility. The optimal exchange rate regime reflects, therefore, a trade-off between reducing exchange rate uncertainty and thereby promoting economic activity on one hand and incurring the cost of committing to an exchange rate band on the other hand. The latter is exacerbated by speculative attacks. This trade-off is in the spirit of the escape clause literature (e.g., Lohmann, 1992 and Obstfeld, 1997).

As noted above, an essential feature of our framework is that the speculators' (ex post) decision on whether or not to attack and the policymaker's (ex ante) choice of an exchange rate regime are interdependent. The explicit recognition of this interdependence yields a number of novel predictions about the optimal exchange rate regime and about the likelihood of a currency crisis.

Two main results concern the effect of a Tobin tax on the optimal regime and on the likelihood of a currency crisis. This tax, proposed by Tobin (1978), is a universal tax on short term inter-currency transactions. The objective of the tax is to reduce the profitability of speculation against the currency, and thereby lower the probability of currency crises. Although Tobin's idea was met with scepticism (mainly because of difficulties of implementation) by and large the conventional wisdom is that, if feasible, such a tax can lower the probability of an attack on the currency.[5]

[5] Recent evaluations of the Tobin tax appear in Eichengreen, Tobin and Wyplosz (1995), Jeanne (1996), Haq, Kaul and Grunberg (1996), Eichengreen (1999) and Berglund et al. (2001).

Our analysis shows that this conventional wisdom is not necessarily correct. Although the tax **does lower** the likelihood of a currency crisis for a given band, it also induces policymakers to install narrower bands in order to achieve more ambitious reductions in exchange rate uncertainty.[6] All else equal, this longer run policy response **raises** the probability of crises. Thus, once the endogeneity of the exchange rate regime is taken into consideration the effect of a Tobin tax on the likelihood of a crisis is generally ambigous. Furthermore, it can be shown that subject to some (reasonable) conditions the latter effect dominates in our framework so that a Tobin tax actually raises rather than lowers the probability of a crisis.

Following are highlights of additional results.[7] First, as the policymaker becomes more concerned with nominal exchange rate uncertainty, the optimal band becomes narrower (and may even become a peg), and currency attacks become more likely. Some of the factors that give rise to cross country variations in the degree of aversion to nominal exchange rate uncertainty are the fraction of financial assets and liabilities and of capital flows that is denominated in foreign exchange, the importance of foreign trade, and the fraction of trade that is invoiced in foreign exchange (McKinnon, 2000; Gylfason, 2000; and Wagner, 2000). In conjunction with this observation, our model predicts that relatively small and open economies are likely to have less flexible exchange rate regimes and more frequent currency attacks.

Second, as the distribution of the fundamentals (i.e., the stochastic shocks to the freely floating exchange rate) becomes more spread out, countries with pegs may move to narrow bands and countries with bands widen them and may even adopt free floats.

Third, the size of the policymaker's loss of credibility following a realignment has an ambiguous effect on the optimal regime. From an ex post point of view, a larger credibility loss induces the policymaker to defend the regime more often. This ex post effect lowers the probability of a speculative attack and therefore induces the policymaker to adopt less flexible regimes. However from an ex ante point of view, when the credibility loss is larger, the policymaker finds it more costly to commit and therefore tends to set a more flexible regime. In general then, the overall effect on the optimal regime is ambiguous. Since the probability of speculative attacks is affected by the flexibility of the regime, the size of the credibility loss has an ambiguous effect on the ex ante probability of currency crises as well, despite the fact that for a given regime, an increases in

[6] This is consistent with the flexibilization of exchange rate regimes following the gradual reductions of restrictions on capital flows in the aftermath of the Bretton Woods system.

[7] Some of these results also rely on the assumption that smaller shocks to the freely floating exchange rate are more likely than larger shocks.

the credibility loss lowers the probability of a crisis. This result underlines again the importance of considering the endogeneity of the regime when analyzing the effect of different parameters on the probability of a currency attack.

Fourth, an increase in the policymaker's reputation (in Barro (1986) sense) about his resolve to prevent the exchange rate from moving outside the band weakens the speculators' incentive to attack the exchange rate and thereby induces policymakers to set a less flexible regime (a narrow band or even a peg). Hong-Kong's currency board is a good example. Since it has never abandoned its currency board in the past, Hong-Kong's currency board enjoys a good reputation. There is thus a "virtuous circle" between good reputation and the performance of a currency board. Moreover, the analysis implies that political events that have an abrupt effect on the policymaker's reputation may transform a state of fragile, but sustainable fundamentals, into an actual currency crisis even if there is no change in the underlying fundamentals and even if given the fundamentals there is a unique equilibrium in the exchange rate market. Thus, our framework can explain currency crises triggered by sudden political events without resorting to multiple equilibria and sunspots.

Apart from being related to the literatures on currency crises and optimal exchange rate regimes, this paper is also related to the literature on exchange rate bands. The methodology that we use in this paper, however, is different from the one that is used in most of the literature on target zones. In their extensive survey on the exchange-rate-bands literature, Garber and Svensson (1995) note that the existence of bands raises two main research questions: "What are the dynamics of exchange rates, interest rates and central bank interventions within exchange rate bands?" and "What are the trade-offs that determine the optimal band width?". As Garber and Svensson say, while the literature has dealt fairly thoroughly with the first question, the second question has hardly been touched upon.[8] To a large extent, this statement is also true today. Our paper provides some new insights on the second question by analyzing a basic trade-off between reduction of exchange rate uncertainty and the costs associated with maintaining a band (or a peg) in the face of potential currency attacks. In order to focus on this question, we abstract from the effect of a band on the behavior of the exchange rate within the band, which is the focus of analysis of the traditional target zone literature. We are aware of three other papers that analyze the optimal width of the band: Sutherland (1995), Miller and Zhang (1996) and Cukierman, Spiegel and Leiderman (2002). The first two papers do not consider the possibility of realignments, nor

[8] The literature on this question started with a seminal paper by Krugman (1991), and continued with many other contributions, for instance, Bertola and Caballero (1992), and Bertola and Svensson (1993).

the interaction between currency attacks and the optimal width of the band. The third paper incorporates the possibility of realignments, but abstracts from the issue of speculative attacks.

The paper is organized as follows. Section 2 presents the basic framework. Section 3 derives the equilibrium behavior of speculators and of the policymaker and characterizes the equilibrium properties of the exchange rate regime. In particular, this section identifies conditions under which the regime is a peg, a free float, or a band, and in the later case, examines the determinants of the band's width and its symmetry. Section 4 provides comparative statics analysis and discusses their empirical implications. Section 5 extends the analysis to the case in which speculators are uncertain about the policymaker's resolve to maintain the band (i.e., the policymaker's reputation). Section 6 concludes. All proofs are in the Appendix.

2 The model

Consider an open economy in which the initial level of the nominal exchange rate (defined as the number of units of domestic currency per one unit of foreign currency) is denoted by e_{-1}. Absent policy interventions and speculations, the new level of the nominal exchange rate, e, is determined in the exchange rate markets. The realization of e reflects various shocks to the current account and to the capital account of the balance of payments excluding the behavior of speculators and government interventions, which are modeled explicitly. For the purpose of this paper, it turns out that it is more convenient to work with the laissez faire rate of change in e, $x \equiv \frac{e-e_{-1}}{e_{-1}}$, rather than with its level, e. We assume that the value of x is drawn from a distribution function $f(x)$ on $\Re$ with c.d.f. $F(x)$.[9] We make the following assumption on $f(x)$:

Assumption 1: $f(x)$ is unimodal with a mode at $x = 0$. That is, $f(x)$ is increasing for all $x < 0$ and decreasing for all $x > 0$.

Assumption 1 states that large rates of change in the freely floating exchange rate (i.e., large devaluations when $x > 0$ and large appreciations when $x < 0$) are less likely than small changes. This is a realistic assumption and, as we shall see later, it is responsible for some of the results of the paper.

[9] We assume that the distribution of x has an unbounded support mainly for convenience. This assumption is not essential and can be relaxed although this would require some additional assumptions to ensure that various parameters of the model are not too large relative to the bounds of the support of x.

2.1 The exchange rate band

A basic premise of the paper is that policymakers dislike nominal exchange rate uncertainty. This is because exporters, importers, as well as lenders and borrowers in foreign currency face higher exchange rate risks when there is more uncertainty about the nominal exchange rate. By raising the foreign exchange risk premium, an increase in exchange rate uncertainty reduces international flows of goods and of financial capital. Policymakers, who wish to promote economic activity, internalize at least part of this aversion to uncertainty and therefore have an incentive to limit it.[10]

In general, there are various conceivable institutional arrangements for limiting exchange rate uncertainty. In this paper we search for an optimal institutional arrangement within the class of bands. This class is quite broad and includes pegs (bands of zero width) and free floats (bands of infinite width) as special cases. Under this class of arrangements, the policymaker sets an exchange rate band $[\underline{e}, \overline{e}]$ around the preexisting nominal exchange rate, e_{-1}. The nominal exchange rate, e, is then allowed to move freely within the band in accordance with market forces, but if the laissez faire exchange rate is outside the band, the policymaker is committed to intervene and keep the exchange rate at one of the boundaries of the band.[11] Given e_{-1} the exchange rate band induces a permissible range of rates of change in the exchange rate, $[\underline{\pi}, \overline{\pi}]$, where $\underline{\pi} = \frac{\underline{e}-e_{-1}}{e_{-1}} < 0$ and $\overline{\pi} = \frac{\overline{e}-e_{-1}}{e_{-1}} > 0$. Within this range, the domestic currency is allowed to appreciate if $x \in [\underline{\pi}, 0)$, and depreciate if $x \in [0, \overline{\pi})$. In other words, $\underline{\pi}$ is the maximal rate of appreciation and $\overline{\pi}$ is the maximal rate of depreciation that the exchange rate band allows.[12]

But leaning against the trends of free exchange rate markets is costly. To defend a currency under attack, policymakers have to deplete their foreign exchange reserves (Krugman, 1979) or put up with substantially higher domestic interest rates (Obstfeld, 1996). If they decide to avoid those costs by exiting the band, they lose some credibility. For example, breaking a commitment can make it harder for the policymaker to achieve other goals in the same period or in the future (e.g.,

[10] Admittedly, some of those risks may be insured by means of future currency markets. However, except perhaps for some of the major key currencies, such markets are largely non-existent, and when they do exist the insurance premia are likely to be prohibitively high.

Rose (2000) presents evidence suggesting that countries with the same currency trade substantially more than comparable countries with their own currencies. Lee (1999) presents evidence from US import markets that is consistent with the view that exchange rate volatility depresses demand for imported consumer durables.

[11] This intervention can be operationalized by buying or selling foreign currency in the market, by changing the domestic interest rate, or by doing some of both.

[12] Note that when $\underline{\pi} = \overline{\pi} = 0$ the band reduces to a peg and when $\underline{\pi} = -\infty$, and $\overline{\pi} = \infty$ it becomes a free float.

commit to a low rate of inflation, commit to low rates of taxation, accomplish structural reforms, etc.). We denote the present value of this loss by δ.

After observing the realization of x, and the fraction of speculators that decide to attack the band, the policymaker can decide to either intervene in order to maintain the band or abandon it. Following Obstfeld (1996) and Morris and Shin (1998), we assume that the cost of maintaining the band increases with the size of the disequilibrium that the policymaker tries to maintain (either $x-\overline{\pi}$ or $\underline{\pi}-x$, depending on whether x is positive or negative) and with the number of speculators that have attacked the band. Specifically, normalizing the mass of speculators to 1, and using α to denote the fraction of speculators that have attacked the band, we assume that the cost of intervention in the exchange rate market is given by

$$C(x,\alpha) = \begin{cases} x-\overline{\pi}+\alpha, & x \geq \overline{\pi}, \\ 0, & \underline{\pi} \leq x \leq \overline{\pi}, \\ \underline{\pi}-x+\alpha, & x \leq \underline{\pi}. \end{cases} \tag{2.1}$$

The assumption that $C(x,\alpha)$ increases with α reflects the idea that as more speculators attack the band, the policymaker has less resources to continue to defend it, and hence it becomes more costly to maintain it. For simplicity, we assume that α enters the cost function additively. The middle line in equation (2.1) states that when the exchange rate is inside the band, the policymaker does not intervene in the market and bears no cost. Obviously, the policymaker will maintain the band only when $C(x,\alpha)$ is less than δ. Otherwise, maintaining the band is too costly so the policymaker will exit the band and incur the cost of realignment.

We formalize the trade-off between uncertainty about the nominal exchange rate and the cost of adopting a band by postulating that the policymaker's objective function is

$$V = -AE\left|\pi\right| - E\left[Min\{C(x,\alpha),\ \delta\}\right], \qquad A > 0, \tag{2.2}$$

where π is the actual rate of change in the nominal exchange rate (under laissez faire, $\pi = x$). We think of this objective function mostly as a positive description of how a rational policymaker might approach the problem of choosing the band width. The first component of V represents the policymaker's aversion to nominal exchange rate uncertainty, measured in terms of the absolute value of π (i.e., the magnitude of nominal depreciations or appreciations). The parameter A represents the relative importance that the policymaker assigns to exchange rate stability. As A increases, the policymaker becomes more concerned with exchange rate uncertainty and is more willing to incur costs in order to limit it.[13] The parameter A is likely to vary substantially across economies depending on factors like the degree of openess of the economy, its size, the fraction of financial assets and liabilities owned by domestic producers and consumers that is denominated in foreign exchange, and the fraction of foreign trade that is invoiced in foreign exchange. All else equal, residents of small open economies are more averse to nominal exchange rate uncertainty than residents of large, relatively closed, economies like the US or the Euro area. Hence the parameter A is larger in small open economies than in large, relatively closed, economies.

The second component of V represents the policymaker's cost of adopting an exchange rate band. This cost is either $C(x, \alpha)$ if the policymaker defends the band or δ if he exits the band and the exchange rate is realigned. It is important to note that in the latter case, the policymaker does not incur an additional cost as a result of the actual change in the exchange rate provided it has been fully anticipated. Since the model focuses on the negative effects of ex ante nominal exchange rate uncertainty on economic activity, the only reason for intervention in the foreign exchange market is to limit the negative effect of this uncertainty on economic activity.[14] This is also the reason that the policymaker commits to a band ex ante: Without committing to a band, the policymaker will not have an incentive to intervene ex post, after the realization of x. Thus, to reduce ex ante uncertainty, the policymaker has to commit to intervene. Although the policymaker's objective function here is different, this is similar to the typical Kydland-Prescott (1977), Barro-Gordon (1983) time inconsistency problem.

2.2 Speculators

We model speculative behavior using the Morris and Shin (1998) framework. There is a continuum of speculators with a total mass of one. When the exchange rate is at one of the boundaries of the band because of intervention, each speculator independently observes a noisy signal on the exchange rate that would prevail under laissez faire. Specifically, we assume that the signal obtained by speculator i is given by

$$\theta_i = x + \varepsilon_i, \tag{2.3}$$

[13]Note that we assume that the policymaker is equally averse to expected depreciations and appreciations. This assumption can be easily relaxed at the cost of more notation.

[14]More generally, variability and uncertainty do not always coincide (Cukierman and Wachtel, 1982).

where ε_i is a white noise, independent across speculators, and distributed uniformly on the interval $[-\varepsilon, \varepsilon]$. The conditional density of x given a signal θ_i is given by:

$$f(x \mid \theta_i) = \frac{f(x)}{F(\theta_i + \varepsilon) - F(\theta_i - \varepsilon)}. \tag{2.4}$$

In what follows, we focus on the case where ε is small so that the signals that speculators observe are "almost perfect."

Based on θ_i, each speculator i decides whether or not to attack the currency. If the current exchange rate is at the lower bound of the band, speculator i can shortsell the foreign currency at the current (high) price of $\underline{e}$ and then buy the same amount on the market to clear his position. If the policymaker fails to defend the band and the exchange rate falls to $e < \underline{e}$, speculator i's profit from shortselling is $\underline{e} - e$. Denoting by t the nominal transaction cost associated with switching between currencies, the speculator's net payoff is $\underline{e} - e - t$. On the other hand, if the exchange rate stays at $\underline{e}$, the net payoff of the speculator is simply $-t$. Likewise, if the current exchange rate is at the upper bound of the band, speculator i can buy the foreign currency at the current (low) price of $\overline{e}$. If the policymaker exits the band and the exchange rate jumps to $e > \overline{e}$, the speculator's net payoff is $e - \overline{e} - t$. Again, if the policymaker successfully defends the band, the speculative attack fails, and the net payoff of the speculator is $-t$. If the speculator does not attack the band, his payoff is 0.[15]

Note that since $x \equiv \frac{e-e_{-1}}{e_{-1}}$ and since $\underline{e} = (1+\underline{\pi})e_{-1}$ and $\overline{e} = (1+\overline{\pi})e_{-1}$, the net payoffs from attacking the lower and upper bounds of the band can be written as $(1+\underline{\pi})e_{-1} - (1+x)e_{-1} - t = (\underline{\pi} - x)e_{-1} - t$, and $(1+x)e_{-1} - (1+\overline{\pi})e_{-1} - t = (x - \overline{\pi})e_{-1} - t$, respectively. Since x has an unbounded support, then so long as $\underline{\pi} > -\infty$ and $\overline{\pi} < \infty$ (i.e., the exchange rate regime is not a free float), there are sufficiently low realizations of x for which $(\underline{\pi} - x)e_{-1} - t$ is positive and sufficiently high realizations of x for which $(x - \overline{\pi})e_{-1} - t$ is positive. We now make the following assumption on t:

Assumption 2: The real transaction cost, $\frac{t}{e_{-1}}$, is small relative to δ (the future credibility loss from exiting the band) in the sense that $\frac{t}{e_{-1}} < \delta$.

As will become clear below, Assumption 2 ensures that speculators will always attack the band if they believe that x is such that the policymaker will exit the band. This rules out the (uninteresting) possibility that speculators do not attack the band even if they know that the policymaker is not going to defend it.

[15]In order to focus on speculation against the band we abstract from speculative trading within the band. Thus, the well-known 'honeymoon effect' (Krugman, 1991) is absent from the model.

2.3 The sequence of events and the structure of information

The sequence of events unfolds as follows:

- Stage 1: The policymaker announces a band around the existing nominal exchange rate and commits to intervene in the exchange rate market when $x < \underline{\pi}$ or $x > \overline{\pi}$.

- Stage 2: The "free float" random shock, x, realizes. There are now two possible cases:

  (i) If $\underline{\pi} \leq x \leq \overline{\pi}$, the nominal exchange rate is determined by market forces. Hence, the exchange rate adjusts to $(1 + x)e_{-1}$.

  (ii) If $x < \underline{\pi}$ or $x > \overline{\pi}$, the exchange rate adjusts to the lower bound or to the upper bound of the band, respectively. Simultaneously, speculator i gets the signal, θ_i, on x and decides whether or not to attack the band. Those decisions determine the fraction, α, of speculators who decide to attack the band.

- Stage 3: The policymaker observes x and α, and evaluates the total cost of defending the band. If the policymaker decides to defend the band, the exchange rate stays at the boundary of the band and the policymaker incurs the cost $C(x, \alpha)$. If the policymaker exits the band, the exchange rate moves to its freely floating rate so the induced rate of change in the exchange rate is x and the policymaker incurs a future credibility loss whose present value is δ.[16]

3 The equilibrium

In this section we characterize the perfect Bayesian equilibrium of the model. To this end, we solve the model backwards. First, whenever $x < \underline{\pi}$ or $x > \overline{\pi}$, then given α, the policymaker decides in stage 3 whether or not to continue to maintain the band. Second, given the signals that they

[16] The events at stages 2 and 3 of the model are similar to the events in Morris and Shin (1998), and follow the implied sequence of events in Obstfeld (1996). The assumptions imply that speculators can make a profit from an attack only if there is a realignment, and that the policymaker realigns only if the proportion of speculators who attack, α, is sufficiently large. These realistic features are captured by the model in a reduced-form manner. One possible way to justify these features within our framework is as follows: Speculators who attack in stage 2 enter the market at a price of either $\underline{e}$ or $\overline{e}$ because the policymaker still defends the band at this stage. In stage 3, given α and the realization of x, the policymaker decides whether to maintain the band for the future or to realign. In the latter case, speculators make a profit on the difference between the price they got in stage 2, and the new price that is set in stage 3. A possible justification for the assumption that the policymaker always defends in stage 2 is a short lag in decision making or in the arrival of information.

observe in stage 2, speculators decide whether or not to attack the band. When $x \in [\underline{\pi}, \overline{\pi}]$, the policymaker does not intervene in the exchange rate market and the exchange rate moves freely within the band. Finally, in stage 1, prior to the realization of x, the policymaker sets the exchange rate regime.

3.1 The choice between defending the currency and realigning

Suppose that $x < \underline{\pi}$ or $x > \overline{\pi}$. Since the cost of maintaining the band after the initial phase is $C(x, \alpha)$, whereas the loss from exiting the band is δ, the policymaker will continue to maintain the band if and only if $C(x, \alpha) \leq \delta$. Using equation (2.1), this inequality implies that the set of states for which the policymaker will defend the band is

$$\underline{\pi} + \alpha - \delta \leq x \leq \underline{\pi}, \quad \text{and} \quad \overline{\pi} \leq x \leq \overline{\pi} - \alpha + \delta. \tag{3.1}$$

When $x < \underline{\pi} + \alpha - \delta$ or $x > \overline{\pi} - \alpha + \delta$, defending the band is too costly, so the policymaker exits the band and allows the exchange rate to be realigned. Note that as α increases (more speculators attack the band in stage 2), realignment occurs for a wider set of realizations of x.

3.2 Speculative attacks

When $x \in [\underline{\pi}, \overline{\pi}]$, the exchange rate is determined solely by market forces. Hence, speculators cannot gain from attacking the currency and their payoff is 0. In contrast, when $x < \underline{\pi}$ or $x > \overline{\pi}$, speculators know that the policymaker will, at least initially, intervene in the foreign exchange market and prevent the exchange rate from moving outside the band. In such cases, the exchange rate no longer reflects market forces and speculators may choose to attack the band in the hope of making a profit in case there is a realignment.

Suppose that the exchange rate reaches the upper bound of the band and recall that the net payoff from attacking the band in this case is $(x - \overline{\pi})e_{-1} - t$ if the policymaker eventually exits the band and $-t$ if he does not. Since the policymaker exits the band if and only if $x > \overline{\pi} - \alpha + \delta$, the net payoff from attacking the upper bound of the band is $(x - \overline{\pi})e_{-1} - t$ if $x > \overline{\pi} - \alpha + \delta$, and $-t$ if $\overline{\pi} \leq x \leq \overline{\pi} - \alpha + \delta$. Analogously, if the exchange rate reaches the lower bound of the band, the net payoff of a speculator who attacks the lower bound of the band is $(\underline{\pi} - x)e_{-1} - t$ if $x < \underline{\pi} + \alpha - \delta$, and $-t$ if $\underline{\pi} + \alpha - \delta \leq x \leq \underline{\pi}$.

Speculators do not observe x directly and need to use the signal θ_i in order to estimate it. Moreover, in order to assess whether the policymaker will or will not defend the band, each speculator needs to estimate α which is the fraction of other speculators that decide to attack the band. Hence, each speculator needs to form a belief about the behavior of other speculators.

The following lemma establishes that in the limit as $\varepsilon \to 0$, the equilibrium behavior of each speculator is to attack the upper (lower) bound of the band if and only if the signal that he observes is above (below) some threshold. This threshold is common to all speculators. The proof of the lemma, as well as the proofs of all other results, are in the Appendix.

Lemma 1 *Suppose that speculators have almost perfect information, i.e., $\varepsilon \to 0$. Then,*

(i) when the exchange rate reaches the upper (lower) bound of the band, there exists a unique perfect Bayesian equilibrium, such that each speculator attacks the band if and only if the signal that he observes is above some threshold $\overline{\theta}^*$ (below some threshold $\underline{\theta}^*$).

(ii) The thresholds $\overline{\theta}^*$ and $\underline{\theta}^*$ are given by $\overline{\theta}^* = \overline{\pi} + r$ and $\underline{\theta}^* = \underline{\pi} - r$, where

$$r = \sqrt{\frac{t}{e_{-1}} + \frac{(\delta - 1)^2}{4}} + \frac{\delta - 1}{2}.$$

(iii) In equilibrium, all speculators attack the upper (lower) bound of the band and the policymaker realigns it if and only if $x > \overline{\theta}^* = \overline{\pi} + r$ ($x < \underline{\theta}^* = \underline{\pi} - r$). Whenever $\underline{\theta}^* \leq x \leq \overline{\theta}^*$ speculators do not attack the band and the band is not realigned. The probability of a speculative attack is therefore

$$P = F(\underline{\pi} - r) + (1 - F(\overline{\pi} + r)).$$

The uniqueness result in part (i) of Lemma 1 follows Carlsson and van Damme (1993) and Morris and Shin (1998). The proof of this part is based on an iterative elimination of dominated strategies. The idea is as follows: the signal θ_i that speculator i observes is distributed on the interval $[x - \varepsilon, x + \varepsilon]$. If $\theta_i > \overline{\theta} \equiv \overline{\pi} + \delta + \varepsilon$, the speculator correctly infers that $x > \overline{\pi} + \delta$. At this level, the cost of defending the upper bound of the band exceeds the associated benefit even if no speculators attack the band. Hence, the speculator correctly infers that the policymaker is surely going to exit the band, so the net payoff from attacking it is $(x - \overline{\pi})e_{-1} - t$. But since $x > \overline{\pi} + \delta$, the net payoff is at least $\delta e_{-1} - t$, which is strictly positive by Assumption 2. Therefore when $\theta_i \geq \overline{\theta}$, it is a dominant strategy for speculator i to attack the upper bound of the band.[17] But now, if

[17]The strategy to attack the upper bound of the band if $\theta_i > \overline{\theta}$ is dominant because it is optimal no matter what other speculators are going to do. The existence of a region in which speculators have dominant strategies is crucial for deriving a unique equilibrium (see Chan and Chiu, 2002).

θ_i is just slightly below $\overline{\theta}$, speculator i realizes that a large fraction of other speculators must have observed signals above $\overline{\theta}$ and will surely attack the band. From that, speculator i concludes that the policymaker will exit the band so it is again optimal to attack it. This chain of reasoning can proceed further where each time we lower the critical signal above which speculator i will attack the upper bound of the band, given that speculators who observe even higher signals surely attack it.

When $\theta_i < \underline{\theta} \equiv \overline{\pi} + \frac{t}{e_{-1}} - \varepsilon$, speculator i correctly infers that $x < \overline{\pi} + \frac{t}{e_{-1}}$. Consequently, even if the policymaker will surely exit the band, the payoff from attacking it is negative as $(x - \overline{\pi})e_{-1} - t < \left((\overline{\pi} + \frac{t}{e_{-1}}) - \overline{\pi}\right) e_{-1} - t = 0$. Hence, it is a dominant strategy for speculator i not to attack the band when $\theta_i < \underline{\theta}$. But then, if θ_i is just slightly above $\underline{\theta}$, speculator i will infer that a large fraction of other speculators must have observed signals below $\underline{\theta}$ and will surely not attack the band. Hence, speculator i concludes that the policymaker will successfully defend the band so it is optimal not to attack it. Again, this chain of reasoning can proceed further where each time the critical signal below which speculator i will not attack the upper bound of the band is raised.

As $\varepsilon \longrightarrow 0$, the critical signal above which speculators attack the band coincides with the critical signal below which they do not attack it. This establishes the existence of a unique threshold signal, $\overline{\theta}^*$, such that all speculators attack the upper bound of the band if and only if they observe signals above $\overline{\theta}^*$. Similar arguments establish the existence of a unique threshold signal, $\underline{\theta}^*$, such that all speculators will attack the lower bound of the band if and only if they observe signals below $\underline{\theta}^*$.

The unique equilibrium obtained in Lemma 1 contrasts with second-generation models of currency crises of the type developed by Obstfeld 1996 and 1997 in which multiple equilibria are possible. The difference arises because in that framework the only constraint on expectations is that they must be consistent with the resulting equilibrium. Here by contrast, each speculator also needs to estimate the information available to other speculators based on his own signal. As a consequence, there is less freedom in choosing the range of possible beliefs that each speculator may entertain about the beliefs of others (and therefore about the probability that each speculator assigns to the event that the policymaker will defend the currency). Since the speculators' beliefs about x and about other speculators' beliefs are tied together by the realization of x, all speculators are led to believe that **either** all of them believe that the policymaker will exit the band, **or** that all of them believe that the policymaker will defend the band. Given the realization of x, one of those beliefs about the beliefs of others is inconsistent with a conditional rational inference about the beliefs of others, which rules out multiple equilibria.[18]

Having solved for the behavior of speculators, we turn next to the implications of this behavior for the exchange rate band. Recall that we are interested in cases where $\varepsilon \to 0$. Part (iii) of Lemma 1 implies that the exchange rate band gives rise to two *Ranges of Effective Commitment* (REC) such that the policymaker intervenes in the exchange rate market and defends the band only if x falls inside one of these ranges. The positive REC is equal to $[\overline{\pi}, \overline{\theta}^*]$ or $[\overline{\pi}, \overline{\pi}+r]$; when $x \in [\overline{\pi}, \overline{\pi}+r]$, the policymaker keeps the rate of depreciation from being above $\overline{\pi}$. The negative REC is equal to $[\underline{\theta}^*, \underline{\pi}]$ or $[\underline{\pi}-r, \underline{\pi}]$; when $x \in [\underline{\pi}-r, \underline{\pi}]$, the policymaker keeps the rate of appreciation from being above the absolute value of $\underline{\pi}$. When $x < \underline{\pi}$ or when $x > \overline{\pi}$, the policymaker exits the band and despite his earlier announcement, tolerates a realignment. On the other hand, when $x \in [\underline{\pi}, \overline{\pi}]$, the policymaker allows the exchange rate to move freely in accordance with market forces. These five ranges of x are illustrated in Figure 1.

Part (ii) of Lemma 1 indicates that r is independent of $\underline{\pi}$ and $\overline{\pi}$. This means that the actual size of the two RECs does not depend on how wide the band is. What the policymaker can do is to shift the two RECs either closer to or away from 0 by choosing $\underline{\pi}$ and $\overline{\pi}$ appropriately. Part (ii) of Lemma 1 also shows that r increases with t, which is the transaction cost associated with speculative attacks, and with δ which is the policymaker's credibility loss from realignment. These properties are intuitive since they imply that a realignment is less likely when it is more costly for speculators to attack the band and when the policymaker is more averse to realignments.

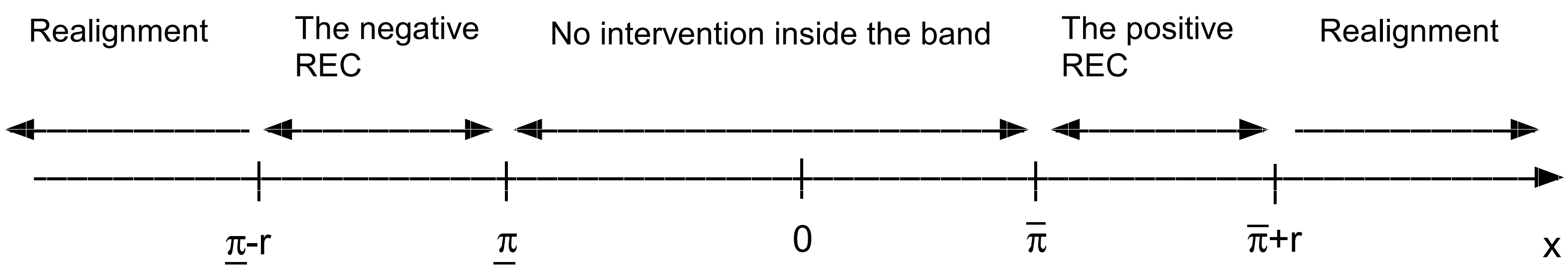
The discussion is now summarized in the following proposition:

Proposition 1 *The exchange rate band gives rise to a positive range of effective commitment (REC), $[\overline{\pi}, \overline{\pi}+r]$, and a negative REC, $[\underline{\pi}-r, \underline{\pi}]$, where r is defined in Lemma 1.*

- *When x falls inside the positive REC, the policymaker defends the currency and ensure that the rate of depreciation is not larger than $\overline{\pi}$.*

[18] By contrast, in the absence of this additional constraint, multiple equilibria do arise for some ranges of x. For example, suppose that $\bar{\pi}+\delta-1 < x < \bar{\pi}+\delta$. If all speculators believe that the policymaker will defend the currency so that none attacks it (i.e., $\alpha = 0$), it indeed pays the policymaker to defend it since the cost of doing that is $C(x,\alpha) = x-\bar{\pi} < (\bar{\pi}+\delta)-\bar{\pi} = \delta$, whereas the cost of exiting the band is δ. Hence, this strategy and the postulated speculators' beliefs constitute an equilibrium. By the same token, if all speculators believe that the policymaker will exit the band and so all of them attack it (i.e., $\alpha = 1$), then $C(x,\alpha) = x-\bar{\pi}+1 > (\bar{\pi}+\delta-1)-\bar{\pi}+1 = \delta$, so indeed, it does not pay the policymaker to defend the band. Hence, having a speculative attack and a realignement is also an equilibrium.

Figure 1: Illustrating the exchange rate band



- *When x falls inside the negative REC, the policymaker defends the currency and ensures that the rate of appreciation is not larger in absolute value than $\underline{\pi}$.*

- *When x falls below the negative REC, above the positive REC, or inside the band, the policymaker lets the exchange rate move freely in accordance with market forces.*

- *The width of the two RECs, r, increases with t and with δ but is independent of the boundaries of the band, $\underline{\pi}$ and $\overline{\pi}$.*

3.3 The choice of band width

We now turn to the policymaker's objective function. First, note that the expected variability of the exchange rate around the current level can be written as follows:

$$\begin{aligned} E\left|\pi\right| &= -\int_{-\infty}^{\underline{\pi}-r} xf(x)dx - \int_{\underline{\pi}-r}^{\underline{\pi}} \underline{\pi}f(x)dx - \int_{\underline{\pi}}^{0} xf(x)dx \\ &\quad + \int_{0}^{\overline{\pi}} xf(x)dx + \int_{\overline{\pi}}^{\overline{\pi}+r} \overline{\pi}f(x)dx + \int_{\overline{\pi}+r}^{\infty} xf(x)dx. \end{aligned} \qquad (3.2)$$

Equation (3.2) shows that the existence of a band has a moderating effect on $|\pi|$ only inside the two RECs. Over these two regions, the policymaker is expected to keep the exchange rate from moving outside the boundaries of the band.

Second, using equation (2.1) and Lemma 1, the cost of intervention in the exchange rate market is $\underline{\pi} - x$ if $x \in [\underline{\pi} - r, \underline{\pi}]$, and $x - \overline{\pi}$ if $x \in [\overline{\pi}, \overline{\pi} + r]$. When either $x < \underline{\pi} - r$ or $x > \overline{\pi} + r$, there are realignments so the policymaker incurs a credibility loss, δ. Hence, using equations (2.2) and (3.2), the expected payoff of the policymaker, given $\underline{\pi}$ and $\overline{\pi}$, is

$$\begin{aligned} V &= A\left[\int_{-\infty}^{\underline{\pi}-r} xf(x)dx + \int_{\underline{\pi}-r}^{\underline{\pi}} \underline{\pi}f(x)dx + \int_{\underline{\pi}}^{0} xf(x)dx \right. \\ &\quad \left. - \int_{0}^{\overline{\pi}} xf(x)dx - \int_{\overline{\pi}}^{\overline{\pi}+r} \overline{\pi}f(x)dx - \int_{\overline{\pi}+r}^{\infty} xf(x)dx\right] \qquad (3.3) \\ &\quad - \int_{-\infty}^{\underline{\pi}-r} \delta f(x)dx - \int_{\underline{\pi}-r}^{\underline{\pi}} (\underline{\pi} - x)f(x)dx - \int_{\overline{\pi}}^{\overline{\pi}+r} (x - \overline{\pi})f(x)dx - \int_{\overline{\pi}+r}^{\infty} \delta f(x)dx. \end{aligned}$$

The first two lines in equation (3.3) represent the policymaker's loss from exchange rate uncertainty while the last line represents the expected cost of adopting a band.

The policymaker chooses the boundaries of the band, $\underline{\pi}$ and $\overline{\pi}$, so as to maximize his expected payoff. The first order conditions for an interior solution to the policymaker's problem (i.e., for $-\infty < \underline{\pi} < 0 < \bar{\pi} < \infty$) are:

$$\frac{\partial V}{\partial \underline{\pi}} = -[r(A-1)+\delta] f(\underline{\pi}-r) + (A-1)\int_{\underline{\pi}-r}^{\underline{\pi}} f(x)dx \tag{3.4}$$
$$= A\int_{\underline{\pi}-r}^{\underline{\pi}} [f(x)-f(\underline{\pi}-r)]\,dx - \left[\int_{\underline{\pi}-r}^{\underline{\pi}} [f(x)-f(\underline{\pi}-r)]\,dx + \delta f(\underline{\pi}-r)\right] = 0,$$

and,

$$\frac{\partial V}{\partial \overline{\pi}} = [r(A-1)+\delta] f(\overline{\pi}+r) - (A-1)\int_{\overline{\pi}}^{\overline{\pi}+r} f(x)dx \tag{3.5}$$
$$= -A\int_{\overline{\pi}}^{\overline{\pi}+r} [f(x)-f(\overline{\pi}+r)]\,dx + \left[\int_{\overline{\pi}}^{\overline{\pi}+r} [f(x)-f(\overline{\pi}+r)]\,dx + \delta f(\overline{\pi}+r)\right] = 0.$$

We prove in the Appendix that $f''(x) \leq 0$ and $A > 1$, along with Assumption 1, are sufficient (but not necessary) conditions for V to be globally concave in $\underline{\pi}$ and $\overline{\pi}$, in which case equations (3.4) and (3.5) are sufficient for a unique maximum. Equations (3.4) and (3.5) show that by altering the bounds of the band, policymakers move along a trade-off between reduction of exchange rate uncertainty and minimization of the costs of maintaining a band. The first term in the second line of (3.5) is the marginal effect of $\overline{\pi}$ on exchange rate uncertainty. This term is multiplied by A which is the weight that the policymaker assigns to exchange rate stability. Since by Assumption 1, $f(x) - f(\overline{\pi}+r) < 0$ for all $x \in [\overline{\pi}, \overline{\pi}+r]$, this term is negative and represents the marginal cost from raising $\overline{\pi}$. This marginal cost arises because when $\overline{\pi}$ is raised, the positive REC over which the exchange rate is kept constant, shifts further away from the center rate. Assumption 1 implies that large changes in the exchange rate are less likely than small ones. Therefore, the shift of the positive REC away from 0 makes the band less effective in reducing exchange rate uncertainty. The second term in equation (3.5) represents the marginal effect of raising $\overline{\pi}$ on the cost of defending the band. By Assumption 1, the integral term in the brackets is positive, implying that raising $\overline{\pi}$ makes it less costly to defend the band. This is because now it is less likely that the policymaker will actually have to defend the band. The δ term is also positive since increasing $\overline{\pi}$ slightly lowers the likelihood that the exchange rate will move outside the positive REC and lead to a realignment. The interpretation of equation (3.4) is analogous except that here, the signs of the various terms are exactly opposite since raising $\underline{\pi}$ slightly shifts the lower bound of the band closer to 0, whereas raising $\overline{\pi}$ slightly shifts the upper bound of the band away from 0.

Using equations (3.4) and (3.5), we can now characterize the equilibrium exchange rate regime.

Proposition 2 *In equilibrium, the exchange rate band has the following properties:*

(i) ***Free float:*** *If* $A \leq 1$*, then* $\underline{\pi} = -\infty$ *and* $\overline{\pi} = \infty$*, so the optimal regime is a free float.*

(ii) ***A nondegenerate band:*** *If*

$$1 < A < \underline{A}(-r) \equiv 1 + \frac{\delta}{\int_{-r}^{0}\left[\frac{f(x)}{f(-r)} - 1\right]dx}, \tag{3.6}$$

then $-\infty < \underline{\pi} < 0$*. Likewise, if*

$$1 < A < \overline{A}(r) \equiv 1 + \frac{\delta}{\int_{0}^{r}\left[\frac{f(x)}{f(r)} - 1\right]dx}, \tag{3.7}$$

then $0 < \bar{\pi} < \infty$*. Hence, the optimal regime is a nondegenerate band.*

(iii) ***A peg:*** *If* V *is concave in* $\underline{\pi}$ *and in* $\bar{\pi}$*, and* $A > Max\{\underline{A}(r), \overline{A}(r)\}$*, then* $\underline{\pi} = \bar{\pi} = 0$*, so the optimal regime is a peg.*

(iv) ***Symmetry:*** *If* $f(x)$ *is symmetric around* 0*, so that* $f(-x) = f(x)$ *for all* x*, the band will be symmetric around* 0 *in the sense that* $-\underline{\pi} = \bar{\pi}$*.*

Proposition 2 shows that depending on the policymaker's aversion to nominal exchange rates variability, the optimal regime can either be a peg, a free float, or a band. When the policymaker is not too concerned with nominal exchange rate variability, i.e., $A \leq 1$, he sets a free float and avoids the cost of maintaining a band. On the other hand, if the policymaker is sufficiently concerned with nominal exchange rate variability, i.e., $A > Max\{\underline{A}(r), \overline{A}(r)\}$, he minimizes it by adopting a peg.[19] In intermediate cases, the policymaker balances the two objectives, namely limiting exchange rate uncertainty and minimizing the cost of intervention by setting a nondegenerate band. Intervention occurs only when x falls inside the negative or the positive RECs. Part (iv) of the proposition states that a sufficient condition for the band to be symmetric is that the distribution of shocks, $f(x)$, is symmetric around 0 (i.e., under laissez faire, depreciations and appreciations are equally likely).[20]

[19] Note that a peg does not mean that the exchange rate is fixed under all circumstances. When the absolute value of x exceeds r, the policymaker abandons the peg and the exchange rate is realigned. Nonetheless, under a peg, the policymaker maintains stability over the range of "small" shocks where $x \in [-r, r]$. Given Assumption 1, such small shocks are more likely than big ones, so when A is large, it is optimal for the policymaker to eliminate these shocks by adopting a peg.

[20] Note that the symmetry of the band is in terms of the permissible rates of revaluations and devaluations of the exchange rate rather than in terms of the gap of the upper and lower bounds of the band from the center rate (i.e., the symmetry is in terms of x rather than e).

4 Comparative statics and empirical implications

4.1 The effects of restrictions on capital flows and of a Tobin tax

During the last three decades there has been a world-wide gradual lifting of restrictions on currency flows and on related capital account transactions. One consequence of this trend is a reduction in the transaction cost of foreign exchange transactions (t in terms of the model) making it easier for speculators to move funds across different currencies, thereby increasing the likelihood of speculative attacks. To counteract this tendency some economists proposed to "throw sand" into the wheels of unrestricted international capital flows. In particular, Tobin (1978) proposed a universal tax on short term inter-currency transactions in order to reduce the profitability of speculation against the currency, and with it the probability of crises. This idea was met with scepticism mainly because of difficulties of implementation. But, by and large, the consensus is that, subject to feasibility, the tax can reduce the probability of attack on the currency. Recent evaluations appear in Eichengreen, Tobin, and Wyplosz (1995), Jeanne (1996), Haq, Kaul and Grunberg (1996), Eichengreen (1999) and Berglund et al. (2001).

The main objective of this subsection is to examine the consequences of such a tax and of the lifting of restrictions on capital flows when the choice of exchange rate regime is endogenous. More formally, the following proposition examines the effect of a reduction in t on the choice of exchange rate regime and on the likelihood of currency crises.

Proposition 3 *Suppose that following a lifting of restrictions on currency flows and capital account transactions, the transaction cost of switching between currencies, t, decreases. Then:*

(i) When the policymaker's problem has a unique interior solution, $\bar{\pi}$ and $\underline{\pi}$ shift away from 0, so the band becomes wider. Moreover, the probability, P, that a speculative attack will occur decreases.

(ii) The bounds $\underline{A}(-r)$ and $\overline{A}(r)$ above which the policymaker adopts a peg increase, implying that policymakers adopts pegs for a narrower range of values of A.

(iii) The equilibrium value of the policymaker's objectives, V, falls.

Part (i) of Proposition 3 states that, in our framework, lifting restrictions on the free flow of capital lowers, on balance, the likelihood of a currency crisis. This counterintuitive result is the outcome of two opposing effects. First, as Proposition 1 shows, the two RECs shrink when

t decreases. Holding the band width constant, this raises the probability of speculative attacks. This effect already appears in the recent literature on international financial crises (e.g., Morris and Shin, 1998). But since the two RECs shrink, the band becomes less effective in guaranteeing exchange-rate stability. The policymaker's reaction to this is to pursues less ambitious stabilization objectives and allow the exchange rate to move freely within a wider band. This lowers, in turn, the probability, P, of speculative attacks. Thus, in general, the effect of reducing t on the probability of a currency attack is ambiguous. Part (i) of proposition 3 and proposition 2 suggest that, for non degenerate bands and unimodal distributions of fundamentals, the second effect dominates so P decreases when t is reduced.[21]

This result implies that, when the endogeneity of the exchange rate regime is recognized, conventional wisdom may be reversed. Dismantling of restrictions on capital flows may reduce rather than raise the probability of currency crisis. The mirror image of this result is that the imposition of a Tobin tax generally has an ambiguous effect on the likelihood of crisis. For a given band width, the tax **does reduce** the likelihood of a currency crisis. However, the imposition of the tax also induces policymakers to install narrower bands in order to achieve more ambitious reductions in exchange rate uncertainty. All else equal, this longer run policy response **raises** the probability of crises.

Part (ii) of Proposition 3 predicts that liberalization of the capital account, as characterized by a reduction in t, should lead to a narrowing of the set of countries that maintain pegs. It also implies that, in spite of this trend, countries with strong preference for stability of the exchange rate (e.g., small open economies with relatively large shares of foreign currency denominated trade and capital flows as well as emerging markets) will continue to peg even in the face of capital market liberalization. By contrast, countries with intermediate preference for exchange rate stability (e.g., more financially mature economies with a larger fraction of domestically denominated debt and capital flows) will move from pegs to bands. These predictions seem to be consistent with casual evidence. Two years following the 1997/8 East Asian crisis, most emerging markets countries in that region are back on pegs (McKinnon, 2001 and Calvo and Reinhart, 2002). On the other hand, following the EMS currency crisis at the beginning of the 90's, the system of cooperative pegs that

[21] This result is partly reminiscent of the discussion in Kupiec (1996) which establishes that, when general equilibrium effects are taken into consideration, a securities transaction tax does not necessarily reduce stock return volatility. By analogy, in our case, when the "general equilibrium" reaction of policymakers is taken into consideration, a Tobin tax does not necessarily reduce the likelihood of a currency crisis.

had existed prior to the crisis was replaced by wide bands until the formation of the EMU at the beginning of 1999.

Finally, part (iii) of Proposition 3 shows that although a decrease in t lowers the likelihood of financial crisis, it nonetheless makes the policymaker worse-off. The reason is that speculative attacks impose a constraint on the policymaker when he chooses the optimal exchange rate regime. A decrease in t strengthen the incentive to mount a speculative attack and thereby makes this constraint more binding.

4.2 The effects of aversion to exchange rate uncertainty

In this subsection we turn to the effects of the parameter A (the relative importance that the policymaker assigns to exchange rate stability) on the choice of regime. As we argued earlier, residents of small open economies are more averse to nominal exchange rate uncertainty than residents of large, relatively closed, economies. Hence the parameter A reflects the size of the economy and the degree to which it is open with larger values of A being associated with smaller and less open economies. In what follows we restrict the comparative statics analysis to the case in which the policymaker's problem has a unique interior solution, i.e., $-\infty < \underline{\pi} < 0 < \bar{\pi} < \infty$. As Proposition 2 indicates, this requires A to be above 1 but not by "too much."

Proposition 4 *Suppose that the policymaker's problem has a unique interior solution. Then as A increases:*

(i) $\bar{\pi}$ and $\underline{\pi}$ shift closer to 0 so the band becomes tighter, and

(ii) the probability, P, that a speculative attack will occur increases.

Proposition 4 says that the policymaker sets a tighter band and thereby allows the exchange rate to move freely within a narrower range around the center rate, as he becomes more concerned with exchange rate stability (i.e., as A increases). Part (ii) of the proposition shows that this tightening of the band raises the likelihood of a speculative attack. This implies that *all else equal*, policymakers in countries with larger values of A are willing to set tighter bands and face a higher likelihood of speculative attacks than policymakers in otherwise similar countries with lower values of A.

Note that as Proposition 2 shows, when A increases above $\underline{A}(r)$ and $\overline{A}(r)$, the optimal band width becomes 0 so the optimal regime is a peg. On the other hand, when A falls below 1, the optimal band width becomes infinite implying that the optimal regime is a free float. Given that a substantial part of international trade is invoiced in US Dollar (McKinnon, 1979), it is likely that policymakers of a key currency country like the US are going to be less sensitive to nominal exchange rate uncertainty and therefore have a smaller A than policymakers in small open economies. Therefore, our model predicts that the US, Japan, and the Euro area should be floating, while Hong-Kong, Panama, Estonia, Lithuania, and Bulgaria, should be on either pegs, currency boards, or even full dollarization. This prediction appears to be consistent with casual observation on the exchange rate systems chosen by those countries..

4.3 The effect of increased variability in fundamentals

Next, we examine how the exchange rate band changes when more extreme realizations of x become more likely. This comparative statics exercise involves shifting probability mass from realizations of x that are either inside the band or inside the two RECs (and therefore do not lead to realignments) to realizations that are either below the negative REC or above the positive REC and therefore lead to realignments.

Proposition 5 *Suppose that $f(x)$ and $g(x)$ are two density functions with a mode at 0 and equal expected values. Moreover, suppose that $g(x)$ lies above $f(x)$ for all $x < \underline{\pi} - r$ and all $x > \bar{\pi} + r$, where $\underline{\pi}$ and $\bar{\pi}$ are the solutions to the policymaker's problem under the original density function $f(x)$ (that is, $g(x)$ has fatter tails than $f(x)$). Then, the policymaker adopts a wider band under $g(x)$ than under $f(x)$.*

Intuitively, when more extreme realizations of x become more likely (the density of x is $g(x)$ rather than $f(x)$), the policymaker is more likely to incur the loss of future credibility associated with realignments. Therefore, the policymaker widens the band to lower the probability that a costly realignment will take place. Moreover, as larger shocks become more likely, the policymaker finds it optimal to shift the two REC's away from 0 in order to shift his commitment to intervene in the market to a range of shocks that are now more probable. This move benefits the policymaker by counteracting part of the increased uncertainty about the free float value of the exchange rate.

4.4 The effects of tightness of commitment to maintain the regime

The degree of commitment to the exchange rate regime is represented in our model by the parameter δ. Differentiating equations (3.4) and (3.5) with respect to δ reveals that in general, δ has an ambiguous effect on the optimal width of the band. On one hand, as δ increases, speculators attack the band for a smaller range of x's. This effect increases the policymaker's incentive to adopt a narrow band. On the other hand, as δ increases, the cost of realignments (when they occur) increases, since they lead to a larger future credibility loss. This effect pushes the policymaker to widen the band. Overall then, the width of the band may either increase or decrease with δ.

Since the probability of speculative attacks, P, is affected by the width of the band, the effect of δ on P is also ambiguous. For a given regime, Proposition 1 implies that P decreases with δ. However, when the endogeneity of the regime is recognized, there is an additional effect that may reverse this result: When δ increases, the policymaker may decide to set a narrower band, knowing that, given the width of the band, he will subsequently decide to maintain the regime for a larger set of x's. This, in turn, may increase the ex ante probability of a speculative attack. Consequently, an increase in the tightness of commitment may increase the probability of speculative attack.[22]

5 The exchange rate band and the policymaker's reputation

Typically the public is not fully informed about the commitment ability of policymakers. In this section, we examine how this uncertainty affects the optimal exchange rate regime. To this end, we assume that there are two possible types of policymakers. The first type, to which we refer as dependable, is identical to the policymaker that we considered so far. The second type, to which we refer as opportunistic, differs from the dependable type in that he does not bear the cost δ if the exchange rate is realigned. Thus, under an opportunistic policymaker, there is always a realignment when x falls outside the band.[23] We assume that speculators assign a probability β to the policymaker's type being dependable and following Barro (1986), interpret β as a measure of the policymaker's "reputation."

We now examine how the optimal exchange rate regime is affected by changes in β. As a point of reference, it should be noted that the analysis so far referred to the case where $\beta = 1$.

[22] We also tried to use the model in order to characterize the optimal degree of commitment to the regime but, since the ratio of economic insights to algebra was low this experiment is not presented. Cukierman, Kiguel and Liviatan (1992) and Flood and Marion (1999) present such an analysis for exogenously given pegs. The analysis here is more complex due to the fact that it involves the simultaneous choice of band width as well as of the degree of commitment to the band.

[23] In our framework, even an opportunistic policymaker that does not intend to defend the band in stage 3 needs to put the initial defense in stage 2 on "automatic pilot" in order to prevent the public from separating him, already in the first stage (when the band is announced), from his dependable counterpart.

Before proceeding further, we modify Assumption 2 as follows:

Assumption 3: The real transaction cost, $\frac{t}{e_{-1}}$, is small relative to δ but not too small in the sense that $\delta(1-\beta) < \frac{t}{e_{-1}} < \delta$.

Assumption 3 ensures that speculators will always attack the band if they believe that x is such that a dependable policymaker will exit the band, but never attack it if they believe that x is such that the dependable policymaker will defend the band.

5.1 Choice of exchange rate regime in the presence of imperfect reputation

With Assumption 3 in place, we examine how the presence of an opportunistic policymaker affects the decisions of speculators on when to attack the band.

Lemma 2 *Suppose that $\varepsilon \to 0$. Then,*

(i) speculators will attack the upper bound of the band if and only if they observe signals above $\overline{\theta}^*_\beta$. They will attack the lower bound of the band if and only if they observe signals below $\underline{\theta}^*_\beta$. The two thresholds, $\overline{\theta}^*_\beta$ and $\underline{\theta}^*_\beta$, are given by $\overline{\theta}^*_\beta = \overline{\pi} + r^\beta$, and $\underline{\theta}^*_\beta = \underline{\pi} - r^\beta$, where

$$r^\beta = \sqrt{\frac{t}{\beta e_{-1}} + \frac{(\delta - \frac{1}{\beta})^2}{4}} + \frac{\delta - \frac{1}{\beta}}{2}. \tag{4.1}$$

(ii) r^β increases with β.

The behavior of speculators implies that the positive REC is now given by $[\overline{\pi}, \overline{\pi} + r^\beta]$ while the negative REC becomes $[\underline{\pi} - r^\beta, \underline{\pi}]$. Since r^β increases with β, it follows that when the policymaker's reputation is imperfect (i.e., $\beta < 1$), the two RECs become narrower relative to the perfect reputation case (i.e., when $\beta = 1$).[24] The intuition underlying this result is that when $\beta < 1$, speculators believe that the policymaker will exit the band with a positive probability. Hence, the expected gain from attacking the band is now larger so a dependable policymaker finds it more difficult to defend the band.

Part (ii) of Lemma 2 can be used to explain how an abrupt change in the policymaker's reputation can generate a currency crisis even if there is no change in the fundamental, x. We illustrate this point by reference to the 1994 Mexican Peso crisis. Prior to the crisis, Mexico

[24] This is analogous to a result in Cukierman and Liviatan (1991) in the context of a Barro-Gordon (1983) inflation bias equilibrium in which the public is uncertain about the dependability of policymakers. Cukierman and Liviatan show that the lower the reputation of a (dependable) policymaker the less ambitious is his inflation target.

maintained a peg for several years and therefore developed a fair amount of reputation about its resolve to defend the peg. This together with, inter alias, substantial interest rate differentials attracted a large capital inflow into Mexico. Then, in March 1994, the ruling party's presidential candidate, Luis Donaldo Colosio, was assassinated. Following that, the Mexican Peso came under attack. The authorities defended the Peso initially but, following a substantial loss of reserves within a short period of time, allowed the Peso to float.[25] Second-generation models of currency crises might interpret Colosio's assassination as a sunspot that, for some unexplained reason, was used by speculators as a coordinating device to move the Mexican economy from a good equilibrium to a bad one.

Within our framework, the same events can be interpreted as follows. Prior to Colosio's assassination, fundamentals were already stretched so that in the absence of intervention the Peso would have depreciated. But, since reputation was high, speculators anticipated that the Mexican government would defend the peg and therefore refrained from attacking it. However, the assassination and the subsequent political instability led to an abrupt decrease in reputation, β. Part (ii) of lemma 2 suggests that this might have narrowed the REC around the Mexican peg and therefore created a new situation in which the free market rate, x, fell outside the REC. Consequently, it became rational for speculators to run on the Peso, and for the Mexican government not to defend it.[26] Note that this interpretation does not rely on sunspots and multiple equilibria and holds even if the assassination had no effect on the economic fundamentals in Mexico.

5.2 Choice of exchange rate regime in the presence of imperfect reputation

We begin the analysis here with the expected variability in the exchange rate. When the policymaker's reputation is imperfect, it is anticipated that with probability β the policymaker is dependable and will defend the band against speculative attacks whenever x falls inside the two RECs, and with probability $1-\beta$ the policymaker is opportunistic and never defends the band.

[25] A factually based analysis of the sequence of political and economic events preceding the Mexican crisis appears in Whitt (1996).

[26] Another example for a crisis triggered by political events that led to an abrupt change in reputation is the rejection of the Maastricht Treaty by Danish voters in summer of 1992 and the evaluation, at that time, that French voters might also reject the Treaty at the subsequent referendum in September 1992. Isard (1995, p. 210) notes that: "Prudent investors who had earlier sought higher yields by placing funds in potentially vulnerable currencies increasingly saw the merit of covering their exposed positions before the French vote."

Hence, the expected variability in the exchange rate around the current level is:

$$\begin{aligned} E^{\beta}|\pi| &= -\int_{-\infty}^{\underline{\pi}-r^{\beta}} xf(x)dx - \int_{\underline{\pi}-r^{\beta}}^{\underline{\pi}} \left[\beta\underline{\pi} + (1-\beta)x\right] f(x)dx - \int_{\underline{\pi}}^{0} xf(x)dx \\ &\quad + \int_{0}^{\overline{\pi}} xf(x)dx + \int_{\overline{\pi}}^{\overline{\pi}+r^{\beta}} \left[\beta\overline{\pi} + (1-\beta)x\right] f(x)dx + \int_{\overline{\pi}+r^{\beta}}^{\infty} xf(x)dx. \end{aligned} \tag{4.2}$$

Note that β affects $E^{\beta}|\pi|$ both through its effect on the width of the two RECs and through its effect on the expected change in the exchange rate inside the two RECs which are now linear combinations of $\underline{\pi}$ and of x inside the negative REC and of $\overline{\pi}$ and of x inside the positive REC.

Given $E^{\beta}|\pi|$, the expected payoff of a dependable policymaker becomes,

$$\begin{aligned} V^{\beta} &= A\Bigg[\int_{-\infty}^{\underline{\pi}-r^{\beta}} xf(x)dx + \int_{\underline{\pi}-r^{\beta}}^{\underline{\pi}} \left[\beta\underline{\pi} + (1-\beta)x\right] f(x)dx + \int_{\underline{\pi}}^{0} xf(x)dx \\ &\quad - \int_{0}^{\overline{\pi}} xf(x)dx - \int_{\overline{\pi}}^{\overline{\pi}+r^{\beta}} \left[\beta\overline{\pi} + (1-\beta)x\right] f(x)dx - \int_{\overline{\pi}+r^{\beta}}^{\infty} xf(x)dx\Bigg] \\ &\quad - \int_{-\infty}^{\underline{\pi}-r^{\beta}} \delta f(x)dx - \int_{\underline{\pi}-r^{\beta}}^{\underline{\pi}} (\underline{\pi}-x)f(x)dx - \int_{\overline{\pi}}^{\overline{\pi}+r^{\beta}} (x-\overline{\pi})f(x)dx - \int_{\overline{\pi}+r^{\beta}}^{\infty} \delta f(x)dx. \end{aligned} \tag{4.3}$$

A dependable policymaker chooses the boundaries of the band, $\underline{\pi}$ and $\overline{\pi}$, so as to maximize his expected payoff. We do not need to specify the expected payoff of an opportunistic policymaker because, given that he does not intend to defend the band, he always wishes to announce the same band as his dependable counterpart in order to prevent his type from being revealed; in turn this lowers the exchange rate uncertainty ex ante relative to the case where the opportunistic policymaker's type is revealed.

The following proposition characterizes some of the factors, including reputation, that affect the choice of exchange rate regime in the presence of imperfect reputation.

Proposition 6 *In equilibrium, the exchange rate band has the following properties:*

(i) ***Free float:*** *If $A\beta \leq 1$, then $\underline{\pi} = -\infty$ and $\overline{\pi} = \infty$, so the optimal regime is a free float.*

(ii) ***A nondegenerate band:*** *If*

$$1 < A\beta < \underline{A}(-r^{\beta}) \equiv 1 + \frac{\delta}{\int_{-r^{\beta}}^{0}\left[\frac{f(x)}{f(-r^{\beta})} - 1\right]dx}, \tag{4.6}$$

then $-\infty < \underline{\pi} < 0$. Likewise, if

$$1 < A\beta < \overline{A}(r^{\beta}) \equiv 1 + \frac{\delta}{\int_{0}^{r^{\beta}}\left[1 - \frac{f(x)}{f(r^{\beta})}\right]dx}, \tag{4.7}$$

then $0 < \bar{\pi} < \infty$. Hence, the optimal regime is a nondegenerate band.

(iii) ***A peg:*** *If V^{β} is concave in $\underline{\pi}$ and in $\bar{\pi}$ and $A\beta > Max\{\underline{A}^{\beta}, \bar{A}^{\beta}\}$ then $\underline{\pi} = \bar{\pi} = 0$, so the optimal regime is a peg.*

(iv) ***The width of the band and the likelihood of speculative attacks:*** *Suppose that the policymaker's problem has a unique interior solution. Then, $\bar{\pi}$ and $\underline{\pi}$ shift closer to 0 as β increases towards 1, implying that as the policymaker's reputation improves, he adopts a tighter band. Moreover, as the policymaker's reputation improves, the likelihood of speculative attack increases ($\frac{\partial P^{\beta}}{\partial \beta} > 0$).*

Parts (i)-(iii) of Proposition 6 modify the corresponding parts of Proposition 2 for the case where the policymaker's reputation is imperfect. Part (iv) of Proposition 6 says that as the policymaker's reputation improves, the exchange rate band becomes tighter. This implies that a good reputation induces the policymaker to be more ambitious in his attempt to reduce exchange rate uncertainty. The reason for that is twofold. First, when the policymaker's reputation improves, a tighter band has a greater moderating effect on the expected variability of the nominal exchange rate. Second, holding the width of the band constant, improved reputation lowers the likelihood of speculative attacks, and therefore makes it less costly for the policymaker to set a tighter band. Hong-Kong's currency board fits into this "box" of the model. Since the peg has never been abandoned in the past, Hong-Kong's currency board has good reputation, which induces the authorities to defend the peg under a wider set of circumstances than is the case under a lower reputation level.

But part (iv) of Proposition 6 also states that when a policymaker has a better reputation there is an overall increase in the likelihood of speculative attacks. Although better reputation leads to wider RECs (i.e., ranges of x for which the policymaker defends the band), it also induces the policymaker to adopt tighter bands, which makes the exchange rate regime more susceptible to speculative attacks. In our model, the second effect is stronger, so overall there is an increase in the likelihood of speculative attack.

6 Concluding reflections

This paper develops a framework for analyzing the interaction between the *ex ante* choice of exchange rate regime and the probability of *ex post* currency attacks. In the model, the policymaker commits to an exchange rate band in order to reduce the ex ante uncertainty about the level of the exchange rate; the policymaker dislikes this uncertainty because it raises the foreign exchange risk premium and disrupts the international flows of goods and of financial capital. This commitment is costly however: maintaining a band or a peg in the face of speculative attacks forces the policymaker to use up foreign exchange reserves or to deviate from the level of the interest rate that is consistent with other objectives. If the policymaker exits the band, he loses credibility. In the model, the ex ante probability of a currency attack decreases with the width of the band. The policymaker, in turn, takes this probability into account when choosing the regime ex ante. Thus, in our framework the exchange rate regime and the likelihood of speculative attacks are interrelated. To the best of our knowledge, this is the first paper that solves endogenously for the optimal regime and for the probability of currency attacks and studies their interrelation.

Our framework generates several novel predictions that are consistent with empirical evidence. First, we find that financial liberalization that lowers the transaction costs of switching between currencies induces the policymaker to adopt a more flexible exchange rate regime. This is broadly consistent with the flexibilization of exchange rate regimes following the gradual reductions of restrictions on capital flows in the aftermath of the Bretton Woods system (see, for example, Isard, 1995). Second, in our model, small open economies with substantial aversion to exchange rate uncertainty are predicted to have narrower bands and more frequent currency attacks than large, relatively closed economies. This is broadly consistent with the fact that large economies with key currencies like the US, Japan and the Euro area chose to float, while small open economies like Argentina (until the beginning of 2002), Thailand, and Korea chose less flexible regimes that are more susceptible to currency attacks like the 1997/8 South-East Asian crisis. Third, the model predicts that policymakers with high reputation tend to set less flexible regimes, and are less vulnerable to speculative attacks. Hong-Kong's currency board is a good example. Since it has never abandoned its currency board in the past, Hong-Kong's currency board enjoys a good reputation, and attracts less speculative pressure.

Another possible prediction of our model, which we did not highlight so far, is related to the bipolar view, according to which following the process of globalization, there has been a gradual shift away from intermediate exchange rate regimes to either hard pegs or freely floating regimes (Fischer, 2001). Globalization is expected to have two opposite effects in our model: On one hand, it lowers the cost of switching between currencies and hence facilitates speculation. This effect induces policymakers to set more flexible regimes. On the other hand, globalization also increases the volume of international trade in goods and financial assets, and increases, therefore, the aversion to nominal exchange rate uncertainty. This effect induces policymakers to set less flexible regimes. The second effect is likely to be large for small open economies whose currencies are not used much for either capital account or current account transaction in world markets, and to be small or even negligible for large key currency economies. Hence, the first effect is likely to be dominant in large, relatively closed blocks, while the second is likely to be dominant in small open economies. All else equal, the process of globalization should therefore induce relatively large currency blocks to move towards more flexible exchange rate arrangements while pushing small open economies in the opposite direction.

Our model also generates some novel counter-intuitive results. The most interesting one is probably the effect of a Tobin tax on the ex ante probability of a currency attack. As in existing literature, our model implies that for a given exchange rate regime, a Tobin tax reduces the probability of a currency attack. However, once the choice of an exchange rate regime is endogenized, a Tobin tax has an additional, opposite, effect: It induces policymakers to set less flexible regimes, and this by itself increases the probability of currency attacks. Thus, in general, a Tobin tax has an overall ambiguous effect on the probability of currency attacks. Similarly, conventional wisdom suggests that when policymakers bear a larger credibility loss following a realignment, they have a stronger incentive to defend the exchange rate regime against speculative attacks and this lowers the probability of this event. However, once the choice of a regime is endogenized, the overall effect becomes ambiguous since ex ante, realizing that speculative attacks are less likely, policymakers may have an incentive to adopt a less flexible regime.

Although our framework captures many empirical regularities regarding exchange rate regimes and speculative attacks, it obviously does not capture all of them. For example, as Calvo and Reinhart (2002) have recently shown, policymakers often intervene in exchange rate markets even in the absence of explicit pegs or bands. We believe that an extension that will analyze the desirability of implicit bands as well as other regimes is a promising direction for future research.[27] Another interesting direction for future research is the development of a dynamic framework in which the fundamentals of the economy are changing over time, and speculators can attack the currency at several points in time. The optimal policy in a dynamic context raises additional interesting issues such as the change in the policymaker's reputation over time.

[27] A recent theoretical discussion of implicit bands appears in Koren (2000).

7 Appendix

Proof of Lemma 1: **(i)** We analyze the behavior of the policymaker and of the speculators after the exchange rate reaches the upper bound of the band. We show that in the limit as $\varepsilon \longrightarrow 0$, there exists in this case a unique perfect Bayesian equilibrium in which each speculator attacks the band if and only if he observe a signal above a unique threshold signal $\overline{\theta}^*$. The proof for the case where the exchange rate reaches the lower bound of the band is analogous.

We start with some notation. From equation (3.1) we know that the policymaker defends the upper bound of the band if and only if $x \leq \overline{\pi} - \alpha + \delta$. Using this expression, let

$$\alpha^*(x) = \begin{cases} 0, & if \quad \overline{\pi} - x + \delta < 0, \\ \overline{\pi} - x + \delta, & if \quad 0 \leq \overline{\pi} - x + \delta \leq 1, \\ 1 & if \quad \overline{\pi} - x + \delta > 1, \end{cases} \tag{A-1}$$

be the critical measure of speculators below which the policymaker defends the upper bound of the band when the laissez faire rate of change in the exchange rate is x. Using the definition of $\alpha^*(x)$, the net payoff from attacking the upper bound of the band is:

$$v(x, \alpha) = \begin{cases} (x - \overline{\pi})\, e_{-1} - t, & if \quad \alpha \geq \alpha^*(x), \\ -t, & if \quad \alpha < \alpha^*(x). \end{cases} \tag{A-2}$$

Equation (A-2) shows that $v(x, \alpha)$ is weakly increasing in α because the assumption that $x \geq \overline{\pi}$ implies that $(x - \overline{\pi})\, e_{-1} - t \geq -t$ (hence, the expression in the top line in (A-2) exceeds the one at the bottom line of (A-2)). Moreover, noting from equation (A-1) that $\alpha^*(x)$ is weakly decreasing in x, equation (A-2) shows that $v(x, \alpha)$ is weakly increasing in x, and strictly increasing in x if $v(x, \alpha) \geq 0$.

Let $\alpha_i(x)$ be speculator i's belief about the measure of speculators who will attack the band for each level of x. We will say that the belief $\alpha_i'(x)$ is higher (lower) than $\alpha_i(x)$ if $(\alpha_i'(x) \geq \alpha_i(x))$ for all x with strict inequality for at least one x.

The decision of speculator i on whether or not to attack the band depends on the signal θ_i that the speculator observes and on the speculator's belief, $\alpha_i(x)$. Since $\theta_i = x + \varepsilon_i$, where $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$, and using equation (2.4), the net expected payoffs of speculator i from attacking the upper bound of the band is:

$$h(\theta_i, \alpha_i(x)) = \int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} v(x, \alpha_i(x)) f(x \mid \theta_i) dx = \frac{\int_{\theta_i - \varepsilon}^{\theta_i + \varepsilon} v(x, \alpha_i(x)) f(x) dx}{F(\theta_i + \varepsilon) - F(\theta_i - \varepsilon)}. \tag{A-3}$$

We establish three properties of $h(\theta_i, \alpha_i(x))$. First, since θ_i affects the numerator of $h(\theta_i, \alpha_i(x))$ only through the boundaries of integration and since $F(\cdot)$ is a continuous function, it follows that $h(\theta_i, \alpha_i(x))$ is continuous in θ_i. Second, recalling that $v(x, \alpha)$ is weakly increasing in α, it follows that if $\alpha_i'(x) \leq \alpha_i(x)$ ($\alpha_i'(x) \geq \alpha_i(x)$), then $h(\theta_i, \alpha_i'(x)) \leq h(\theta_i, \alpha_i(x))$ ($h(\theta_i, \alpha_i'(x)) \geq h(\theta_i, \alpha_i(x))$) for all θ_i. Third, note that:

$$\begin{aligned}
\frac{\partial h(\theta_i, \alpha_i(x))}{\partial \theta_i} &= \frac{\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} f(x)dx \left[v\left(\theta_i+\varepsilon, \alpha_i(\theta_i+\varepsilon)\right) f(\theta_i+\varepsilon) - v\left(\theta_i-\varepsilon, \alpha_i(\theta_i-\varepsilon)\right) f(\theta_i-\varepsilon)\right]}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2} \\
&\quad - \frac{\int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} v(x, \alpha_i(x)) f(x)dx \left[f(\theta_i+\varepsilon) - f(\theta_i-\varepsilon)\right]}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2} \\
&= \frac{f(\theta_i+\varepsilon) \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \left[v\left(\theta_i+\varepsilon, \alpha_i(\theta_i+\varepsilon)\right) - v(x, \alpha_i(x))\right] f(x)dx}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2} \\
&\quad + \frac{f(\theta_i-\varepsilon) \int_{\theta_i-\varepsilon}^{\theta_i+\varepsilon} \left[v(x, \alpha_i(x)) - v\left(\theta_i-\varepsilon, \alpha_i(\theta_i-\varepsilon)\right)\right] f(x)dx}{\left(F\left(\theta_i+\varepsilon\right) - F\left(\theta_i-\varepsilon\right)\right)^2}.
\end{aligned} \tag{A-4}$$

Recalling from above that $v(x, \alpha)$ is weakly increasing in both x and α, it follows that if $\alpha_i(x)$ is non-decreasing in x, then $h(\theta_i, \alpha_i(x))$ is weakly increasing in θ_i. Next, we show that whenever it is nonnegative, $h(\theta_i, \alpha_i(x))$ must be strictly increasing in θ_i. To this end, note that since $h(\theta_i, \alpha_i(x))$ is the expected value of $v(x, \alpha_i(x))$ when $x \in [\theta_i - \varepsilon, \theta_i + \varepsilon]$, then $h(\theta_i, \alpha_i(x)) \geq 0$ implies that there exists at least one value of x in the interval $[\theta_i - \varepsilon, \theta_i + \varepsilon]$ for which $v(x, \alpha_i(x)) > 0$ (otherwise $h(\theta_i, \alpha_i(x)) \leq 0$). Since we showed above that $v(x, \alpha)$ is strictly increasing in x if $v(x, \alpha) \geq 0$, it follows that $v(x, \alpha)$ is strictly increasing in x for at least one value of $x \in [\theta_i - \varepsilon, \theta_i + \varepsilon]$. But since $v(x, \alpha)$ is weakly increasing in x and strictly increasing in x for at least one value of x, it follows that $v\left(\theta_i + \varepsilon, \alpha_i(\theta_i + \varepsilon)\right) > v\left(\theta_i - \varepsilon, \alpha_i(\theta_i - \varepsilon)\right)$. Consequently, $h(\theta_i, \alpha_i(x))$ is strictly increasing in θ_i whenever $h(\theta_i, \alpha_i(x)) \geq 0$.

In equilibrium, the strategy of speculator i is to attack the upper bound of the band if $h(\theta_i, \alpha_i(x)) > 0$ and not attack it if $h(\theta_i, \alpha_i(x)) < 0$. Moreover, the equilibrium belief of speculator i, $\alpha_i(x)$, must be consistent with the equilibrium strategies of all other speculators (for short we will simply say that in equilibrium, the belief of speculator i is consistent). To characterize the equilibrium strategies of speculators, we first show that there exists a range of sufficiently large signals for which speculators have a dominant strategy to attack the band and likewise, there exists a range of sufficiently small signals for which speculators have a dominant strategy not to attack the band. Then, we use an iterative process of elimination of dominated strategies to establish the existence of a unique signal, $\overline{\theta}^*$, such that speculator i attacks the upper bound of the band if and only if $\theta_i > \overline{\theta}^*$.

Suppose that speculator i observes a signal $\theta_i > \overline{\theta} \equiv \overline{\pi} + \delta + \varepsilon$. Then speculator i realizes that $x > \overline{\pi} + \delta$. Using equation (A-1), this means that $\alpha^*(x) = 0$ so the policymaker is surely going to exit the band. By equation (A-2), the net payoff from attacking the band is therefore $v(x, \alpha) = (x - \overline{\pi})e_{-1} - t$, for all α. But since $x > \overline{\pi} + \delta$, it follows that $v(x, \alpha) > \delta e_{-1} - t$ for all α, which is strictly positive by Assumption 2. Hence, the net expected payoff of the speculator is such that $h(\theta_i, \alpha_i(x)) > 0$ for all $\theta_i > \overline{\theta}$ and all $\alpha_i(x)$, implying that it is a dominant strategy for a speculator who observes a signal above $\overline{\theta}$ to attack the upper bound of the band. Similarly, suppose that speculator i observes a signal $\theta_i < \underline{\theta} \equiv \overline{\pi} + \frac{t}{e_{-1}} - \varepsilon$ (since we focus on the case where $\varepsilon \to 0$ and since $t > 0$, such signals are observed with a positive probability whenever $x > \overline{\pi}$), the speculator realizes that $x < \overline{\pi} + \frac{t}{e_{-1}}$. Consequently, even if the policymaker surely exits the band, the payoff from attacking it is negative as $v(x, \alpha) = (x - \overline{\pi})e_{-1} - t < \left((\overline{\pi} + \frac{t}{e_{-1}}) - \overline{\pi}\right) e_{-1} - t = 0$. This implies in turn that $h(\theta_i, \alpha_i(x)) < 0$ for all $\theta_i < \underline{\theta}$ and all $\alpha_i(x)$, so it is a dominant strategy for speculator i not to attack the band after observing a signal below $\underline{\theta}$.

Now, we start an iterative process of elimination of dominated strategies from $\overline{\theta}$, in order to expand the range of signals for which speculators will surely attack the band. To this end, let $\alpha(x, \theta)$ represent a speculator's belief regarding the measure of speculators who will attack the band for each level of x, when the speculator believes that all speculators will attack the upper bound of the band if and only if they observe signals above some level θ. Since $\varepsilon_i \sim U[-\varepsilon, \varepsilon]$, it follows that

$$\alpha(x, \theta) = \begin{cases} 0, & if \quad x < \theta - \varepsilon, \\ \frac{x - (\theta - \varepsilon)}{2\varepsilon}, & if \quad \theta - \varepsilon \le x \le \theta + \varepsilon, \\ 1, & if \quad x > \theta + \varepsilon. \end{cases} \tag{A-5}$$

The iterative process of elimination of dominated strategies works as follows. Above, we already established that $h(\theta_i, \alpha_i(x)) > 0$ for all $\theta_i > \overline{\theta}$ and all $\alpha_i(x)$. But since $h(\theta_i, \alpha_i(x))$ is continuous in θ_i, it follows that $h(\overline{\theta}, \alpha_i(x)) \ge 0$ for all $\alpha_i(x)$, and in particular for $\alpha_i(x) = \alpha(x, \overline{\theta})$. Thus, $h(\overline{\theta}, \alpha(x, \overline{\theta})) \ge 0$. Note that since in equilibrium, the beliefs of speculators are consistent, only beliefs that are higher than or equal to $\alpha(x, \overline{\theta})$ can hold in equilibrium (because all speculators attack the band when they observe signals above $\overline{\theta}$). Thus, we say that $\alpha(x, \overline{\theta})$ is the "lowest" consistent belief on α.

Let $\overline{\theta}^1$ be the value of θ_i for which $h(\theta_i, \alpha(x, \overline{\theta})) = 0$. That is, $h(\overline{\theta}^1, \alpha(x, \overline{\theta})) \equiv 0$. Note that $\overline{\theta}^1 \le \overline{\theta}$, and that $\overline{\theta}^1$ is defined uniquely because we showed above that $h(\theta_i, \alpha_i(x))$ is strictly increasing in θ_i whenever $h(\theta_i, \alpha_i(x)) \ge 0$. Using the second and third properties of $h(\theta_i, \alpha_i(x))$

and recalling that $\alpha(x,\overline{\theta})$ is the lowest consistent belief on α, it follows that $h(\theta_i,\alpha_i(x))>0$ for any $\theta_i>\overline{\theta}^1$ and any consistent belief $\alpha_i(x)$. Thus, in equilibrium, speculators must attack the band if they observe signals above $\overline{\theta}^1$. As a result, $\alpha(x,\overline{\theta}^1)$ becomes the lowest consistent belief on $\alpha_i(x)$.

Starting from $\overline{\theta}^1$, we can now repeat the process along the following steps (these steps are similar to the ones that were used in order to establish $\overline{\theta}^1$). First, note that since $h(\overline{\theta}^1,\alpha(x,\overline{\theta}))\equiv 0$ and since $\alpha(x,\theta)$ is weakly decreasing with θ and $h(\theta_i,\alpha_i(x))$ is weakly increasing with $\alpha_i(x)$, it follows that $h(\overline{\theta}^1,\alpha(x,\overline{\theta}^1))\geq 0$. Second, find a $\theta_i\leq\overline{\theta}^1$ for which $h(\theta_i,\alpha(x,\overline{\theta}^1))=0$, and denote it by $\overline{\theta}^2$. Using the same arguments as above, $\overline{\theta}^2$ is defined uniquely. Third, since $\alpha(x,\overline{\theta}^1)$ is the lowest consistent belief on $\alpha_i(x)$ and using the second and third properties of $h(\theta_i,\alpha_i(x)))$, it follows that speculators must attack the band if they observe signals above $\overline{\theta}^2$. The lowest possible belief on $\alpha_i(x)$ becomes $\alpha(x,\overline{\theta}^2)$.

We repeat this process over and over again (each time lowering the value of θ above which speculators will attack the upper bound of the band), until we reach a step n such that $\overline{\theta}^{n+1}=\overline{\theta}^n$, implying that the process cannot continue further. Let $\overline{\theta}^\infty$ denote the value of θ at which the process stops. (Clearly, $\overline{\theta}^\infty\leq\overline{\theta}$.) By definition, speculators will attack the band if they observe signals above $\overline{\theta}^\infty$. Since $\overline{\theta}^\infty$ is the point where the process stops, it must be the case that $h(\overline{\theta}^\infty,\alpha(x,\overline{\theta}^\infty))=0$ (otherwise, we can find some $\theta_i<\overline{\theta}^\infty$ for which $h(\theta_i,\alpha(x,\overline{\theta}^\infty))=0$, and the iterative process could have been continued further).

Starting a similar iterative process from $\underline{\theta}$ and following the exact same steps, we also obtain a signal $\underline{\theta}^\infty(\geq\underline{\theta})$ such that speculators will never attack the band if they observe signals below $\underline{\theta}^\infty$. At this signal, it must be the case that $h(\underline{\theta}^\infty,\alpha(x,\underline{\theta}^\infty))=0$. Since we proved that in equilibrium speculators attack the upper bound of the band if they observe signals above $\overline{\theta}^\infty$ and do not attack it if they observe signals below $\underline{\theta}^\infty$, it must be the case that $\overline{\theta}^\infty\geq\underline{\theta}^\infty$.

The last stage of the proof involves showing that $\overline{\theta}^\infty=\underline{\theta}^\infty$. We do that for the case where $\varepsilon\to 0$. First, note that $\overline{\theta}^\infty$ is defined by the equation $h(\overline{\theta}^\infty,\alpha(x,\overline{\theta}^\infty))=0$. Using equations (A-3) and (A-5), this equality can be written as

$$\frac{\int_{\overline{\theta}^\infty-\varepsilon}^{\overline{\theta}^\infty+\varepsilon} v(x,\frac{x-(\overline{\theta}^\infty-\varepsilon)}{2\varepsilon})f(x)dx}{F\left(\overline{\theta}^\infty+\varepsilon\right)-F\left(\overline{\theta}^\infty-\varepsilon\right)}=0. \tag{A-6}$$

Using the equality $\alpha=\frac{x-(\overline{\theta}^\infty-\varepsilon)}{2\varepsilon}$ to change variables in the integration, equation (A-6) can be written as:

$$\frac{2\varepsilon\int_0^1 v(\overline{\theta}^{\infty}+2\varepsilon\alpha-\varepsilon,\alpha)f(\overline{\theta}^{\infty}+2\varepsilon\alpha-\varepsilon)d\alpha}{F\left(\overline{\theta}^{\infty}+\varepsilon\right)-F\left(\overline{\theta}^{\infty}-\varepsilon\right)} \tag{A-7}$$

$$=\frac{\int_0^1 v(\overline{\theta}^{\infty}+2\varepsilon\alpha-\varepsilon,\alpha)f(\overline{\theta}^{\infty}+2\varepsilon\alpha-\varepsilon)d\alpha}{\frac{F(\overline{\theta}^{\infty}+\varepsilon)-F(\overline{\theta}^{\infty}-\varepsilon)}{2\varepsilon}}=0.$$

At the limit as $\varepsilon \longrightarrow 0$, this equation becomes $\int_0^1 v(\overline{\theta}^{\infty},\alpha)d\alpha = 0$ (by L'Hôpital's rule, the denominator approaches $f\left(\overline{\theta}^{\infty}\right)$ as $\varepsilon \longrightarrow 0$). Similarly, note that $\underline{\theta}^{\infty}$ is defined by the equation $h(\underline{\theta}^{\infty},\alpha(x,\underline{\theta}^{\infty}))=0$, which at the limit as $\varepsilon \longrightarrow 0$, can be written as $\int_0^1 v(\underline{\theta}^{\infty},\alpha)d\alpha = 0$. Now, assume by way of negation that $\overline{\theta}^{\infty} > \underline{\theta}^{\infty}$. Since $v(x,\alpha)$ is weakly increasing in x and strictly increasing in x when $v(x,\alpha)\geq 0$, it follows that $\int_0^1 v(\overline{\theta}^{\infty},\alpha)d\alpha > \int_0^1 v(\underline{\theta}^{\infty},\alpha)d\alpha$ (the strict inequality follows because the equation $\int_0^1 v(\overline{\theta}^{\infty},\alpha)d\alpha = 0$ implies that $v(\overline{\theta}^{\infty},\alpha) > 0$ for at least some values of α). This inequality contradicts the fact that $\int_0^1 v(\overline{\theta}^{\infty},\alpha)d\alpha = 0$ and $\int_0^1 v(\underline{\theta}^{\infty},\alpha)d\alpha = 0$.

Using the notation $\overline{\theta}^{*} \equiv \overline{\theta}^{\infty} = \underline{\theta}^{\infty}$, we proved that at the limit as $\varepsilon \longrightarrow 0$, there exists a unique threshold signal, $\overline{\theta}^{*}$, such that all speculators will attack the upper bound of the band if and only if they observe signals above $\overline{\theta}^{*}$.

(ii) We now characterize $\overline{\theta}^{*}$. The characterization of $\underline{\theta}^{*}$ is then completely analogous. Suppose that the exchange rate is at the upper bound of the band and suppose that absent intervention, the rate of change in the exchange rate is $x < \overline{\theta}^{*} - \varepsilon$. Recalling that the signals that speculators observe are drawn from the interval $[x-\varepsilon, x+\varepsilon]$, it is clear that the highest signal that a speculator can observe in this case is less than $\overline{\theta}^{*}$. Hence, no speculator will attack the band so $\alpha = 0$. On the other hand, if $x > \overline{\theta}^{*} + \varepsilon$, then the lowest signal that a speculator can observe is above $\overline{\theta}^{*}$. Hence, all speculators will attack the band and $\alpha = 1$. In intermediate cases where $\overline{\theta}^{*} - \varepsilon \leq x \leq \overline{\theta}^{*} + \varepsilon$, some speculators will observe signals above $\overline{\theta}^{*}$ and will attack the band while others will observe signals below $\overline{\theta}^{*}$ and will not attack the band. Given that ε_i is distributed uniformly on the interval $[-\varepsilon, \varepsilon]$, the density of speculators who observe signals above $\overline{\theta}^{*}$ and attack the band is $\frac{x-(\overline{\theta}^{*}-\varepsilon)}{2\varepsilon}$. In sum, given x and given $\overline{\theta}^{*}$, the fraction of speculators who choose to attack the upper bound of the band is given by:

$$\alpha(x,\overline{\theta}^{*}) = \begin{cases} 0, & x < \overline{\theta}^{*} - \varepsilon, \\ \frac{x-(\overline{\theta}^{*}-\varepsilon)}{2\varepsilon}, & \overline{\theta}^{*} - \varepsilon \leq x \leq \overline{\theta}^{*} + \varepsilon, \\ 1, & x > \overline{\theta}^{*} + \varepsilon. \end{cases} \tag{A-8}$$

Given $\alpha(x,\overline{\theta}^{*})$, the cost that the policymaker incurs when defending the upper bound of the band against a speculative attack is $C(x,\alpha(x,\overline{\theta}^{*}))$, where $C(x,\cdot)$ is given by equation (2.1). Since the policymaker incurs a future credibility loss, δ, if a realignment takes place, it follows that his optimal policy is to defend the band so long as $C(x, \alpha(x, \overline{\theta}^*)) \leq \delta$, and exit it if $C(x, \alpha(x, \overline{\theta}^*)) > \delta$. Since $\alpha(x, \overline{\theta}^*)$ is weakly increasing in x, a realignment will occur if and only if x is above some threshold level $\overline{x}(\overline{\theta}^*)$. Note that in a perfect Bayesian equilibrium, $\overline{x}(\overline{\theta}^*)$ cannot be below $\overline{\theta}^* - \varepsilon$, because then, the number of speculators that attack the band at $x = \overline{x}(\overline{\theta}^*)$ is zero, and the policymaker should strictly prefer not to exit the band (unless $\overline{x}(\overline{\theta}^*) \geq \overline{\pi} + \delta$, but this means that $\overline{\theta}^*$ is above $\overline{\pi} + \delta + \varepsilon$, which is a contradiction to the fact that speculators have a dominant strategy to attack the band when they observe signals above $\overline{\pi} + \delta + \varepsilon$). Similarly, one can show that $\overline{x}(\overline{\theta}^*)$ cannot be above $\overline{\theta}^* + \varepsilon$. Then, using equations (3.1) and (A-8), we get that when the exchange rate reaches the upper bound of the band, a realignment takes place if and only if

$$x > \overline{x}(\overline{\theta}^*) \equiv \frac{\varepsilon(2\overline{\pi} + 2\delta - 1) + \overline{\theta}^*}{2\varepsilon + 1}. \tag{A-9}$$

Next, consider the decision problem that speculator i faces after observing the signal θ_i. Given that θ_i is drawn from the interval $[x - \varepsilon, x + \varepsilon]$, the speculator realizes that x is distributed on the interval $[\theta_i - \varepsilon, \theta_i + \varepsilon]$, and its conditional density is $f(x \mid \theta_i)$ as given by equation (2.4). But, since the speculator anticipates that the policymaker will defend the band whenever $x < \overline{x}(\overline{\theta}^*)$, it follows that he expects a net payoff of $(x - \overline{\pi})e_{-1} - t$ if $x > \overline{x}(\overline{\theta}^*)$ and $-t$ if $x < \overline{x}(\overline{\theta}^*)$. Part (i) of the lemma implies that, in equilibrium, speculators attack the band if and only if they observe signals above $\overline{\theta}^*$. A speculator that observes exactly $\overline{\theta}^*$ is indifferent between attacking the band and not attacking the band. Using this indifference condition, we get the following equation:

$$\int_{\overline{x}(\overline{\theta}^*)}^{\overline{\theta}^* + \varepsilon} (x - \overline{\pi}) e_{-1} f(x \mid \overline{\theta}^*) dx - t = 0. \tag{A-10}$$

Substituting the expression for $f(x \mid \theta_i)$ from equation (2.4) into equation (A-10) and letting $\varepsilon \to 0$, we get:[28]

$$\lim_{\varepsilon \to 0} \frac{\int_{\overline{x}(\overline{\theta}^*)}^{\overline{\theta}^* + \varepsilon} (x - \overline{\pi}) e_{-1} f(x) dx}{F\left(\overline{\theta}^* + \varepsilon\right) - F\left(\overline{\theta}^* - \varepsilon\right)} = t. \tag{A-11}$$

Substituting the expression for $\overline{x}(\overline{\theta}^*)$ from equation (A-9), using L'Hôpital's rule, and recalling from equation (A-9) that $\overline{x}(\overline{\theta}^*) \to \overline{\theta}^*$ as $\varepsilon \to 0$, we obtain:

$$(\overline{\theta}^* - \overline{\pi}) \left[1 - \delta + (\overline{\theta}^* - \overline{\pi})\right] = \frac{t}{e_{-1}}. \tag{A-12}$$

Solving this equation for $\overline{\theta}^*$ yields the expression in the statement of the proposition.

[28] Note that when ε tends to zero both numerator and denominator in equation (3.5) go to zero.

(iii): Suppose that $\varepsilon \to 0$. Then, equation (A-9) shows that $\overline{x}(\overline{\theta}^*) \to \overline{\theta}^*$ and equation (A-5) shows that $\alpha = 0$ if $x \leq \overline{\theta}^*$ and $\alpha = 1$ if $x > \overline{\theta}^*$. Applying the same logic to the lower bound of the band, it follows that $\underline{x}(\overline{\theta}^*) \to \underline{\theta}^*$ and $\alpha = 0$ if $x \geq \underline{\theta}^*$ and $\alpha = 1$ if $x < \underline{\theta}^*$. **Q.E.D**.

Sufficient conditions for the policymakers' problem to be globally concave in $\underline{\pi} \leq x \leq \pi$: Using equation (3.4), we get

$$\begin{aligned} \frac{\partial^2 V}{\partial \underline{\pi}^2} &= -\left[r(A-1)+\delta\right] f'(\underline{\pi}-r) + (A-1)\left[f(\underline{\pi}) - f(\underline{\pi}-r)\right] \\ &= -\delta f'(\underline{\pi}-r) - r(A-1)\left[f'(\underline{\pi}-r) - \frac{f(\underline{\pi}) - f(\underline{\pi}-r)}{r}\right]. \end{aligned} \tag{A-13}$$

By Assumption 1, the first term on the second line of (A-13) is negative. Now, if $A > 1$ and $f"(\cdot) \leq 0$, the second term on the second line of (A-13) is nonpositive so $\frac{\partial^2 V}{\partial \underline{\pi}^2} < 0$, implying that V is concave in $\underline{\pi}$.

Likewise, using equation (3.5), we get

$$\begin{aligned} \frac{\partial^2 V}{\partial \overline{\pi}^2} &= \left[r(A-1)+\delta\right] f'(\overline{\pi}+r) - (A-1)\left[f(\overline{\pi}+r) - f(\overline{\pi})\right] \\ &= \delta f'(\overline{\pi}+r) + r(A-1)\left[f'(\overline{\pi}+r) - \frac{f(\overline{\pi}+r) - f(\overline{\pi})}{r}\right]. \end{aligned} \tag{A-14}$$

By Assumption 1, the first term on the second line of (A-14) is negative. Now, if $A > 1$ and $f"(\cdot) \leq 0$, the second term on the second line of (A-14) is nonpositive so $\frac{\partial^2 V}{\partial \overline{\pi}^2} < 0$, implying that V is concave in $\overline{\pi}$. **Q.E.D**.

Proof of Proposition 2: (i) By Assumption 1, $f(x) > f(\underline{\pi} - r)$ for all $x \in [\underline{\pi} - r, \underline{\pi}]$. Hence, if $A \leq 1$, $\frac{\partial V}{\partial \underline{\pi}} < 0$ for all $\underline{\pi} < 0$, implying that the policymaker will push $\underline{\pi}$ all the way to $-\infty$. Likewise, by Assumption 1, $f(x) > f(\overline{\pi} + r)$ for all $x \in [\overline{\pi}, \overline{\pi} + r]$; if $A \leq 1$, then $\frac{\partial V}{\partial \overline{\pi}} > 0$ for all $\overline{\pi} > 0$, implying that the policymaker will push $\overline{\pi}$ all the way to ∞.

(ii) To establish that $\underline{\pi} < 0$, it is sufficient to show that evaluated at $\underline{\pi} = 0$, $\frac{\partial V}{\partial \underline{\pi}} < 0$ (the policymaker will not push $\underline{\pi}$ all the way up to 0). Using equation (3.4) we obtain that

$$\begin{aligned} \left.\frac{\partial V}{\partial \underline{\pi}}\right|_{\underline{\pi}=0} &= -\delta f(-r) + (A-1)\int_{-r}^{0}\left[f(x) - f(-r)\right]dx \\ &= -\int_{-r}^{0}\left[f(x) - f(-r)\right]dx\left[\frac{\delta}{\int_{-r}^{0}\left[\frac{f(x)}{f(-r)} - 1\right]dx} + 1 - A\right]. \end{aligned} \tag{A-15}$$

By Assumption 1, the integral term outside the square brackets on the second line of equation (A-15) is negative. If $A < \underline{A}(-r)$ then the term in square brackets is positive, so it is optimal to set $\underline{\pi} < 0$. To show that $\overline{\pi} > 0$, it is sufficient to show that evaluated at $\overline{\pi} = 0$, $\frac{\partial V}{\partial \overline{\pi}} > 0$ (the policymaker will increases $\overline{\pi}$ above 0). Using equation (3.5) we obtain that,

$$\left.\frac{\partial V}{\partial \overline{\pi}}\right|_{\overline{\pi}=0} = \delta f(r) - (A-1)\int_0^r [f(r) - f(x)]\,dx \tag{A-16}$$
$$= \int_0^r [f(r) - f(x)]\,dx \left[\frac{\delta}{\int_0^r \left[\frac{f(x)}{f(r)} - 1\right] dx} + 1 - A\right].$$

By Assumption 1, the integral term outside the square brackets on the second line of equation (A-16) is positive. If $A < \overline{A}(r)$ then the square bracketed term is positive. Hence, it is optimal to set $\overline{\pi} > 0$.

(iii) If V is concave in $\underline{\pi}$, then a sufficient condition for $\underline{\pi} = 0$ is that,evaluated at $\underline{\pi} = 0$, $\frac{\partial V}{\partial \underline{\pi}} \geq 0$ (the policymaker would like to push $\underline{\pi}$ all the way up to 0). From part (ii) of the proposition it is obvious that this occurs when $A > \underline{A}(-r)$. Likewise, if V is concave in $\overline{\pi}$, then a sufficient condition for $\overline{\pi} = 0$ is that, evaluated at $\overline{\pi} = 0$, $\frac{\partial V}{\partial \overline{\pi}} \leq 0$ (the policymaker would not like to increase $\overline{\pi}$ above 0). From part (ii) of the proposition it is obvious that this is the case when $A > \overline{A}(r)$.

(iv) If $f(x)$ is symmetric around 0, then for $0 < a < b$,

$$f(x) = f(-x), \quad \textit{and} \quad \int_{-b}^{-a} f(x)dx = \int_a^b f(x)dx. \tag{A-17}$$

Using these properties, equation (3.4) can be written as:

$$\delta f(-\underline{\pi} + r) - (A-1)\int_{-\underline{\pi}}^{-\underline{\pi}+r} [f(x) - f(-\underline{\pi} + r)]\,dx = 0. \tag{A-18}$$

Replacing $\underline{\pi}$ with $-\bar{\pi}$ in the last expression we obtain equation (3.14), implying that $\underline{\pi} = -\bar{\pi}$. **Q.E.D.**

Proof of Proposition 3: **(i)** Proposition 1 implies that r decreases when t decreases. Straightforward differentiation of equations (3.4) and (3.5) and use of Assumption 1 show that $\underline{\pi}$ decreases and $\bar{\pi}$ increases when r decreases. Hence a decrease in t leads to a decrease in $\underline{\pi}$ and to an increase in $\bar{\pi}$.

By Lemma 1, the probability of a speculative attack is $P = F(\underline{\pi} - r) + (1 - F(\overline{\pi} + r))$. Differentiating this expression with respect to t yields:

$$\frac{\partial P}{\partial t} = f(\underline{\pi} - r)\left[\frac{\partial \underline{\pi}}{\partial r}\frac{\partial r}{\partial t} - \frac{\partial r}{\partial t}\right] - f(\overline{\pi} + r)\left[\frac{\partial \overline{\pi}}{\partial r}\frac{\partial r}{\partial t} + \frac{\partial r}{\partial t}\right] \tag{A-19}$$
$$= \left[f(\underline{\pi} - r)\left[\frac{\partial \underline{\pi}}{\partial r} - 1\right] - f(\overline{\pi} + r)\left[\frac{\partial \overline{\pi}}{\partial r} + 1\right]\right]\frac{\partial r}{\partial t}.$$

By Proposition 1, $\frac{\partial r}{\partial t} > 0$. Hence it is sufficient to establish that $\frac{\partial \underline{\pi}}{\partial r} > 1$ and $\frac{\partial \overline{\pi}}{\partial r} < -1$. Using equation (3.4), it follows that

$$\frac{\partial \underline{\pi}}{\partial r} = \frac{\frac{\partial^2 V}{\partial r \partial \underline{\pi}}}{\frac{\partial^2 V}{\partial \underline{\pi}^2}} = \frac{-\left((A-1)\,r+\delta\right) f'(\underline{\pi}-r)}{-\left[r(A-1)+\delta\right] f'(\underline{\pi}-r) + (A-1)\left[f(\underline{\pi}) - f(\underline{\pi}-r)\right]}, \tag{A-20}$$

which exceeds 1 because of Assumption 1 and because $A > 1$. Likewise, using equation (3.5), it follows that,

$$\frac{\partial \overline{\pi}}{\partial r} = \frac{\frac{\partial^2 V}{\partial r \partial \overline{\pi}}}{\frac{\partial^2 V}{\partial \overline{\pi}^2}} = \frac{-\left((A-1)\,r+\delta\right) f'\left(\overline{\pi}+r\right)}{-\left((A-1)\,r+\delta\right) f'\left(\overline{\pi}+r\right) + (A-1)\left(f\left(\overline{\pi}+r\right) - f\left(\overline{\pi}\right)\right)}, \tag{A-21}$$

which is less than -1 due to Assumption 1 and because $A > 1$.

(ii) $\underline{A}(-r)$ and $\overline{A}(r)$ are given respectively by equations (3.6) and (3.7). Differentiating these expressions with respect to t yields:

$$\frac{\partial \underline{A}(-r)}{\partial t} = -\frac{\delta f'(-r) \int_{-r}^{0} \frac{f(x)}{f(-r)^2} dx}{\left(\int_{-r}^{0} \left[\frac{f(x)}{f(-r)} - 1\right] dx\right)^2} \frac{\partial r}{\partial t} < 0,$$

and

$$\frac{\partial \overline{A}(r)}{\partial t} = \frac{\delta f'(r) \int_{0}^{r} \frac{f(x)}{f(r)^2} dx}{\left(\int_{0}^{r} \left[1 - \frac{f(x)}{f(r)}\right] dx\right)^2} \frac{\partial r}{\partial t} < 0.$$

The signs of those expressions follow from the fact that, by Proposition 1, $\frac{\partial r}{\partial t} > 0$ and since by Assumption 1, $f'(r) < 0$ and $f'(-r) > 0$. Part (iii) of Proposition 2 implies that the policymaker prefers to set a peg when $A > Max\{\underline{A}(r), \overline{A}(r)\}$. Since both $\underline{A}(-r)$ and $\overline{A}(r)$ fall with t, this condition is more likely to hold when t is larger.

(iii) Using equation (3.3) and the envelope theorem it follows that:

$$\frac{\partial V}{\partial t} = \left(r\left(A-1\right)+\delta\right)\left[f(\underline{\pi}-r) + f(\overline{\pi}+r)\right] \frac{\partial r}{\partial t} > 0, \tag{A-22}$$

where the inequality follows from the restriction $A > 1$ and because by Proposition 1, $\frac{\partial r}{\partial t} > 0$. **Q.E.D**

Proof of Proposition 4: **(i)** The proof follows by straightforward differentiation of equations (3.4) and (3.5) and by using Assumption 1.

(ii) By Lemma, the probability of a speculative attack is $P = F(\underline{\pi} - r) + (1 - F(\overline{\pi} + r))$. Straightforward differentiation of this expression with respect to A along with part (i) of the proposition establish the result. **Q.E.D.**

Proof of Proposition 5: Let $\underline{\pi}^f$ and $\bar{\pi}^f$ be the solutions to the policymaker's maximization problem when the density function is $f(x)$ and let $\underline{\pi}^g$ and $\bar{\pi}^g$ be the corresponding solutions when the density function is $g(x)$. $\underline{\pi}^g$ is defined by equation (3.4) with $g(x)$ replacing $f(x)$. Now, let's evaluate $\frac{\partial V}{\partial \underline{\pi}}$ when the density is $g(x)$ at $\underline{\pi}^f$:

$$\begin{aligned} \left.\frac{\partial V}{\partial \underline{\pi}}\right|_{\underline{\pi}=\underline{\pi}^f} &= -\delta g(\underline{\pi}^f - r) + (A-1)\int_{\underline{\pi}^f - r}^{\underline{\pi}^f} \left[g(x) - g(\underline{\pi}^f - r)\right] dx \\ &= -\delta f(\underline{\pi}^f - r) + (A-1)\int_{\underline{\pi}^f - r}^{\underline{\pi}^f} \left[g(x) - f(\underline{\pi}^f - r)\right] dx \qquad \text{(A-23)} \\ &< -\delta f(\underline{\pi}^f - r) + (A-1)\int_{\underline{\pi}^f - r}^{\underline{\pi}^f} \left[f(x) - f(\underline{\pi}^f - r)\right] dx = 0, \end{aligned}$$

where the first equality follows because by assumption, $g(\underline{\pi}^f - r) = f(\underline{\pi}^f - r)$. The inequality follows because $f(x)$ lies above $g(x)$ whenever $x > \underline{\pi}^f - r$, and the second equality follows from equation (3.4). Since $\left.\frac{\partial V}{\partial \underline{\pi}}\right|_{\underline{\pi}=\underline{\pi}^f} < 0$, it follows that $\underline{\pi}^f > \underline{\pi}^g$. The proof that $\bar{\pi}^f < \bar{\pi}^g$ is analogous. Hence, $\underline{\pi}^g < \underline{\pi}^f < \bar{\pi}^f < \bar{\pi}^g$, so the band becomes wider under $g(x)$. **Q.E.D.**

Proof of Lemma 2: (i) Suppose the exchange rate reaches the upper bound of the band. Given x, a dependable policymaker will defend the upper bound of the band if and only if $\alpha < \alpha^*(x)$, where $\alpha^*(x)$ is given by equation (A-1). Hence, the net payoff from attacking the upper bound of the band is:

$$v^{\beta}(x,\alpha) = \begin{cases} (x - \overline{\pi})\, e_{-1} - t, & if \quad \alpha \geq \alpha^*(x), \\ (1-\beta)(x - \overline{\pi})\, e_{-1} - t, & if \quad \alpha < \alpha^*(x). \end{cases}$$

Since $v^{\beta}(x,\alpha)$ has the same properties as $v(x,\alpha)$ in equation (A-2), the equilibrium analysis here is exactly as in the proof of part (i) of Lemma 1. Hence, once again we have a unique equilibrium in which speculators attack the upper bound of the band if and only if they observe a signal above a unique threshold, $\overline{\theta}^*_{\beta}$.

We turn next to a characterization of the behavior of the dependable policymaker in equilibrium. Since in equilibrium, $\alpha(x)$ is increasing in x ($\alpha(x)$ is given by (A-8), where $\overline{\theta}^*_{\beta}$ replaces $\overline{\theta}^*$), and since $C(x, \alpha(x))$ is increasing in both x and $\alpha(x)$, then the dependable policymaker will exit the band if and only if x is above some threshold level, $\overline{x}_{\beta}(\overline{\theta}^*_{\beta})$.

In order to establish that $\overline{x}_{\beta}(\overline{\theta}^*_{\beta}) \to \overline{\theta}^*_{\beta}$ as $\varepsilon \to 0$, we now show that $\overline{x}_{\beta}(\overline{\theta}^*_{\beta})$ must be in the interval $\left[\overline{\theta}^*_{\beta} - \varepsilon, \overline{\theta}^*_{\beta} + \varepsilon\right]$. In order to see this, suppose by way of negation that $\overline{x}_{\beta}(\overline{\theta}^*_{\beta}) > \overline{\theta}^*_{\beta} + \varepsilon$. Then, speculators who observe $\overline{\theta}^*_{\beta}$ know that a dependable policymaker will defend the band. Thus, the payoff they expect to get from attacking the band is lower than $(1-\beta)\left(\overline{\theta}^*_{\beta} + \varepsilon - \overline{\pi}\right) e_{-1} - t$. By equilibrium conditions and continuity, we know that speculators who observe $\overline{\theta}^*_\beta$ must be indifferent between attacking the band and not attacking it. This means that $(1-\beta)\left(\overline{\theta}^*_\beta+\varepsilon-\overline{\pi}\right)e_{-1}-t>0$. However, using Assumption 3, this condition will hold only if $\overline{\theta}^*_\beta+\varepsilon>\overline{\pi}+\delta$. Since by assumption, $\overline{x}_\beta(\overline{\theta}^*_\beta)>\overline{\theta}^*_\beta+\varepsilon$, this implies in turn that $\overline{x}_\beta(\overline{\theta}^*_\beta)>\overline{\pi}+\delta$, thereby contradicting the fact that a dependable policymaker always exits the band when $x>\overline{\pi}+\delta$. Thus, $\overline{x}_\beta(\overline{\theta}^*_\beta)$ cannot be above $\overline{\theta}^*_\beta+\varepsilon$. Next, suppose by way of negation that $\overline{x}_\beta(\overline{\theta}^*_\beta)<\overline{\theta}^*_\beta-\varepsilon$. Then, at $\overline{x}_\beta(\overline{\theta}^*_\beta)$, a dependable policymaker knows that no speculator attacks the band. By equilibrium conditions and continuity, at $\overline{x}_\beta(\overline{\theta}^*_\beta)$, a dependable policymaker must be indifferent between existing the band and maintaining it, that is, $\overline{x}_\beta(\overline{\theta}^*_\beta)=\overline{\pi}+\delta$. Since by assumption, $\overline{x}_\beta(\overline{\theta}^*_\beta)<\overline{\theta}^*_\beta-\varepsilon$, this means that $\overline{\theta}^*_\beta>\overline{\pi}+\delta+\varepsilon$, which contradicts the fact that speculators have a dominant strategy to attack the band when they observe signals above $\overline{\pi}+\delta+\varepsilon$. Thus, $\overline{x}_\beta(\overline{\theta}^*_\beta)$ cannot be below $\overline{\theta}^*_\beta-\varepsilon$

Given the fact that $\overline{x}_\beta(\overline{\theta}^*_\beta)$ is in the interval $\left[\overline{\theta}^*_\beta-\varepsilon,\overline{\theta}^*_\beta+\varepsilon\right]$ and using (A-8), it follows that:

$$\overline{x}_\beta(\overline{\theta}^*_\beta)=\frac{\varepsilon(2\overline{\pi}+2\delta-1)+\overline{\theta}^*_\beta}{2\varepsilon+1}.$$

Since a speculator that observes $\overline{\theta}^*_\beta$ is indifferent between attacking the band and not attacking it, the equation that defines $\overline{\theta}^*_\beta$ is given by:

$$\beta\int_{\overline{x}_\beta(\overline{\theta}^*_\beta)}^{\overline{\theta}^*_\beta+\varepsilon}(x-\overline{\pi})e_{-1}f(x\mid\overline{\theta}^*_\beta)dx+(1-\beta)\int_{\overline{\theta}^*_\beta-\varepsilon}^{\overline{\theta}^*_\beta+\varepsilon}(x-\overline{\pi})e_{-1}f(x\mid\overline{\theta}^*_\beta)dx=t, \tag{A-24}$$

where $f(x\mid\overline{\theta}^*_\beta)$ is defined by equation (2.4). This equation coincides with equation (A-8) if $\beta=1$. Substituting from equation (2.4) for $f(x\mid\overline{\theta}^*_\beta)$ into (A-24), and taking the limit as $\varepsilon\to0$, yields:

$$\lim_{\varepsilon\to0}\frac{\beta\int_{\overline{x}_\beta(\overline{\theta}^*_\beta)}^{\overline{\theta}^*_\beta+\varepsilon}(x-\overline{\pi})e_{-1}f(x)dx+(1-\beta)\int_{\overline{\theta}^*_\beta-\varepsilon}^{\overline{\theta}^*_\beta+\varepsilon}(x-\overline{\pi})e_{-1}f(x)dx}{F\left(\overline{\theta}^*_\beta+\varepsilon)-F(\overline{\theta}^*_\beta-\varepsilon\right)}=t. \tag{A-25}$$

Using L'Hôpital's rule, and the expression for $\overline{x}_\beta(\overline{\theta}^*_\beta)$, and recalling that $\overline{x}_\beta(\overline{\theta}^*_\beta)\to\overline{\theta}^*_\beta$ as $\varepsilon\to0$, we obtain:

$$(\overline{\theta}^*_\beta-\overline{\pi})(1-\beta\delta+\beta(\overline{\theta}^*_\beta-\overline{\pi}))e_{-1}=t. \tag{A-26}$$

Solving this equation for $\overline{\theta}^*_\beta$ reveals that $\overline{\theta}^*_\beta=\overline{\pi}+r^\beta$, where r^β is defined by equation (4.1). Using similar arguments, it follows that $\underline{\theta}^*_\beta=\underline{\pi}-r^\beta$.

(ii) Differentiating r^β with respect to β and we obtain:

$$\frac{\partial r^\beta}{\partial\beta}=\frac{1}{2\beta^2}\left[1-\frac{\frac{t}{e_{-1}}-\frac{\delta-\frac{1}{\beta}}{2}}{\sqrt{\frac{t}{\beta e_{-1}}+\frac{(\delta-\frac{1}{\beta})^2}{4}}}\right]. \tag{A-27}$$

This derivative is positive if and only if the expression inside the brackets is positive. This is the case, in turn, if and only if

$$\frac{t}{\beta e_{-1}} + \frac{(\delta - \frac{1}{\beta})^2}{4} > \left[\frac{t}{e_{-1}} - \frac{\delta - \frac{1}{\beta}}{2}\right]^2. \tag{A-28}$$

Further rearrangement of the last inequality shows that it is equivalent to Assumption 2. Hence, r^β increases with β. **Q.E.D**.

Proof of Proposition 6: The first order conditions for an interior solution for the problem of a dependable policymaker are:

$$\begin{aligned}\frac{\partial V^\beta}{\partial \underline{\pi}} &= -\left[r^\beta(A\beta-1)+\delta\right] f(\underline{\pi}-r^\beta) + (A\beta-1)\int_{\underline{\pi}-r^\beta}^{\underline{\pi}} f(x)dx \qquad \text{(A-29)}\\ &= A\beta\int_{\underline{\pi}-r^\beta}^{\underline{\pi}}\left[f(x)-f(\underline{\pi}-r^\beta)\right]dx - \left[\int_{\underline{\pi}-r^\beta}^{\underline{\pi}}\left[f(x)-f(\underline{\pi}-r^\beta)\right]dx + \delta f(\underline{\pi}-r^\beta)\right] = 0,\end{aligned}$$

and,

$$\begin{aligned}\frac{\partial V^\beta}{\partial \overline{\pi}} &= \left[r^\beta(A\beta-1)+\delta\right] f(\overline{\pi}+r^\beta) - (A\beta-1)\int_{\overline{\pi}}^{\overline{\pi}+r^\beta} f(x)dx \qquad \text{(A-30)}\\ &= -A\beta\int_{\overline{\pi}}^{\overline{\pi}+r^\beta}\left[f(x)-f(\overline{\pi}+r^\beta)\right]dx + \left[\int_{\overline{\pi}}^{\overline{\pi}+r^\beta}\left[f(x)-f(\overline{\pi}+r^\beta)\right]dx + \delta f(\overline{\pi}+r^\beta)\right] = 0.\end{aligned}$$

As in the case where $\beta < 1$, it can be shown that $f''(x) \leq 0$ and $A\beta > 1$, along with Assumption 1, are sufficient for V^β to be globally concave in $\underline{\pi}$ and $\overline{\pi}$ so equations (A-29) and (A-30) are sufficient for a unique maximum.

(i)-(iii) The proofs follow the corresponding proofs in Proposition 2 with r replaced by r^β.

(iv) Differentiating $\frac{\partial V^\beta}{\partial \overline{\pi}}$ with respect to β and $\overline{\pi}$, and using the implicit function theorem,yields:

$$\frac{\partial \overline{\pi}}{\partial \beta} = -\frac{A\int_{\overline{\pi}}^{\overline{\pi}+r^\beta}\left[f(\overline{\pi}+r^\beta)-f(x)\right]dx + \left[r^\beta(A\beta-1)+\delta\right] f'(\overline{\pi}+r^\beta)\frac{\partial r^\beta}{\partial \beta}}{-(A\beta-1)\left[f(\overline{\pi}+r^\beta)-f(\overline{\pi})\right] + \left[r^\beta(A\beta-1)+\delta\right] f'(\overline{\pi}+r^\beta)}. \tag{A-31}$$

Assumption 1 ensures that the integral term in the numerator is negative and it also ensures that $f'(\overline{\pi}+r^\beta) < 0$. Since by Lemma 2, $\frac{\partial r^\beta}{\partial \beta} > 0$, it follows that the numerator is negative. By the second order conditions for maximization, the denominator is negative so $\overline{\pi}$ decreases towards 0. Similarly, it can be shown that as β increases, $\underline{\pi}$ increases towards 0. Hence, an increase in β leads to a tighter band.

Finally, the probability of a speculative attack is now $P^\beta = F(\underline{\pi} - r^\beta) + (1 - F(\overline{\pi} + r^\beta))$. Differentiating this expression with respect to β yields:

$$\frac{\partial P^\beta}{\partial \beta} = f(\underline{\pi} - r)\left[\frac{\partial \underline{\pi}}{\partial \beta} - \frac{\partial r^\beta}{\partial \beta}\right] - f(\overline{\pi} + r)\left[\frac{\partial \overline{\pi}}{\partial \beta} + \frac{\partial r^\beta}{\partial \beta}\right].$$

To determine the sign of this expression, note that using (A-30) we obtain

$$\frac{\partial \overline{\pi}}{\partial \beta} + \frac{\partial r^\beta}{\partial \beta} = -\frac{A\int_{\overline{\pi}}^{\overline{\pi}+r^\beta}\left[f(\overline{\pi}+r^\beta) - f(x)\right]dx + (A\beta - 1)\left[f(\overline{\pi}+r^\beta) - f(\overline{\pi})\right]\frac{\partial r^\beta}{\partial \beta}}{-(A\beta - 1)\left[f(\overline{\pi}+r^\beta) - f(\overline{\pi})\right] + \left[r^\beta(A\beta - 1) + \delta\right]f'(\overline{\pi}+r^\beta)}. \tag{A-32}$$

Assumption 1 ensures that the integral term in the numerator as well as $f(\overline{\pi} + r^\beta) - f(\overline{\pi})$ are both negative. Since by assumption, $A\beta > 1$ and since by Lemma 2, $\frac{\partial r^\beta}{\partial \beta} > 0$, it follows that the numerator is negative. The denominator is also negative by the second order conditions for maximization. Hence, $\frac{\partial \overline{\pi}}{\partial \beta} + \frac{\partial r^\beta}{\partial \beta} < 0$. Similar calculations establish that $\frac{\partial \underline{\pi}}{\partial \beta} - \frac{\partial r^\beta}{\partial \beta} > 0$. Hence, $\frac{\partial P^\beta}{\partial \beta} > 0$, implying that as the policymaker's reputation improves, there is a greater likelihood of speculative attacks. **Q.E.D.**

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