Euro-Dollar Real Exchange Rate Dynamics in an Estimated Two-Country Model: What is Important and What is Not

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Abstract

Central puzzles in international macroeconomics are why fluctuations of the real exchange rate are so volatile with respect to other macroeconomic variables, and the contradiction of efficient risk-sharing. Several theoretical contributions have evaluated alternative forms of pricing under nominal rigidities along with different asset markets structures to explain real exchange dynamics. In this paper, we use a Bayesian approach to estimate a standard two-country New Open Economy Macroeconomics (NOEM) using data for the United States and the Euro Area, and perform model comparisons to study the importance of departing from the law of one price and complete markets assumptions. Our results can be summarized as follows. First, we find that the baseline model does a good job in explaining real exchange rate volatility, but at the cost of implying too high volatility in output and consumption. Second, the introduction of incomplete markets allows the model to better match the volatilities of all real variables. Third, introducing sticky prices in local currency pricing (LCP) improves the fit of the baseline model, but not by as much as by introducing incomplete markets. Finally, we show that monetary shocks have played a minor role in explaining the behavior of the real exchange rate, while both demand and technology shocks have been important.

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I. Introduction

Most puzzles in international macroeconomics are related to real exchange rate dynamics. Fluctuations in real exchange rates are very large and persistent, when compared to other real variables. In addition, there is clear evidence of lack of consumption risk-sharing across countries, which is at odds with the assumption of complete markets. In order to replicate these features of the data, the New Open Economy Macroeconomics (NOEM) literature has incorporated either nominal rigidities, alternative structures of assets markets, or both.

The real exchange rate (q_t) between two countries is defined as the ratio of price levels expressed in a common currency.[2] When all the components of the price level, namely domestically produced and imported goods, are sticky, it can be possible to explain some empirical features, like the high correlation between nominal and real exchange rates, and real exchange rate volatility. In the literature, pricing of imports goods are assumed to be governed either by Producer Currency Pricing (PCP), where the law of one price holds and there is perfect pass-through; or Local Currency Pricing (LCP), where the pass-through is zero in the short run.

Moreover, it is well known that under complete markets, the real exchange rate should be equal to the ratio of the marginal utility of consumption across countries, because it reflects the relative price of foreign goods in terms of domestic goods. For example, assuming separable preferences and log utility, for simplicity, the following relationship should hold as an equilibrium condition: $q_t = c_t - c_t^*$, where c_t and c_t^* are the levels of domestic and foreign consumption. This relationship, which implies a correlation of one between the real exchange rate and the ratio of consumption levels in two countries, does not hold for many bilateral relationships in general.[3] For the bilateral euro-U.S. dollar exchange rate in particular, the correlation between these variables (HP-filtered) is -0.17. Hence, models that incorporate complete markets are bound to fail, regardless of the presence of other nominal or real rigidities. A common assumption in the literature is one where agents do not have access to complete markets to insure their wealth against idiosyncratic and country-specific shocks. Another possibility is to introduce preference shocks that affect the marginal utility of consumption, as in Stockman and Tesar (1995).

Following this line of research, a recent paper by Chari, Kehoe and McGrattan (2002, hereafter CKM) attempts to explain the volatility and persistence of the real exchange rate by constructing a model with sticky prices and local currency pricing. Their main finding is that monetary shocks and complete markets, along with a high degree of risk aversion and price stickiness of one year are enough to account for real exchange rate volatility, and to less extent for its persistence. However, their model found it difficult to account for the observed negative

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[2] In linear terms, the real exchange rate is defined as $q_t = s_t + p_t^* - p_t$, where s_t is the nominal exchange in units of domestic currency per unit of foreign currency, p_t^* is the foreign price level, and p_t is the domestic price level.

[3] See Chari, Kehoe and McGrattan (2002).

correlation between real exchange rates and relative consumption across countries, a fact that they labeled the *consumption-real exchange rate anomaly*. In addition, CKM show that the most widely used forms of asset market incompleteness and habit persistence do not eliminate the anomaly.[4]

In this paper, we use a Bayesian approach to estimate and compare two-country NOEM models under different assumptions of imports goods pricing and asset markets structures, thereby testing some of the key implications of CKM. Unlike them, we find that monetary policy shocks have a minor role in explaining real exchange rate volatility, and that both demand and technology shocks have had some importance. Using the Bayes factor to compare between competing alternatives, we find that what turns out to be crucial to explain real exchange rate dynamics and the exchange rate-consumption anomaly is the introduction of incomplete markets with stationary net foreign asset positions. Somewhat surprisingly, we find that in a complete markets set up, the introduction of LCP improves the fit of the model, while when incomplete markets are allowed for, LCP actually lowers the overall fit, implying too high real exchange rate volatility and a lower fit for the correlation between the real exchange rate and the ratio of relative consumptions.

The main contributions of the present paper are on the estimation side. First, we focus on the relationship between relative consumptions and the real exchange rate by introducing data on consumption for the two economic areas in the estimation. Second, while our model is quite rich in shocks (we need nine shocks because we try to explain nine variables), we have left aside uncovered interest-rate parity (UIP)-type shocks, which tend to explain a large fraction of real exchange rate variability. We do so because under complete markets these shocks, at least conceptually, should not be included and also because we want to study more carefully the role of "traditional" shocks (technology, demand, monetary and so on) in explaining real exchange rate fluctuations.[5] Third, we believe this is the first paper to evaluate the merits of the incomplete markets assumption with stationary net foreign assets in a two-country NOEM model. Last, but not least, we perform an in-sample forecast exercise and find that the preferred model does a good job in forecasting compared to the other NOEM models, but is still far away from the performance of a vector autorregression (VAR) model.

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[4] Alternative ways to explain this anomaly typically include models with traded and nontraded goods. Selaive and Tuesta (2003a) and Benigno and Thoenissen (2005) have shown that this anomaly can be successfully addressed by models with incomplete markets and nontraded goods, with the traditional Balassa-Samuelson effect and sector-specific productivity shocks. Similarly, Ghironi and Melitz (2005) rely on aggregate productivity shocks and also find that the Balassa-Samuelson effect help to explain the consumption real exchange rate anomaly. Finally, Corsetti, Dedola and Leduc (2004) have shown that distribution services can help to account for the real exchange rate-consumption correlation by lowering the import demand elasticity.

[5] Our benchmark model, unlike the International Real Business Cycle (IRBC) literature, always includes nominal rigidities, because we want to evaluate the relative importance of monetary shocks in explaining real exchange rate fluctuations.

The literature on estimating NOEM models in the spirit of CKM and Galí and Monacelli (2005) has grown rapidly, with the adoption of the Bayesian methodology to an open economy setting already used in a closed economy environment.[6] For example, Lubik and Schorfheide (2003) estimate small open economy models with data for Australia, New Zealand, Canada and the U.K., focusing on whether the monetary policy rules of those countries have targeted the nominal exchange rate. Justiniano and Preston (2004) also estimate and compare small open economy models with an emphasis on the consequences of introducing imperfect pass-through. Adolfson et al. (2005) estimate a medium-scale (15 variable) small open economy model for the euro area, while Lubik and Schorfheide (2005) and Batini et al. (2005) estimate a small-scale two-country model using U.S. and euro area data.

The rest of the paper is organized as follows. In the next section we outline the baseline model, and we describe the LCP and the incomplete markets extensions. In section III we explain the data and in section IV the econometric strategy. The estimation results can be found in section V. First, we present the parameter estimates of the baseline model. Then, we analyze the parameter estimates of all the extensions along with the second moments implied by each model. We select our preferred model based on the comparison of Bayes factors, and analyze its dynamics by studying the impulse response functions. Finally, we evaluate the importance of shocks through variance decompositions. We also compare the forecasting performance of all the DSGE models with respect to VAR models. In section V we conclude.

II. The Model

In this section we present the stochastic two country New Open Economy Macroeconomics (NOEM) model that we will use to analyze real exchange rate dynamics.[7] We first outline a baseline model with complete markets and where the law of one price holds, in the spirit of Clarida, Galí and Gertler (2002), Benigno and Benigno (2003) and Galí and Monacelli (2005). In order to obtain a better fit, we incorporate the following assumptions: home bias, habit formation in consumption, and staggered price setting a la Calvo (1983) with backward looking indexation. Later, we introduce the two main extensions we are interested in, namely incomplete markets and sticky prices of imported goods in local currency.

We assume that there are two countries, home and foreign, of equal size. Each country produces a continuum of intermediate goods, indexed by $h \in [0,1]$ in the home country and $f \in [0,1]$ in the foreign country. Preferences over these goods are of the Dixit-Stiglitz type, implying that producers operate under monopolistic competition, and all goods are internationally tradable. In order to assist the reader with the notation, in Table 1 we present a list with all the variables of

[6] Examples of closed economy applications of this methodology are Rabanal and Rubio-Ramírez (2005), and Galí and Rabanal (2004) for the United States, and Smets and Wouters (2003) for the Euro Area.

[7] This type of model that has been the *workhorse* the NOEM literature after Obstfeld and Rogoff (1995). Since the model is now fairly standard, we only outline the main features here, and refer the reader to an appendix available upon request for a full version of the model.

the model. The model contains nine shocks: a world technology shock that has a unit root, and country-specific stationary technology, monetary, demand and preference shocks. All stationary shocks are AR(1), except for the monetary shocks that are iid.

Table 1. Variables in the Home and Foreign Countries

	Home		Foreign	
	Quantity	Price	Quantity	Price
Consumption goods:				
Aggregate	C_t	P_t	C_t^*	P_t^*
Imports	$C_{F,t}$	$P_{F,t}$	$C_{H,t}^*$	$P_{H,t}^*$
Domestically Produced	$C_{H,t}$	$P_{H,t}$	$C_{F,t}^*$	$P_{F,t}^*$
Intermediate Goods				
Imports	$c_t(f)$	$p_t(f)$	$c_t^*(h)$	$p_t^*(h)$
Domestically Produced	$c_t(h)$	$p_t(h)$	$c_t^*(f)$	$p_t^*(f)$
Production:				
Aggregate (GDP)	$Y_{H,t}$	$P_{H,t}$	$Y_{F,t}^*$	$P_{F,t}^*$
Intermediate Goods	$y_t(h)$	$p_t(h)$	$y_t^*(f)$	$p_t^*(f)$

	Home		Foreign
Labor Markets:			
Hours worked	N_t		N_t^*
Real Wage	ω_t		ω_t^*
Firms' labor demand	$N_t(h)$		$N_t^*(f)$
Terms of Trade	T_t		T_t^*
Interest Rates	R_t		R_t^*
Bonds	B_t		B_t^*
Real Exchange Rate		Q_t	
Nominal Exchange Rate		S_t	
Shocks			
World Technology		A_t	
Country Technology	X_t		X_t^*
Preference	G_t		G_t^*
Monetary	z_t		z_t^*
Demand	η_t		η_t^*

A. Households

In each country there is a continuum of infinitely lived households in the unit interval, who obtain utility from consuming the final good and disutility from supplying hours of labor. It is assumed that consumers have access to complete markets at the country level and at the world level, which implies that consumer's wealth is insured against country specific and world shocks, and hence all consumers face the same consumption-savings decision.[8]

In the home country, households' lifetime utility function is:

$$E_0 \sum_{t=0}^{\infty} \beta^t G_t [\log(C_t - bC_{t-1}) - \frac{N_t^{1+\gamma}}{1+\gamma}]. \tag{1}$$

E_0 denotes the rational expectations operator using information up to time $t=0$. $\beta \in [0,1]$ is the discount factor. The utility function displays *external* habit formation. $b \in [0,1]$ denotes the importance of the habit stock, which is last period's aggregate consumption. $\gamma>0$ is inverse elasticity of labor supply with respect to the real wage.

Table 2 contains additional variable definitions and functional forms. C_t denotes the consumption of the final good, which is a CES aggregate of consumption bundles of home and foreign goods. The parameter $1-\delta$ is the fraction of home-produced goods in the consumer basket, and denotes the degree of home bias in consumption. Its analogous in the foreign country is $1-\delta^*$. The elasticity of subtitution between domestically produced and imported goods in both countries is θ, while the elasticity of substitution between types of intermediate goods is $\varepsilon>1$.

In our baseline case, we assume that the law of one price holds for each intermediate good. This implies that $P_{H,t} = S_t P^*_{H,t}$, and $P_{F,t} = S_t P^*_{F,t}$. Note, however, that purchasing power parity (a constant real exchange rate) does not necessarily hold because of the presence of home bias in preferences. The home-bias assumption allows to generate real exchange rate dynamics in a model, like this one, with only tradable goods. From previous definitions we can express the real exchange rate as a function of the terms of trade:

$$Q_t = \frac{S_t P_t^*}{P_t} = \left[\frac{\delta^* + (1-\delta^*)T_t^{1-\theta}}{(1-\delta) + \delta T_t^{1-\theta}} \right]^{\frac{1}{1-\theta}} \tag{2}$$

[8] Baxter and Crucini (1993) have used the same assumption in an IRBC model in order to explain the saving-investment correlation.

Table 2: Definitions and Functional Forms

Consumption	$C_t \equiv \left[(1-\delta)^{\frac{1}{\theta}}(C_{H,t})^{\frac{\theta-1}{\theta}} + \delta^{\frac{1}{\theta}}(C_{F,t})^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$ $C_t^* \equiv \left[(\delta^*)^{\frac{1}{\theta}}(C_{H,t}^*)^{\frac{\theta-1}{\theta}} + (1-\delta^*)^{\frac{1}{\theta}}(C_{F,t}^*)^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}$
Consumption components	$C_{H,t} \equiv \left\{\int_0^1 [c_t(h)]^{\frac{\varepsilon-1}{\varepsilon}} dh\right\}^{\frac{\varepsilon}{\varepsilon-1}}$, $C_{H,t}^* \equiv \left\{\int_0^1 [c_t^*(h)]^{\frac{\varepsilon-1}{\varepsilon}} dh\right\}^{\frac{\varepsilon}{\varepsilon-1}}$ $C_{F,t} \equiv \left\{\int_0^1 [c_t(f)]^{\frac{\varepsilon-1}{\varepsilon}} df\right\}^{\frac{\varepsilon}{\varepsilon-1}}$, $C_{F,t}^* \equiv \left\{\int_0^1 [c_t^*(f)]^{\frac{\varepsilon-1}{\varepsilon}} df\right\}^{\frac{\varepsilon}{\varepsilon-1}}$
Consumer Price Indices	$P_t \equiv \left[(1-\delta)(P_{H,t})^{1-\theta} + \delta(P_{F,t})^{1-\theta}\right]^{\frac{1}{1-\theta}}$, $P_t^* \equiv \left[\delta^*(P_{H,t}^*)^{1-\theta} + (1-\delta^*)(P_{F,t}^*)^{1-\theta}\right]^{\frac{1}{1-\theta}}$.
Price subindices	$P_{H,t} \equiv \left\{\int_0^1 [p_t(h)]^{1-\varepsilon} dh\right\}^{\frac{1}{1-\varepsilon}}$, $P_{H,t}^* \equiv \left\{\int_0^1 [p_t^*(h)]^{1-\varepsilon} dh\right\}^{\frac{1}{1-\varepsilon}}$ $P_{F,t} \equiv \left\{\int_0^1 [p_t(f)]^{1-\varepsilon} df\right\}^{\frac{1}{1-\varepsilon}}$, $P_{F,t}^* \equiv \left\{\int_0^1 [p_t^*(f)]^{1-\varepsilon} df\right\}^{\frac{1}{1-\varepsilon}}$
Terms of Trade	$T_t = P_{F,t} / P_{H,t}, T_t^* = P_{H,t}^* / P_{F,t}^*$
Real Exchange Rate	$Q_t = \frac{S_t P_t^*}{P_t}$
Production Functions	$y_t(h) = A_t X_t N_t(h)$, $y_t^*(f) = A_t X_t^* N_t^*(f)$
World Technology Shock	$\log(A_t) = \Gamma + \log(A_{t-1}) + \varepsilon_t^a$
Country Tech. Shocks	$\log(X_t) = \rho_x \log(X_{t-1}) + \varepsilon_t^x$, $\log(X_t^*) = \rho_x^* \log(X_{t-1}^*) + \varepsilon_t^{x^*}$
Preference Shocks	$\log(G_t) = \rho_g \log(G_{t-1}) + \varepsilon_t^g$, $\log(G_t^*) = \rho_g^* \log(G_{t-1}^*) + \varepsilon_t^{g^*}$

B. Asset Market Structure, the Budget Constraint, and the Consumer's Optimizing Conditions

We model complete markets by assuming that households have access to a complete set of state contingent nominal claims which are traded domestically and internationally. We represent the asset structure by assuming a complete set of contingent one-period nominal bonds denominated in home currency.[9] Hence, households in the home country maximize their utility (1) subject to the following budget constraint:

$$C_t = \omega_t N_t + \frac{E_t\{\xi_{t,t+1} B_{t+1}\} - B_t}{P_t} + \int_0^1 \Pi_t(h)dh, \tag{3}$$

where B_{t+1} denotes nominal state-contigent payoffs of the portfolio purchased in domestic currency at t, and $\xi_{t,t+1}$ is the stochastic discount factor.[10] The real wage is deflated by the country's CPI. The last term of the right hand side of the previous expression denotes the profits from the monopolistically competitive intermediate goods producers firms, which are ultimately owned by households in each country.

The first order conditions for labor supply and consumption/savings decisions are as follows:

$$\omega_t = (C_t - bC_{t-1})N_t^{\gamma}, \tag{4}$$

and

$$\beta \frac{G_{t+1}}{G_t} \frac{(C_t - bC_{t-1})}{(C_{t+1} - bC_t)} \frac{P_t}{P_{t+1}} = \xi_{t,t+1}. \tag{5}$$

By taking expectations to the above equation across all possible states, and by using the fact that $E_t\{\xi_{t,t+1}\} = 1/R_t$ we can obtain the traditional Euler equation in consumption.

Moreover combining equation (5) with the intertemporal efficiency condition in the foreign country we obtain that under complete markets the ratio of marginal utilities of the two countries is equal to:

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[9] Given these assumptions, it is not necessary to characterize the current account dynamics in order to determine the equilibrium allocations, and the currency denomination of the bonds is irrelevant.

[10] $\xi_{t,t+1}$ is a price of one unit of nominal consumption of time *t+1*, expressed in units of nominal consumption at *t*, contingent on the state at *t+1* being s_{t+1}, given any state s_t in *t*. The complete market assumptions implies that there exists a unique discount factor with the property that the price in period *t* of the portfolio with random value B_{t+1} is $E_t\{\xi_{t,t+1} B_{t+1}\}$.

$$Q_t = \nu \frac{(C_t - bC_{t-1})}{(C_t^* - b^* C_{t-1}^*)} \frac{G_t^*}{G_t}, \tag{6}$$

where ν is a constant that depends on initial conditions (see CKM, and Galí and Monacelli, 2005). The risk-sharing condition (6) differs with respect to the one in CKM because of the presence of both preference shocks and habit persistence.

C. Intermediate Goods Producers and Pricesetting

In each country, there is a continuum of intermediate goods producers, each producing a type of good that is an imperfect substitute of the others. As shown in Table 2, the production function is linear in the labor input, and has two technology shocks. The first one is a world technology shock, that affects the two countries that same way: it has a unit root, as in Galí and Rabanal (2004) and Ireland (2004), and it implies that real variables in both countries grow at a rate Γ. In addition, there is a country-specific technology shock that evolves as an AR(1) process.

Firms face a modified Calvo (1983)-type restriction when setting their prices. When they receive the Calvo-type signal, which arrives with probability $1-\alpha$ in the home country, firms reoptimize their price. When they do not receive that signal, a fraction τ of intermediate goods producers index their price to last period's inflation rate, and a fraction $1-\tau$ indexes their price to the steady-state inflation rate. This assumption is needed to incorporate trend inflation, as in Yun (1996). The equivalent parameters in the foreign country are $1-\alpha^*$ and τ^*.

Cost minimization by firms implies that the real marginal cost of production is $\omega_t/(A_t X_t)$. Since the real marginal cost depends only on aggregate variables, it is the same for all firms in each country. The overall demand for an intermediate good produced in *h* comes from optimal choices by consumers at home and abroad:

$$D_t(h) = c_t(h) + c_t^*(h) = \left(\frac{p_t(h)}{P_{H,t}}\right)^{-\varepsilon} \left(\frac{P_{H,t}}{P_t}\right)^{-\theta} \left[(1-\delta)C_t + \delta^* C_t^* Q_t^{\theta}\right]$$

Hence, whenever intermediate-goods producers are allowed to reset their price, they maximize the following profit function, which discounts future profits by the probability of not being able to reset prices optimally every period:

$$\underset{p_t(h)}{Max}\, E_t \sum_{k=0}^{\infty} \alpha^k \xi_{t,t+k} \left\{ \left[\frac{p_{t,t+k}(h)}{P_{t+k}} - \frac{\omega_{t+k}}{A_{t+k} X_{t+k}} MC_{t+k} \right] D_{t,t+k}(h) \right\}. \tag{7}$$

where $p_{t,t+k}(h)$ is the price prevailing at *t*+*k* assuming that the firm last reoptimized at time *t*, and whose evolution will depend on whether the firm indexes its price to last period's inflation rate or to the steady-state rate of inflation, $D_{t,t+k}(h)$ the demand associated to that price, and $\xi_{t,t+k}$ is the k periods ahead stochastic discount factor.

The evolution of the aggregate consumption bundle price produced in the home country is:

$$P_{H,t}^{1-\varepsilon} = (1-\alpha)(\hat{P}_{H,t})^{1-\varepsilon} + \alpha\left[P_{H,t-1}\left(\frac{P_{H,t-1}}{P_{H,t-2}}\right)^{\tau}\overline{\Pi}_{H}^{1-\tau}\right]^{1-\varepsilon}. \tag{8}$$

where $\hat{P}_{H,t}$ is the optimal price set by firms in a symmetric equilibrium.

D. Closing the Model

In order to close the model, we impose market clearing conditions for all home and foreign intermediate goods. For each individual good, market clearing requires $y_t(h) = c_t(h) + c_t^*(h)$ for all $h \in [0,1]$. Defining aggregate real GDP as $Y_{H,t} = \left[\int_0^1 p_t(h) y_t(h) dh\right] / P_{H,t}$, the following market clearing condition holds at the home-country level:

$$Y_{H,t} = \left[(1-\delta)C_t + \delta^* C_t^* Q_t^{\theta}\right]\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} + \eta_t \tag{9}$$

The analogous expressions for the foreign country are, $y_t^*(f) = c_t(f) + c_t^*(f)$, for all $f \in [0,1]$ and for aggregate foreign real GDP:

$$Y_{F,t}^* = \left[\delta C_t + (1-\delta^*)C_t^* Q_t^{\theta}\right]\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} + \eta_t^* \tag{10}$$

We also introduce an exogenous demand shock for each country (η_t, η_t^*) that be interpreted as government purchases, and/or trade with third countries that are not part of in the model.

E. Symmetric Equilibrium

Since we have assumed a world-wide technology shock that grows at a rate Γ, output, consumption, real wages, and the level of exogenous demand in the two economies grow at that same rate. In order to render these variables stationary, we divide them by the level of world technology A_t. Real marginal costs, hours, inflation, interest rates, the real exchange rate and the terms of trade are stationary.

F. Dynamics

We obtain the model's dynamics by taking a linear approximation to the steady state values with zero inflation. We impose a symmetric home bias, such that $\delta = \delta^*$. We denote by lower case variables percent deviations from steady state values. Moreover, variables with a tilde have been normalized by the level of technology to render them stationary. For instance, $\tilde{c}_t = (\tilde{C}_t - \tilde{C}) / \tilde{C}$, where $\tilde{C}_t = C_t / A_t$. The relationship between the transformed variables in the model (normalized by the level of technology) and the first-differenced variables is as follows:

$$\tilde{c}_t = \tilde{c}_{t-1} + \Delta c_t - \varepsilon_t^a, \tilde{y}_t = \tilde{y}_{t-1} + \Delta y_t - \varepsilon_t^a, \tilde{c}_t^* = \tilde{c}_{t-1}^* + \Delta c_t^* - \varepsilon_t^a, \text{and } \tilde{y}_t^* = \tilde{y}_{t-1}^* + \Delta y_t^* - \varepsilon_t^a.$$

where Δ denotes the first difference operator. These relationships are used in the estimation strategy, since we include first-differenced real variables in the set of observable variables.

In this subsection, we focus the discussion on the equations that influence the behavior of the real exchange rate, and that will change once we assume imperfect pass-through and incomplete markets. In Table 3, we present the rest of the equations of the model, which are fairly standard given our assumptions. The only exception are the Taylor rules, which modify the original formulation by reacting to output growth instead of the output gap, incorporating interest rate smoothing, and an iid monetary shock.

Table 3: Linearized equations

Euler equations	$b\Delta c_t = -(1+\Gamma-b)(r_t - E_t\Delta p_{t+1}) + (1+\Gamma)E_t\Delta c_{t+1} + (1+\Gamma-b)(1-\rho_g)g_t$
	$b^*\Delta c_t^* = -(1+\Gamma-b^*)(r_t^* - E_t\Delta p_{t+1}^*) + (1+\Gamma)E_t\Delta c_{t+1}^* + (1+\Gamma-b^*)(1-\rho_g^*)g_t^*$
Labor supply	$\tilde{\omega}_t = \gamma n_t + \left[\frac{(1+\Gamma)\tilde{c}_t - b\tilde{c}_{t-1} + b\varepsilon_t^a}{1+\Gamma-b}\right]$
	$\tilde{\omega}_t^* = \gamma n_t^* + \left[\frac{(1+\Gamma)\tilde{c}_t^* - b^*\tilde{c}_{t-1}^* + b^*\varepsilon_t^a}{1+\Gamma-b^*}\right]$
Goods market clearing	$\tilde{y}_{H,t} = \theta\left[\frac{2\delta(1-\delta)}{1-2\delta}\right]q_t + (1-\delta)\tilde{c}_t + \delta\tilde{c}_t^* + \eta_t$
	$\tilde{y}_{F,t}^* = -\theta\left[\frac{2\delta(1-\delta)}{1-2\delta}\right]q_t + \delta\tilde{c}_t + (1-\delta)\tilde{c}_t^* + \eta_t^*.$
Production functions	$\tilde{y}_{H,t} = x_t + n_t$
	$\tilde{y}_{F,t}^* = x_t^* + n_t^*$
Taylor rules	$r_t = \rho_r r_{t-1} + (1-\rho_r)\gamma_p\Delta p_{H,t} + (1-\rho_r)\gamma_y\Delta y_{H,t} + z_t$
	$r_t^* = \rho_r^* r_{t-1}^* + (1-\rho_r^*)\gamma_p^*\Delta p_{F,t}^* + (1-\rho_r^*)\gamma_y^*\Delta y_{F,t}^* + z_t^*$
Terms of trade	$\Delta t_t = \Delta s_t + \Delta p_{F,t}^* - \Delta p_{H,t}$

The risk sharing condition delivers the following relationship between consumption in the two countries, the preference shocks, and the real exchange rate:

$$q_t = \left[\frac{(1+\Gamma)\widetilde{c}_t - b\widetilde{c}_{t-1}}{1+\Gamma-b}\right] - \left[\frac{(1+\Gamma)\widetilde{c}_t^* - b^*\widetilde{c}_{t-1}^*}{1+\Gamma-b^*}\right] - \left(g_t - g_t^*\right) + \left(\frac{1+\Gamma}{1+\Gamma-b} - \frac{1+\Gamma}{1+\Gamma-b^*}\right)\varepsilon_t^a. \tag{11}$$

As in CKM, the real exchange rate depends on the ratio of marginal utilities of consumption, which in our case include the habit stock in each country, and the preference shocks. Note that the innovation to world growth enters as long as the effect on the ratio of marginal utilities is different in the two countries, due to differences in the habit formation parameters.

Domestic (GDP deflator) inflation dynamics in each country are given by:

$$\Delta p_{H,t} = \gamma_b \Delta p_{Ht-1} + \gamma_f E_t \Delta p_{Ht+1} + \kappa[\widetilde{\omega}_t - x_t + \delta t_t)], \tag{12}$$

$$\Delta p_{F,t}^* = \gamma_b^* \Delta p_{F,t-1}^* + \gamma_f^* E_t \Delta p_{F,t+1}^* + \kappa^*[\widetilde{\omega}_t^* - x_t^* - \delta^* t_t]. \tag{13}$$

where for the home country, the backward and forward looking components are $\gamma_b \equiv \tau/(1+\beta\tau)$, $\gamma_f \equiv \beta/(1+\beta\tau)$, and the slope is given by $\kappa \equiv (1-\alpha\beta)(1-\alpha)/[(1+\beta\tau)\alpha]$. Similar expressions with asterisks deliver the coefficients γ_b^*, γ_f^*, and κ^*. Domestic inflation is determined by unit labor costs (the real wage), productivity shocks, and the terms of trade. This last variable appears because real wages are deflated by the CPI: an increase in imports goods prices will cause real wages to drop, and households will demand higher wages. As a result, domestic inflation will also increase.

When the law of one price holds, the real exchange rate and the terms of trade are linked as follows: $q_t = (1-2\delta)t_t$. The symmetric home bias assumption implies a positive comovement between the real exchange rate and the terms of trade which is consistent with the data. Thus, in this model, the real exchange rate inherits the properties of the terms of trade. With no home bias (δ=1/2), the real exchange rate is constant and purchasing power parity holds. The degree of home bias is crucial to account for the volatility of the real exchange rate: the larger the degree of home bias (smaller δ), the larger the volatility of the real exchange rate.[11]

Finally, the CPI inflation rates are a combination of domestic inflation and imported goods. Since prices are set in the producer currency, and the law of one price holds, the nominal exchange rate has a direct inflationary impact on CPI inflation:

[11] In a model with non-tradable goods this proportionality is broken down so that the real exchange rate will depend upon to the relative price of tradable to non tradable goods across countries.

$$\Delta p_t = (1-\delta)\Delta p_{H,t} + \delta \Delta p^*_{F,t} + \delta \Delta s_t \tag{14}$$

and

$$\Delta p^*_t = \delta^* \Delta p^*_{H,t} - \delta^* \Delta s_t + (1-\delta^*)\Delta p^*_{F,t}. \tag{15}$$

III. Extensions to the Baseline Model

A. Incomplete Markets with Stationary Net Foreign Assets

In this section we introduce the incomplete markets assumption a simple and tractable way. We assume that home-country households are able to trade in two nominal riskless bonds denominated in domestic and foreign currency, respectively. These bonds are issued by home-country residents in the domestic and foreign currency to finance their consumption. Home-country households face a cost of undertaking positions in the foreign bonds market.[12] For simplicity, we further assume that foreign residents can only allocate their wealth in bonds denominated in foreign currency. In each country, firms are still assumed to be completely owned by domestic residents, and profits are distributed equally across households.

The real budget constraint of home-country households is now given by:

$$C_t + \frac{B_t}{P_t R_t} + \frac{S_t B^*_t}{P_t R^*_t \phi\left(\frac{S_t B^*_t}{P_t}\right)} = \omega_t N_t + \frac{B_{t-1}}{P_t} + \frac{S_t B^*_{t-1}}{P_t} + \int_0^1 \Pi_t(h)dh, \tag{3'}$$

where the $\phi(.)$ function depends on the real holdings of the foreign assets in the entire economy, and therefore is taken as given by individual households.[13]

We further assume that the initial level of wealth is the same across households belonging to the same country. This assumption combined with the fact that households within a country equally share the porfits of intermediate goods producers, implies that within a country all households face the same budget constraint. In their consumption decisions, they will choose the same path of consumption.

[12] This cost allows to achieve stationarity in the net foreign asset position. See Schmitt-Grohe and Uribe (2001) and Kollman (2002) for applications in small open economies, and Benigno (2001) and Selaive and Tuesta (2003a) for applications in two-country models. Heathcote and Perri (2002) have used the same transaction cost in a two-country IRBC model.

[13] In order to achieve stationarity $\phi(.)$ has to be differentiable and decreasing in a neighborhood of zero. We further assume that $\phi(.)$ equals zero when B^*_t=0.

Dynamics

Under incomplete markets, the net foreign asset (NFA) position for the home country consists of the holding of foreign bonds (since domestic bonds are in net supply in the symmetric equilibrium). By definition, the NFA position of the foreign country equals the stock of bonds outstanding with the home country. The risk sharing condition holds in expected first difference terms and depends on the NFA position and preference shocks:

$$E_t q_{t+1} - q_t = \left[\frac{(1+\Gamma)E_t \Delta c_{t+1} - b\Delta c_t}{1+\Gamma-b}\right] - \left[\frac{(1+\Gamma)E_t \Delta c^*_{t+1} - b^* \Delta c^*_t}{1+\Gamma-b^*}\right] + (1-\rho_g)g_t - (1-\rho^*_g)g^*_t + \chi b^*_t \tag{11'}$$

where $\chi = -\phi'(0)Y_H$ and $b^*_t = \left(\frac{S_t B^*_t}{Y_{H,t} P_t}\right)$, which substitutes equation (11) in section II.F.

The net foreign asset position becomes a state variable: its evolution depends on the stock of previous debt and on the trade deficit (or surplus):

$$\beta b^*_t = b^*_{t-1} + \delta\left[\frac{2\theta(1-\delta)-1}{1-2\delta}\right]q_t - \delta\left(\tilde{c}_t - \tilde{c}^*_t\right). \tag{16}$$

Note that the effect of the real exchange rate on the NFA critically depends on the size of the elasticity of substitution: with a low elasticity, a real depreciation will imply that volumes increase less than prices decline, and hence the value of net exports declines after a real devaluation.

B. Local Currency Pricing by Intermediate Goods Producers

We assume price stickiness in each country's imports prices in terms of local currency. Each firm chooses a price for the domestic market and a price for the foreign market under the same conditions of the modified Calvo lottery with indexation as above. This assumption allows to generate deviations from the law of one price at the border, and nominal exchange rate movements generate ex-post deviations from the law of one price.[14] Importantly, under the assumption of local currency pricing, even without home bias it is possible to generate real exchange rate fluctuations.

The overall demand (domestic and exports) for an intermediate good produced in *h*, is given by:

[14] Monacelli (2005) assumes that it is retail importers those are subject to sticky prices, rather than the exporting firms in the country of origin. In his model, the law of one price holds at the border, but the pass-through is slow.

$$c_t(h) = (1-\delta)\left(\frac{p_t(h)}{P_{H,t}}\right)^{-\varepsilon}\left(\frac{P_{H,t}}{P_t}\right)^{-\theta} C_t \quad \text{and} \quad c_t^*(h) = \delta\left(\frac{p_t^*(h)}{P_{H,t}^*}\right)^{-\varepsilon}\left(\frac{P_{H,t}^*}{P_t^*}\right)^{-\theta} C_t^*.$$

Hence, whenever domestic intermediate-goods producers are allowed to reset their prices in the home and the foreign country, they maximize the following profit function:

$$\underset{p_t(h), p_t^*(h)}{Max} \; E_t \sum_{k=0}^{\infty} \alpha^k \xi_{t,t+k} \left\{ \frac{\left[p_{t,t+k}(h) - \omega_t /(A_t X_t)\right] c_{t,t+k}(h) + \left[p_{t,t+k}^*(h) S_{t+k} - \omega_t /(A_t X_t)\right] c_{t,t+k}^*(h)}{P_{t+k}} \right\}.$$

where $p_{t,t+k}(h)$ and $p_{t,t+k}^*(h)$ are prices of the home good set at home and abroad prevailing at $t+k$ assuming that the firm last reoptimized at time t, and whose evolution will depend on whether the firm indexes to last period's inflation rate (a fraction τ of firms) or to the steady-state rate of inflation (a fraction 1- τ of firms) when it is not allowed to reoptimize. $c_{t,t+k}(h)$ and $c_{t,t+k}^*(h)$ are the associated demands for good h in each country.

To obtain the log-linear dynamics, we first need to redefine the terms of trade:

$$t_t \equiv p_{F,t} - p_{H,t}, \text{and } t_t^* \equiv p_{H,t}^* - p_{F,t}^*.$$

These ratios represent the relative price of imported goods in terms of the domestically produced goods expressed in local currency, for each country.[15]

Dynamics

The following new equations arise with respect to the baseline (PCP) case. The inflation equations for home-produced goods are:

$$\Delta p_{H,t} = \gamma_b \Delta p_{Ht-1} + \gamma_f E_t \Delta p_{Ht+1} + \kappa[\widetilde{\omega}_t - x_t + \delta t_t)], \tag{12'}$$

$$\Delta p_{H,t}^* = \gamma_b \Delta p_{H,t-1}^* + \gamma_f E_t \Delta p_{H,t+1}^* + \kappa[\widetilde{\omega}_t - x_t - (1-\delta) t_t^* - q_t)], \tag{12b'}$$

$$\Delta p_{F,t}^* = \gamma_b^* \Delta p_{F,t-1}^* + \gamma_f^* E_t \Delta p_{F,t+1}^* + \kappa^*[\widetilde{\omega}_t^* - x_t^* + \delta t_t^*], \tag{13'}$$

$$\Delta p_{F,t} = \gamma_b^* \Delta p_{F,t-1} + \gamma_f^* E_t \Delta p_{F,t+1} + \kappa^*[\widetilde{\omega}_t^* - x_t^* - (1-\delta) t_t + q_t], \tag{13b'}$$

[15] Note that if the law of one price holds, $t_t = -t_t^*$, but now it is no longer the case.

Similarly to the baseline case, real wages are deflated by the CPI which causes the terms of trade for each country, as well as the real exchange rate, to matter in the determination of unit labor costs and of domestic inflation.

The CPI inflation rates under LCP do not include the nominal exchange rate as a direct determinant of imported goods inflation, because the pass-through is low and imports prices are sticky in domestic currency:

$$\Delta p_t = (1-\delta)\Delta p_{H,t} + \delta \Delta p_{F,t} \tag{14'}$$

and

$$\Delta p_t^* = \delta \Delta p_{H,t}^* + (1-\delta)\Delta p_{F,t}^* \tag{15'}$$

which substitute equations (12)-(15). In addition, the market-clearing conditions in Table 3 become:

$$\tilde{y}_{H,t} = (1-\delta)\theta\delta\left(t_t - t_t^*\right) + (1-\delta)\tilde{c}_t + \delta\tilde{c}_t^* + \eta_t \text{, and}$$

$$\tilde{y}_{F,t}^* = -(1-\delta)\theta\delta\left(t_t - t_t^*\right) + \delta\tilde{c}_t + (1-\delta)\tilde{c}_t^* + \eta_t^* .$$

C. Incomplete Markets and Sticky Prices in Local Currency Pricing

Under incomplete markets and local currency pricing, the equations of the model are given by those in section II.F, Table 3, and modified by those in section III.B. The additional change is that while the behavior of the real exchange rate is the same than under incomplete markets (equation 11' in section III.A), the NFA dynamics is given by:

$$\beta b_t^* = b_{t-1}^* + \delta(1-\delta)(\theta-1)(t_t - t_t^*) + \delta q_t - \delta\left(\tilde{c}_t - \tilde{c}_t^*\right). \tag{16'}$$

which substitutes (16) in section III.A.

IV. ESTIMATION AND MODEL COMPARISON

In this section, we describe the data for the United States and the euro area. We also explain the Bayesian methodology used to estimate the parameters of each model, and to compare the different versions of the NOEM model.

A. Data

Data sources for the United States are as follows (pnemonics are in parenthesis as they appear in the Haver USECON database): we use quarterly real GDP (GDPH), the GDP deflator (DGDP), real consumption (CH), and the 3-month T-bill interest rate (FTB3) as the relevant short-run interest rate. Since we want to express real variables in per capita terms, we divide real GDP and consumption by total population of 16 years and over (LN16).

Data for the Euro area as a whole come from the Fagan, Henry and Maestre (2001) dataset, with pnemonics in parenthesis as they appear in this dataset. This dataset is a synthetic dataset constructed by the Econometric Modeling Unit at the European Central Bank, and should not be viewed as an "official" series. We extract from that database real consumption (PCR), real GDP (YER), the GDP deflator (YED), and short-term interest rates (STN). The euro zone population series is taken from Eurostat. Since it consists of annual data, we transform it to quarterly frequency by using linear interpolation.

The convention we adopt is that the home country is the euro area, and the foreign country is the United States, such that the real exchange rate consists of the nominal exchange rate in euros per U.S. dollar, converted to the real exchange rate index by multiplying it by the U.S. CPI and dividing it by the Euro area CPI. The "synthetic" euro/U.S. dollar exchange rate prior to the launch of the euro in 1999 also comes from Eurostat, while the U.S. CPI comes from the Haver USECON database (PCU) and the euro area CPI comes from the Fagan, Henry and Maestre data base (HICP).

Table 4: Properties of the Data for the United States and the Euro Area

Raw Data, Quarterly Growth Rates					
	Consumption Euro	Output Euro	Consumption USA	Output USA	Real Exch. Rate
Mean	0.47	0.47	0.53	0.48	-0.14
Std. Dev.	0.57	0.58	0.67	0.85	4.59
Raw Data, Quarterly Rates					
	Interest Rate Euro	Inflation Euro	Interest Rate USA	Inflation USA	
Mean	2.08	1.44	1.59	1.00	
Std. Dev.	0.83	0.93	0.73	0.67	
First Autocorr.	0.96	0.89	0.94	0.90	
HP-Filtered Data					
	Consumption Euro	Output Euro	Consumption USA	Output USA	Real Exch. Rate
Std. Dev.	0.91	1.01	1.28	1.58	7.83
Corr. with RER	-0.26	-0.06	-0.02	-0.08	1.00
First Autocorr.	0.84	0.86	0.87	0.87	0.83
	Consumption Euro, USA	Output Euro, USA	Relative Cons., RER	Relative Outputs, RER	
Other Correlations	0.33	0.47	-0.17	0.04	

Note: Relative variables are the ratio between the euro area variable and its US counterpart.

Our sample period goes from 1973:1 to 2003:4, at quarterly frequency, which is when the euro area data set ends. To compute per capita output and consumption growth rates, and inflation, we take natural logs and first differences of per capita output and consumption, and the GDP deflator respectively. We divide the short term interest rate by four to obtain its quarterly equivalent. We also take natural logs and first differences of the euro/dollar real exchange rate.

Table 4 presents some relevant statistics. Interestingly, the raw data show that per capita output growth rates in the United States and the euro area are not that different (0.48 versus 0.47), while per capita consumption and output in the euro area grow at the same rate (0.47). Consumption growth in the U.S. displays a higher sample mean growth rate (0.53) which is not surprising given current recent trends. Interestingly, growth rates in the euro area are less volatile than in the U.S. The real exchange rate displays a small appreciating trend mean during the sample period, and is much more volatile than any other series.

Out of the HP-filtered statistics, it is important to highlight the well-known fact that the real exchange rate is much more volatile than any other series: the bilateral real exchange rate has a standard deviation of 7.83, while output and consumption in the U.S. has a standard deviation of 1.58 and 1.28 respectively. Output and consumption in the euro area are less volatile, with a standard deviation of about 1 percent. Interest rates and inflation rates display high persistence, and so do all real variables when they are HP-filtered. Interestingly, only consumption in the euro area displays some non-zero correlation with the real exchange rate, which is -0.26. The correlation of output in Europe, and output and consumption in the U.S. with the real exchange rate is basically zero.

Finally, it is worth noting that the correlation between consumptions is smaller than between outputs (0.33 versus 0.47), although the size of the two correlations are smaller than those obtained using shorter sample periods (ending in the early 1990s), as in Backus, Kehoe and Kydland (1992).[16] The correlation of relative output with the real exchange rate is fairly small, while the correlation between the real exchange rate and relative consumptions across countries is negative (-0.17) which is at odds with efficient risk-sharing.[17]

B. Bayesian Estimation of the Model's Parameters

According to Bayes' rule, the posterior distribution of the parameters is proportional to the product of the prior distribution of the parameters and the likelihood function of the data. An appealing feature of the Bayesian approach is that additional information about the model's parameters (i.e. micro-data evidence, features of the first moments of the data) can be introduced via the prior distribution.

To implement the Bayesian estimation method, we need to be able to evaluate numerically the prior and the likelihood function. The likelihood function is evaluated using the state-space representation of the law of motion of the model, and the Kalman filter. Then, we use the

[16] Heathcote and Perri (2004) document that in recent years the U.S. economy has become less correlated with the rest of the world.

[17] All the facts related to the U.S. economy are very similar to the ones presented in CKM.

Metropolis-Hastings algorithm to obtain random draws from the posterior distribution, from which we obtain the relevant moments of the posterior distribution of the parameters.[18]

Let ψ denote the vector of parameters that describe preferences, technology, the monetary policy rules, and the shocks in the two countries of the model. The vector of observable variables consists of $x_t = \{\Delta y_t, \Delta c_t, r_t, \Delta p_{H,t}, \Delta y_t^*, \Delta c_t^*, r_t^*, \Delta p_{F,t}^*, \Delta q_t\}$. The assumption of a world technology shock with a unit root makes the real variables stationary in the model in first differences. Hence, we use consumption and output growth per country, which are stationary in the data and in the model. We first-difference the real exchange rate, while inflation and the nominal interest rate in each country enter in levels.[19] We express all variables as deviations from their sample mean. We denote by $L(\{x_t\}_{t=1}^T \mid \psi)$ the likelihood function of $\{x_t\}_{t=1}^T$.

Priors

Table 5 shows the prior distributions for the model's parameters, that we denote by Π(ψ). For the estimation, we decide to fix only two parameters. The first one is the steady-state growth rate of the economy. Based on the evidence presented in section III.A, we set Γ=0.5 percent, which implies that the world growth rate of per capita variables is about 2 percent per year. In order to match a real interest rate in the steady state of about 4 percent per year, we set the discount factor to $\beta=0.995$. For reasonable parameterizations of these two variables the parameter estimates do not change significantly. For the remainder of parameters, gamma distributions are used as priors when non-negativity constraints are necessary, and uniform priors when we are mainly interested in estimating fractions or probabilities. Normal distributions are used when more informative priors seem to be necessary.

Unlike other two-country model papers (i.e. Lubik and Schorfheide, 2005; and CKM), we do not impose that the parameter values be the same in the two countries. However, we do use the same prior distributions for parameters across countries. We use normal distributions for the coefficients of habit formation and inverse elasticity of labor supply with respect to the real wage, centered at conventional values in the literature (0.7 and 1, respectively). We truncate the habit formation parameter to be between 0 and 1, which on the upper bound it would be six standard deviations away from the prior mean. We assume that the average duration of price contracts has a prior mean of 3 in the two countries, following empirical evidence reported in Taylor (1999). In this case, a gamma distribution is used.[20] The prior on the fraction of price setters that follow a backward looking indexation rule is less informative and takes the form of a uniform distribution between zero and one.

[18] See the Appendix for some details on the estimation. Lubik and Schorfheide (2003, 2005) also provide useful details on the estimation procedure.

[19] Hence, we avoid the discussion on which detrending method (linear, quadratic or HP-filter) to use.

[20] To keep the probability of the Calvo lottery between 0 and 1, the prior distribution is specified as average duration of price contracts minus one: D=1/(1-α)-1. The shape of the prior is not much different than assuming a beta distribution for α.

Table 5: Prior Distributions of the Model's Parameters

Parameter		Distribution	Mean	Std. Dev.
Habit formation	b, b^*	Normal	0.70	0.05
Labor supply	γ, γ^*	Normal	1.00	0.25
Average Price Duration	$(1-\alpha)^{-1}, (1-\alpha^*)^{-1}$	Gamma	3.00	1.42
Indexation	τ, τ^*	Uniform(0,1)	0.50	0.29
Fraction of imported goods	δ	Normal	0.20	0.03
Elasticity of substitution between home and foreign goods	θ	Normal	1.50	0.25
Elasticity of the real exchange rate to the NFA position	χ	Gamma	0.02	0.014
Taylor rule: inflation	$\gamma_p,\ \gamma_p^*$	Normal	1.50	0.25
Taylor rule: output growth	$\gamma_y,\ \gamma_y^*$	Normal	1.00	0.20
Taylor rule: smoothing	ρ_r, ρ_r^*	Uniform(0,1)	0.50	0.29
AR coefficents of shocks	$\rho_x, \rho_x^*, \rho_g,$ $\rho_g^*, \rho_\eta, \rho_\eta^*$	Uniform(0,0.96)	0.48	0.28
Std. Dev. technology shocks	$\sigma_x, \sigma_x^*, \sigma_a$	Gamma	0.007	0.003
Std. Dev. preference shocks	σ_g, σ_g^*	Gamma	0.010	0.005
Std. Dev. monetary shocks	σ_z, σ_z^*	Gamma	0.004	0.002
Std. Dev. demand shocks	$\sigma_\eta, \sigma_\eta^*$	Gamma	0.010	0.005

The parameters that incorporate the open economy features of the model take the following distributions. The parameter δ, which captures the implied home bias, has a prior distribution with mean 0.2 and standard deviation 0.03, implies a smaller home-bias than suggested by Heathcote and Perri (2002) and CKM. The elasticity of substitution between home and foreign goods (θ) is source of controversy. We center it at a value of 1.5 as suggested by CKM, but with a large enough standard deviation to accommodate other feasible parameters, even those below one.[21] Finally, the parameter χ, that measures the elasticity of the risk premium with respect to the net foreign asset position, is assumed to have a gamma distribution with mean 0.02 and standard deviation 0.014, following the evidence in Selaive and Tuesta (2003a and 2003b).

[21] Trade studies typically find values for the elasticity of import demand to respect to price (relative to the overall domestic consumption basket) in the neighborhood of 5 to 6, see Trefler and Lai (1999). Most of the NOEM models consider values of 1 for this elasticity which implies Cobb-Douglas preferences in aggregate consumption.

For the coefficients of the interest rate rule, we center the coefficients to the values suggested by Rabanal (2004b) who estimates rules with output growth for the United States. Hence, γ_p has a prior mean of 1.5, and γ_y has a prior mean of 1. The same values are used for the monetary policy rule in the Euro area, and we use uniform priors for the autoregressive processes between zero and one. We also truncate the prior distributions of the Taylor rule coefficients such that the models deliver a unique, stable solution.

Regarding the priors for the shocks of the model, we use also uniform priors on the autoregressive coefficients of the six AR(1) shocks. We truncate the upper bound of the distribution to 0.96, because we want to examine how far can the models go in endogenously replicating persistence. We choose gamma distributions for the priors on the standard deviations of the shocks, to stay in positive numbers. The prior means are chosen to match previous studies. For instance, the prior mean for the standard deviation of all technology shocks is set to 0.007, close to the values suggested by Backus, Kehoe and Kydland (1992), while the prior mean of the standard deviation of the monetary shocks comes from estimating the monetary policy rules using OLS. The standard deviation of the prior are set to reflect the uncertainty over these parameters.

Drawing from the Posterior and Model Comparison

We implement the Metropolis-Hastings algorithm to draw from the posterior. The results are based on 250,000 draws from the posterior distribution. The definition of the marginal likelihood for each model is as follows:

$$L(\{x_t\}_{t=1}^T) = \int_{\psi \in \Psi} L(\{x_t\}_{t=1}^T \mid \psi)\Pi(\psi)d\psi \tag{17}$$

The marginal likelihood averages all possible likelihoods across the parameter space, using the prior as a weight. Multiple integration is required to compute the marginal likelihood, making the exact calculation impossible. We use a technique known as modified harmonic mean to estimate it.[22]

Then, for two different models (A and B), the posterior odds ratio is

$$\frac{P(A \mid \{x_t\}_{t=1}^T)}{P(B \mid \{x_t\}_{t=1}^T)} = \frac{\Pr(A)L(\{x_t\}_{t=1}^T \mid \text{model} = A)}{\Pr(B)L(\{x_t\}_{t=1}^T \mid \text{model} = B)}.$$

If there are $m \in M$ competing models, and one does not have strong views on which model is the best one (i.e. *Pr(A)=Pr(B)=1/M*) the posterior odds ratio equals the Bayes factor (i.e. the ratio of marginal likelihoods).

[22] See Fernández-Villaverde and Rubio-Ramírez (2004).

V. RESULTS

We present our results in the following way. First, we present the posterior estimates obtained for a closed economy vis-à-vis the four specifications considered for open economy models. Second, we perform a model comparison by evaluating the marginal likelihood for each model. Third, we compute the standard deviations and correlations of each model at the mode posterior values. Fourth, we discuss the dynamics of our preferred model by analyzing the importance of the structural shocks for real exchange rate fluctuations. Finally, we look at the one-step ahead in-sample forecast performance of all models, and compare their performance to VARs.

A. Posterior Distributions for the Parameters

In Table 6, we present the means and standard deviations of the posterior parameters of all the models. In order to have a benchmark for the open economy estimates, we first provide the results from estimating each country as a closed economy. For the closed economy specification we assume that within each country agents only consume home produced goods ($\delta=\theta=0$), and are not allowed to trade bonds internationally. In addition, the real exchange rate is dropped from the set of observed variables.

In column I of Table 6, we report the mean and standard deviation of the posterior distributions of the parameters for the euro area and U.S., treating each of them as a closed economy. Overall, our estimates are in the line of previous contributions. The average duration of price contracts implied by the point estimate of the price stickiness parameters are above four (5.94) and six quarters (7.09), respectively. The implied proportion of firms that index their prices to the inflation rate are 0.06 and 0.09 percent for the euro area and U.S. respectively, which, as in Galí and Rabanal (2004), suggests that with highly correlated shocks the pure forward-looking model seems to be valid. The habit formation parameters are around 0.5 and 0.6 in the euro area and the U.S. respectively, which are in line with the values found by Smets and Wouters (2003) and slightly above to the ones of Galí and Rabanal (2004) for the U.S. economy. The estimates of the monetary policy rule parameters are similar to what it is usually assumed in the literature. Thus, the estimated coefficients over inflation and output are 1.62 and 1.10 for the euro area and 1.85 and 0.92 for the U.S. respectively. Our results also suggest a high degree of interest rate smoothing (0.87 and 0.83 for the euro area and U.S. respectively). Finally, the estimated processes of the shocks suggest that all of them are highly autocorrelated, except for the productivity shock in the euro area.

Table 6: Posterior Distributions

	I. Two Closed Economies		II. Complete Markets, PCP		III. Complete Markets, LCP		IV. Incomplete Markets, PCP		V. Incomplete Markets, LCP	
	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.	Mean	Std. Dev.
b	0.57	0.04	0.78	0.02	0.72	0.03	0.62	0.03	0.64	0.03
b^*	0.61	0.04	0.69	0.04	0.68	0.04	0.55	0.04	0.54	0.03
γ	1.09	0.23	1.25	0.21	1.47	0.20	1.18	0.20	0.79	0.23
γ^*	1.00	0.21	0.86	0.25	1.02	0.23	1.16	0.20	1.03	0.23
$(1-\alpha)^{-1}$	5.94	0.89	4.77	0.46	6.29	0.62	4.28	0.39	4.12	0.47
$(1-\alpha^*)^{-1}$	7.09	0.92	14.74	1.68	12.66	1.36	5.74	0.63	4.95	0.63
τ	0.06	0.07	0.93	0.06	0.41	0.17	0.94	0.06	0.84	0.13
τ^*	0.09	0.08	0.04	0.04	0.06	0.05	0.09	0.07	0.09	0.08
δ	-	-	0.13	0.02	0.23	0.02	0.12	0.01	0.06	0.01
θ	-	-	0.04	0.03	0.01	0.01	0.44	0.00	0.91	0.01
χ	-	-	-	-	-	-	0.007	0.004	0.013	0.008
ρ_r	0.87	0.02	0.88	0.01	0.87	0.01	0.89	0.01	0.88	0.01
γ_y	1.08	0.16	0.07	0.06	0.16	0.10	1.04	0.16	0.98	0.17
γ_p	1.59	0.13	2.24	0.15	2.03	0.13	1.90	0.15	1.71	0.16
ρ_r^*	0.82	0.02	0.85	0.02	0.85	0.01	0.81	0.02	0.82	0.02
γ_y^*	0.91	0.15	1.24	0.13	1.34	0.14	1.16	0.14	1.01	0.15
γ_p^*	1.81	0.17	1.67	0.13	1.71	0.13	1.85	0.13	1.86	0.15
ρ_x	0.80	0.27	0.63	0.04	0.69	0.04	0.17	0.08	0.39	0.07
ρ_x^*	0.93	0.02	0.96	0.002	0.96	0.002	0.92	0.02	0.96	0.002
ρ_g	0.91	0.02	0.96	0.001	0.96	0.001	0.93	0.02	0.93	0.02
ρ_g^*	0.82	0.05	0.84	0.03	0.89	0.03	0.87	0.03	0.88	0.03
ρ_η	0.93	0.03	0.89	0.04	0.86	0.04	0.96	0.00	0.93	0.02
ρ_η^*	0.93	0.03	0.94	0.02	0.93	0.04	0.95	0.01	0.94	0.01
σ_x (in %)	1.93	0.45	4.45	0.52	4.27	0.48	4.00	0.55	4.23	0.54
σ_x^* (in %)	1.91	0.29	4.68	0.43	3.32	0.33	0.92	0.20	0.62	0.09
σ_g (in %)	2.32	0.29	5.72	0.40	4.78	0.31	3.64	0.45	3.41	0.49
σ_g^* (in %)	2.16	0.24	2.73	0.27	3.07	0.30	2.13	0.26	2.17	0.27
σ_z (in %)	0.20	0.02	0.23	0.02	0.22	0.02	0.20	0.01	0.19	0.01
σ_z^* (in %)	0.23	0.02	0.22	0.02	0.22	0.02	0.25	0.02	0.23	0.02
σ_a (in %)	0.70	0.02	2.58	0.21	2.03	0.17	1.39	0.12	1.44	0.14
σ_η (in %)	0.46	0.03	0.43	0.03	0.44	0.03	0.67	0.05	0.50	0.03
σ_η^* (in %)	0.69	0.05	0.67	0.04	0.66	0.04	0.89	0.06	0.70	0.05
Log-Marginal			3981.6		4033.2		4106.8		4070.9	

Our benchmark open economy model is the one that assumes complete markets and PCP. The results are displayed in column II of Table 6. Interestingly, the results change in important ways with respect to the closed economy case. First, the proportion of firms that index their prices to the lagged inflation rate increases to almost one in the euro area, while inflation remains almost purely forward looking in the United States. The average duration of price contracts decreases for the euro area to 4.77 quarters and increases significantly for the United States to 14.74 quarters, which is a fairly large number.[23] The habit persistence parameters increase both in the euro area (to 0.78) and the U.S. (to 0.69). Estimates of the Taylor rule for the U.S. obtained from the two-country model are more or less the same to the one obtained from the closed economy counterpart. However, we observe significant changes in the euro area, thus the estimated coefficient on inflation rises from 1.59 to 2.24 and the one on output decreases from 1.08 to 0.07. The degree of interest rate smoothing for each block presents minor changes with respect to the closed economy estimations.

The persistence and volatility of all shocks increases greatly when the model tries to match the behavior of the real exchange rate. Except for the monetary and the demand shocks, the standard deviation of the shocks doubles or triples with respect to the closed economy estimates. Also, the autocorrelation of technology shocks in the United States and of preference shocks in the euro area increases to 0.96, which is the upper bound allowed for in the estimation. Thus, there is a tension in the model between matching a highly volatile real exchange rate and the less volatile output and consumption series. Below, we examine how well the models match the second moments of the data.[24]

We now turn to analyze the parameters that are critical in NOEM models and which are key in shaping real exchange rate dynamics: the implied degree of home bias captured by $1-\delta$, the intratemporal elasticity of substitution between goods across countries, θ, and the real exchange-rate elasticity with respect to the stock of foreign debt, χ, that arises from the incomplete markets assumption. In our benchmark NOEM model we find that the implied degree of home bias towards home goods is 0.87 which is below 0.984, the value used by CKM (2002) and Heathcote and Perri (2002). The baseline two-country model delivers a very small estimate for the elasticity θ, close to zero. This result can be understood from the market clearing conditions, because output and consumption are much less volatile than the real exchange rate. Another

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[23] This result comes from the assumption of a production function that is linear in labor input. If we assumed, as Galí, Gertler and López-Salido (2001) that the production function is concave in labor, we would obtain smaller average price durations. The same would happen if we introduced firm-specific capital or real demand rigidities, as in Altig et al. (2005) or Eichenbaum and Fischer (2004).

[24] In our estimation we assume that shocks are orthogonal. However, the world aggregate shock indirectly adds some form of spillovers. Baxter and Crucini (1995) highlight the importance of the structure of shocks for international asset market structures in IRBC models. In particular, they find that if shocks are stationary and with substantial spillovers both complete and incomplete markets perform similarly. But if shocks are very persistent without spillovers, adding incomplete markets changes significantly the prediction of IRBC models.

reason to expect such low elasticity is that the real exchange rate displays close-to-zero correlations with consumption and output in each country.

In order to gain more intuition about this result, let's assume for simplicity that the utility function does not exhibit habit persistence and there are no preference or world technology shocks. Then, the risk-sharing condition under complete markets can be expressed as: $q_t = c_t - c_t^*$. This risk-sharing condition, combined with the market clearing conditions in both countries (see Table 3) and the fact that the real exchange rate and the terms of trade under PCP are related as $q_t = (1-2\delta)t_t$ delivers the following relation between relative outputs and the real exchange rate:

$$y_{H,t} - y^*_{F,t} = \left[\frac{4\delta(1-\delta)(\theta-1)-1}{(1-2\delta)^2} \right] q_t \tag{18}$$

Equation (18) ilustrates the relationship between the volatility of relative outputs and the volatility of the real exchange rate, and highlights the need of a low value of the intratemporal elasticity of substitution between tradable goods, θ, in order to match the data. For a given volatility of the real exchange rate, the volatility of relative outputs is increasing in θ: a low value of θ will help in fitting the data better. Our prior distribution was centered at a value of 1.5, so the data clearly provide evidence that the value is much smaller.[25] Under PCP, we should observe the largest demand substitution towards home goods after a devaluation. A value close to zero for θ is neglecting this expenditure-switching effect.

When we relax the PCP assumption (column III) allowing for deviations from the law of one price by using local currency pricing (LCP), the results only change marginally. The parameter θ stays close to zero. Two differences in the results are worth mentioning. First, the implied degree of home-bias in preferences drops from 0.87 to 0.77 in the euro area, and second, the proportion of firms that index their prices to the inflation rate drops from 0.93 to 0.41. Given the unreasonably low values obtained for θ, our estimation does not provide support for the complete asset market structure.

The LCP assumption adds endogenous volatility and persistence to the real exchange rate dynamics, so a smaller degree of home bias is needed in order to match the data. The real exchange rate under LCP can be decomposed as

$$q_t = lop_t - (1-\delta)t^*_t - \delta t_t \tag{19}$$

$$lop_t - lop_{t-1} = \Delta s_t + \Delta p^*_{H,t} - \Delta p_{H,t} \tag{20}$$

[25] On the other hand, Batini et al. (2005) allow for different elasticities of substitution of home and foreign goods for the United States and the euro area, and find that this elasticity is above one in both countries under complete markets.

where lop_t denotes deviations from the law of one price which arise from the LCP assumption.[26] When the law of one price holds, then lop_t =0. The LCP assumption introduces both endogenous persistence and volatility to the real exchange rate that can be summarized by the variable lop_t . Given the larger endogenous volatility of the real exchange rate and given the unchanged estimated low values for θ, a reduction of home bias parameter (i.e. increased δ) is needed.

In columns IV and V of Table 6 we present the estimates of the model under incomplete markets with both PCP and LCP, respectively. There are some important differences to highlight with respect to the models with complete markets. First, the estimates of θ increase significantly, with point estimates of 0.45 and 0.91 under PCP and LCP, respectively. The intuition for this result can be seen from the law of motion of the NFA position (equation (16) or (16')). A real depreciation has to lead to a positive income effect to avoid having explosive NFA dynamics. For that to happen, θ has to be in the neighborhood of ½ under PCP and 1 under LCP. This implicit restriction pushes the value of the elasticity up, although in both cases it seems to stay around the lowest possible value that delivers stable dynamics.

Given the low estimated values of the elasticity of substitution between home and foreign goods in complete market models compared to incomplete market models (which are at odds with both macro and micro empirical evidence), our results give support for the latter asset market structure. We conclude that the degree of financial integration (or lack thereof) is central for understanding the international transmission of business cycles and real exchange rate dynamics.

Table 6 also provides an estimate for χ. We find values of 0.007 and 0.013 under PCP and LCP, respectively. These values are larger to the ones found by Bergin (2004) for the G7 countries (0.0038) and smaller to the ones obtained by Lane and Milesi-Ferreti (2001) from a panel of OECD countries (0.0254). Selaive and Tuesta (2003a and 2003b), by using GMM, estimate a risk-sharing condition similar to ours, with estimates in a range between 0.004 and 0.071 for a sample of OECD countries. From the above results it seems that the data give support for an incomplete asset market structure with a stationary net foreign asset position.

It is worth noting that the volatility of the shocks affecting the U.S. economy becomes much smaller under incomplete markets, and even for the productivity shock they are half the size of those under a closed economy set up. For the case of the euro area, the estimated volatility of the shocks is smaller, although the reduction is not as important as in the U.S. case. Finally, the estimates of the Taylor rule for the euro area become closer to what was obtained under a closed economy. Hence, it seems that the introduction of incomplete markets does help improve the internal dynamics of the model by requiring smaller shocks.

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[26] That it, lop_t denotes deviations from the following variable $LOP_t = S_t P^*_{H,t} / P_{H,t}$ under LCP from its PCP counterpart value of one.

B. Model Comparison

The last row of Table 6 shows the marginal likelihood of the four open economy models.[27] While introducing incomplete markets is always better than complete markets (either under PCP or LCP), and while LCP improves the fit of the model under complete markets, the model that ranks highest is the incomplete markets model with PCP. The model that ranks second is the incomplete markets model with LCP, while the two models with complete markets rank last.

The differences are very important. In all cases the (log) differences are of similar magnitude, and such differences would imply "decisive" evidence for the model with highest log marginal likelihood, using the Bayesian model comparison language (Kass and Raftery, 1995). For instance the difference between the log-marginals of the first and the second model is about 36. This means that we would need a prior that favors the second model over the first by a factor of $3.9*10^{15}$ in order to accept it after observing the data. Since this is a large number, we conclude that the incomplete market model with PCP outperforms the incomplete market model with LCP, which in turn outperforms the two models with complete markets.

C. Second Moments

To understand why the model with more features, that *a priori* should be the "best model", does not rank first in terms of the Bayes factor comparison, in this subsection we present some selected second moments. In all the models, the evaluation is done at the mode of the posterior distribution. Table 7 presents some selected second moments implied by our estimations and are compared with those in the actual data.[28]

We find that the baseline model does a good job in explaining real exchange rate volatility and persistence, but at the cost of implying too high volatility and persistence in output and consumption in both countries, and too high volatility of interest rates and inflation in the euro area. Extending the benchmark complete markets model by allowing for deviations from the law of one price, gives a slightly better fit of the rest of the real variables although it delivers a slightly less volatile and persistent real exchange rate. The introduction of incomplete markets allows the model to better match the volatilities of all real variables best. This is a consequence of the size of all shocks being smaller in this case, as discussed above. While the model fits all U.S. variables and the nominal interest rate and inflation in the euro area fairly well, it still predicts output and consumption volatility in the euro area by as twice as much as what observe

[27] The marginal likelihood of the closed economies is not computed because of the different observed variables used in the estimation.

[28] All model-based standard deviations and autocorrelations of nominal variables are based on simulating the model at the posterior mode, to be consistent with the observable counterpart. Autocorrelations and cross-correlations of real variables come from simulating the model 1000 times with 124 periods at the posterior mode and applying the HP filter.

in the data. Adding sticky prices in local currency pricing to the incomplete markets model results is an overprediction of the real exchange rate volatility (the standard deviations rises from 4.90 to 6.83 percent, while in the data it is 4.59 percent), and a mild worsening of other features of the data. Hence, this is why this model, which is the one analyzed by CKM, does not rank best using the Bayes factor.

Table 7: Selected Second Moments in the Data and in the Models

	Euro Area				United States				
Std. Dev. (in percent)	Δc	Δy	r	Δp	Δc*	Δy*	r*	Δp*	Δq
Data	**0.57**	**0.58**	**0.83**	**0.93**	**0.67**	**0.85**	**0.73**	**0.67**	**4.59**
Closed Economy	0.73	0.81	0.59	0.74	0.85	1.00	0.57	0.56	-
Complete, PCP	1.30	1.27	1.43	1.31	1.24	1.27	0.63	0.79	4.96
Complete, LCP	1.30	1.18	1.10	0.90	1.17	1.16	0.77	0.72	5.32
Incomplete, PCP	1.04	1.07	0.83	1.11	0.83	0.98	0.61	0.62	4.90
Incomplete, LCP	1.03	1.11	0.82	1.22	0.86	0.97	0.60	0.63	6.83
Autocorrelations	c	y	r	Δp	c*	y*	r*	Δp*	q
Data	**0.84**	**0.86**	**0.96**	**0.89**	**0.87**	**0.87**	**0.94**	**0.90**	**0.83**
Closed Economy	0.87	0.85	0.91	0.73	0.88	0.85	0.89	0.77	-
Complete, PCP	0.92	0.91	0.95	0.82	0.91	0.89	0.89	0.85	0.77
Complete, LCP	0.90	0.89	0.93	0.71	0.91	0.88	0.91	0.84	0.71
Incomplete, PCP	0.87	0.85	0.94	0.79	0.88	0.83	0.87	0.72	0.72
Incomplete, LCP	0.87	0.86	0.93	0.78	0.87	0.83	0.88	0.72	0.70
Other Correlations	c,c*	c,y	y,y*	c*,y*	c-c*,q	y-y*,q			
Data	**0.33**	**0.81**	**0.47**	**0.85**	**-0.17**	**0.04**			
Complete, PCP	0.32	0.97	0.48	0.91	0.03	0.09			
Complete, LCP	0.21	0.93	0.60	0.87	0.20	0.13			
Incomplete, PCP	0.53	0.86	0.60	0.76	-0.37	0.09			
Incomplete, LCP	0.53	0.90	0.44	0.78	0.04	0.26			

Note: all model-based standard deviations and autocorrelations of nominal variables are computed by simulating the model at the posterior mode. Autocorrelations and cross-correlations of real variables come from simulating the model 1000 times with 124 periods at the posterior mode and applying the HP filter.

The bottom panel of Table 7 shows some selected cross-correlations of the HP-filtered data of real variables. We focus on these because they are typically central in international business cycle analysis. In terms of consumption and output correlations across countries both the PCP and LCP complete market models perform quite well, but they are not good in explaining the correlation between the real exchange rate and relative consumptions across countries. In the data this correlation is negative (-0.17) and we obtain positive values (0.03 and 0.20 for the PCP and LCP models, respectively). CKM (2002) refer to this discrepancy between the models and the data as *the consumption real exchange rate anomaly*. CKM find hard to explain this anomaly even when introducing incomplete markets. Remarkably, once we extend the models allowing for incomplete markets we get closer to the data without affecting other moments significantly. In particular, we obtain a negative correlation between the relative consumption across-countries and the real exchange rate (-0.37). Once again, extending the incomplete markets model allowing for deviations from the law of one price give us a correlation close to zero, worsening the fit with respect to the incomplete market and PCP model.

We give some intuition why incomplete markets helps to explain the lack of risk-sharing reported in Table 4. Let's assume again for simplicity that the utility function does not exhibit habit persistence and there are not preference, demand or permanent technology shocks. Under incomplete markets, the risk-sharing condition can be written as

$$E_t\left(q_{t+1} - q_t\right) = E_t\left[c_{t+1} - c_t - \left(c_{t+1}^* - c_t^*\right)\right] + \chi b_t^*$$

while the net foreign asset position becomes:

$$\beta b_t^* = b_{t-1}^* + \delta\left[q_t - \left(c_t - c_t^*\right)\right] + \delta \frac{(\theta - 1)(1-\delta)}{1-2\delta} q_t$$

.

Let's first assume that $\theta = 1$. If the initial level of net foreign assets is equal to zero, the above two equations yield that $q_t = c_t - c_t^*$ for all t. Thus, under these simplyfing conditions, the incomplete markets model will still deliver perfect risk-sharing and no wealth effects of a devaluation. Thus, in order to break risk-sharing across countries, and induce a negative correlation between relative consumptions and the real exchange rate, values for the elasticity of substitution between tradable goods θ between zero and one are needed. In addition, preference shocks will also help at capturing the lack of risk sharing.

Table 7 shows that each model matches a particular moment of the data better than the others. The advantage of the Bayesian approach to model comparison is that it is a likelihood-based method: all the implications of each model for fitting the data are contained in the likelihood function. The good news is that the model that ranks highest using the marginal likelihood criterion seems to deliver the best fit to most features of the data.

D. Shocks and Real Exchange Rate Dynamics

In this subsection, we investigate what is the importance of the different shocks for explaining real exchange dynamics. We perform this exercise only for our "preferred" model, which is the incomplete markets model with PCP and where the law of one price holds. Table 8 reports the contribution of each shock to the standard deviation of the observable variables in the model.[29]

The shock that explains most of real exchange rate variance is the demand shock. It explains 49.2 percent of the RER variance. The second largest component are the country-specific

[29] Lubik and Schorfheide (2005) add error terms to either the UIP or the PPP equations to assess the degree of model mispecification in explaining real exchange rate dynamics.

technology shocks which explain 35.5 percent of the variance of the real exchange rate.[30] Interestingly, the estimated model shows that real exchange rate fluctuations have fairly little to do with either monetary or preference shocks (9.0 and 5.8 percent, respectively).[31]

Table 8: Contributions of the Shocks to Selected Second Moments in the Preferred Model

	Euro Area				United States				
Percent Variance	Δc	Δy	r	Δp	Δc*	Δy*	r*	Δp*	Δq
Monetary shocks	10.1	14.7	2.9	14.4	10.5	14.4	5.3	14.9	9.0
Country tech. shocks	12.7	16.2	2.9	24.7	2.8	12.9	2.2	11.4	35.5
World tech. shocks	31.3	33.6	0.9	15.8	48.5	31.8	0.1	33.6	0.6
Preference shocks	41.8	19.3	90.9	44.7	31.8	16.8	79.2	37.4	5.8
Demand shocks	4.1	16.2	2.5	0.4	6.4	24.2	13.1	2.6	49.2
Autocorrelations	c	y	r	Δp	c*	y*	r*	Δp*	q
Data	**0.84**	**0.86**	**0.96**	**0.89**	**0.87**	**0.87**	**0.94**	**0.90**	**0.83**
Monetary shocks	0.77	0.71	0.45	0.87	0.77	0.70	0.44	0.72	0.49
Country tech. shocks	0.89	0.83	0.82	0.53	0.91	0.84	0.88	0.67	0.73
World tech. shocks	0.93	0.93	0.92	0.85	0.92	0.93	0.83	0.71	0.86
Preference shocks	0.78	0.78	0.97	0.89	0.81	0.76	0.92	0.74	0.85
Demand shocks	0.88	0.62	0.87	0.94	0.87	0.63	0.80	0.97	0.69
Other Correlations	c,c*	c,y	c,c*	c*,y*	c-c*,q	y-y*,q			
Data	**0.33**	**0.81**	**0.47**	**0.85**	**-0.17**	**0.04**			
Monetary shocks	0.33	0.97	0.01	0.96	0.79	0.94			
Country tech. shocks	0.72	0.75	-0.03	0.61	-0.24	0.90			
World tech. shocks	0.99	1.00	0.99	1.00	0.42	0.86			
Preference shocks	-0.30	0.93	0.38	0.82	-0.80	-0.40			
Demand shocks	0.25	-0.10	0.35	-0.35	-0.74	-0.97			

Note: all model-based standard deviations and autocorrelations of nominal variables are computed by simulating the model at the posterior mode. Autocorrelations and cross-correlations of real variables come from simulating the model 1000 times with 124 periods at the posterior mode and applying the HP filter.

Only the world technology shock and the preference shocks are able to generate a highly persistent response of the real exchange rate, but as we argued they have trouble explaining its variability. Monetary policy shocks on their own cannot account for real exchange rate persistence, which is only 0.49 under these shocks. Our results suggest that, unlike CKM, it is very difficult that monetary shocks and sticky prices with LCP might help to account for the

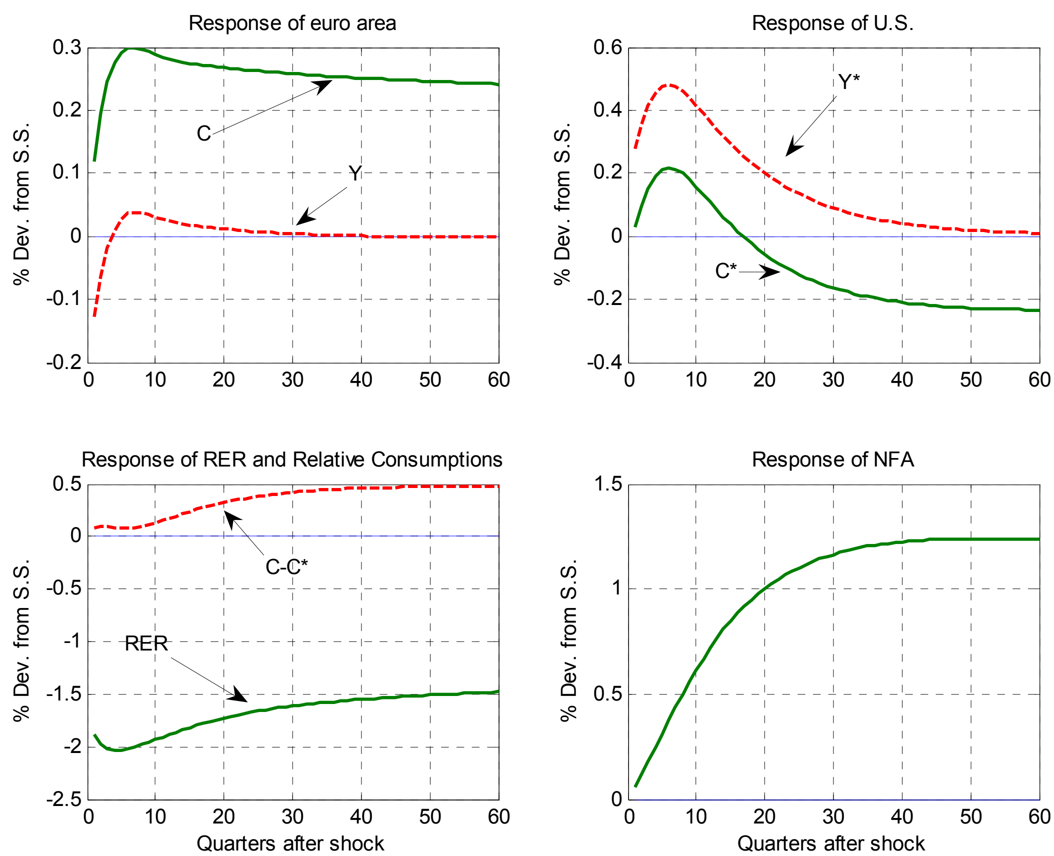
[30] Note, however, that this is an upper bound for the importance of technology shocks. Since the model does not have capital in the production function or sticky wages, then price markup, wage markup and temporary technology shocks have the same effect and cannot be separately identified.

[31] Faust and Rogers (2003) find that monetary shocks explain a small share of the volatility in the nominal exchange rate. Clarida and Galí (1994) in an earlier contribution find that demand shocks explain most of the variance in the real exchange rate fluctuations.

observed real exchange rate volatility. However, monetary shocks have some importance at explaining domestic inflation rates (14.4 and 14.9 percent for the euro area and U.S., respectively). A model with either monetary or world technology shocks being the only driving force, delivers positive correlations between relative consumptions and the RER, of 0.79 and 0.42, respectively. Our results show that in order to simultaneously account for the real exchange rate volatility and the *consumption-real exchange rate anomaly* we need technology, preference and demand shocks.

Next, we proceed to better understand real exchange rate dynamics by analyzing the posterior impulse response functions. Given the major importance of both technology and demand shocks in explaining the real exchange rate volatility, we focus our analysis on those two shocks.[32] We plot the responses of consumption and output in both countries, relative consumption, the real exchange rate and the net foreign asset position.

Figure 2: Impulse Response to a U.S. technology shock



[32]We use the mode of the posterior distribution of the model's parameters to compute the impulse responses. The impulse responses to monetary, preference and world technology shocks are available upon request.

Figure 2 displays the effects of one standard deviation of a U.S. transitory technology shock. The effects of technology shocks are broadly similar in both economies but they have more persistent effects in the case of the U.S., because the persistence parameter is higher. A U.S. technology shock expands both output and consumption in the U.S., it also expands consumption in the euro area, but it reduces euro area output. The technology shock also generates a persistent real exchange rate apreciation of the euro (i.e. real depreciation of the dollar) which is consistent with a decrease in U.S. domestic prices (worsening in the terms of trade) due to an improvement in productivity. Because of this, we observe that the euro area net foreign asset position improves, and then returns to its the steady state value very slowly. The model also predicts an increase in relative consumptions. Euro area technology shocks imply a similar pattern, but with the opposite sign and lower persistence.[33].

Figure 3: Impulse Response to a U.S. demand shock

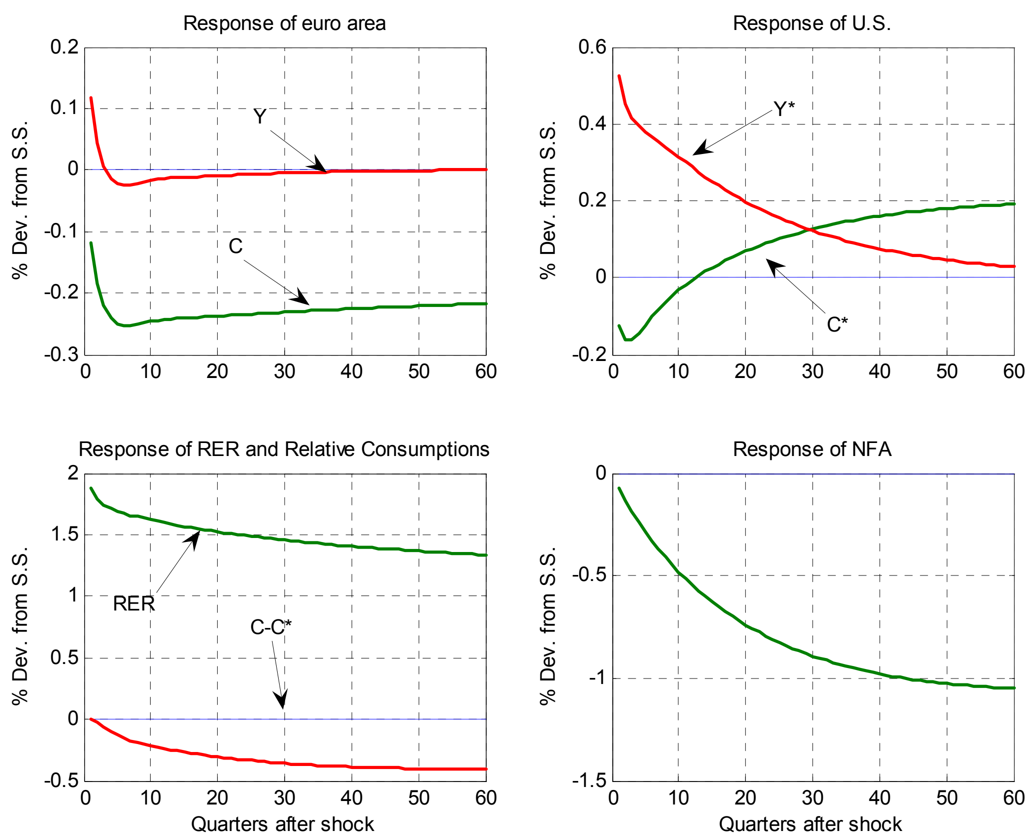


Figure 3 presents the impulse response to a demand shocks in the U.S. A positive demand shock in the U.S. generates an increase in output and a decrease in consumption in the two areas.

[33] To save space we omit this figure, which is available upon request.

Consequently, to restore the balance in the U.S., we observe a real euro depreciation (i.e. dollar appreciation), along with a persistent NFA deccumulation in the euro area due to wealth effects. It takes several periods for consumption in the U.S. to recover, such that the euro area NFA slowly increases to its steady state level. The strong euro depreciation boosts output in the euro area but causes consumption to decline. A positive demand shock in the euro area would generate the exact opposite result. Remarkably, the impulse responses generated by the demand shocks will imply in both cases a negative co-movement between the real exchange rate and relative consumptions, as it is observed in the data. Therefore, that demand shocks are crucial to explain real exchange rate behavior.

E. Forecasts

Table 9 presents the mean squared errors (MSE) for one-step ahead in-sample forecasts of all models. We also estimated VARs with the same nine observable variables as the DSGE models, with up to 6 lags. The bad news is that all versions of the NOEM model perform very poorly when trying to forecast the real exchange rate. The preferred model with incomplete markets and PCP is the only one of the four that beats a simple random walk with drift: the "preferred model has a MSE of 4.56, while the random walk with drift has a MSE of 4.59 percent. Interestingly, a VAR with just one lag does not perform better, than the DSGE models, and it is only after including 4 lags in the VAR that the MSE for forecasting the real exchange rate drops significantly.

Table 9: Mean Squared Errors of One Period Ahead Forecasts (in percent)

	Euro Area				United States				
	Cons.	Output	Int.Rate	Inflation	Cons.	Output	Int.Rate	Inflation	RER
Closed Economy	0.46	0.53	0.09	0.21	0.38	0.73	0.11	0.14	-
Complete, PCP	0.33	0.52	0.19	0.38	0.35	0.72	0.11	0.08	4.59
Complete, LCP	0.34	0.53	0.14	0.35	0.39	0.75	0.11	0.08	4.95
Incomplete, PCP	0.45	0.59	0.11	0.43	0.42	0.73	0.11	0.18	4.56
Incomplete, LCP	0.43	0.59	0.11	0.31	0.41	0.71	0.12	0.17	4.76
VAR(1)	0.83	0.82	0.47	0.62	0.93	1.03	0.49	0.54	4.63
VAR(2)	0.49	0.47	0.22	0.30	0.57	0.66	0.23	0.25	4.03
VAR(4)	0.44	0.41	0.20	0.27	0.48	0.52	0.19	0.22	3.70
VAR(6)	0.38	0.35	0.10	0.21	0.40	0.45	0.13	0.17	3.24

While none of the DSGE models does a good job in forecasting real exchange rates, they can claim some victory in forecasting several macro variables. In particular, for consumption and interest rates in both areas, and inflation in the U.S., some version of the NOEM model outperforms the VAR model that includes 6 lags. Finally, the forecasting performance of the five DSGE models is quite similar, with no single model standing out as having the best forecasting performance for all variables.[34]

VI. Concluding remarks

In this paper we have estimated a two-country NOEM model for the U.S. and the euro area with a particular focus on the implications for real exchange rate dynamics. We have used a Bayesian approach to estimate the models' parameters and to compare a baseline two-country model with complete markets and producer currency pricing with two main extensions, namely incomplete markets and sticky prices in imported goods with local currency pricing.

Our results suggest that the complete markets assumption ends up attributing a very small role to international trade. In particular, we obtain a very low estimated parameter for the elasticity of substitution between home and foreign goods. Our close-to-zero estimate implies that the expenditure-switching effect of a real devaluation as a transmission mechanism is negligible. By contrast, the Bayesian estimation gives empirical support to the incomplete markets assumption. We find that the baseline model with complete markets and law of one price performs well in explaining real exchange rate dynamics, but at the cost of implying too large volatilities in other real variables, especially in the euro area. The extension with incomplete markets in which the law of one price holds performs best at fitting the data. In particular, this model is able to simultaneously account the real exchange rate volatility and persistence along with the negative correlation between the real exchange rate and relative consumptions. Interestingly, a model with both incomplete markets and sticky imports prices in local currency does not perform best, but it still outperforms models with complete markets. We show that both demand and technology shocks have played a major role in explaining the behavior of the real exchange rate, while monetary shocks have not.

There are some interesting avenues for future research, some of which we are exploring in ongoing work. We believe that the failure of the LCP assumption could be due to the fact that we are not explicitly using imports price series. Hence, if we want to explore the implications of these types of models for aggregate data, some other form of deviations from the law of one price should be explored. A promising line of research consists in incorporating distribution services in two-sector (with tradable and nontradable goods), two-country models. Campa and Goldberg (2004) provide evidence that deviations from the law of one price at the border due to the presence of distribution services helps explain a lower exchange rate pass-through at the consumer level than at the producer level. Finally, analyzing the out-of-sample forecasting performance of competing NOEM models, along the lines of the exercise performed by Del Negro et al. (2004) in the closed economy, and exploiting the information content of the net

[34] Using net foreign asset position data would enrich the real exchange rate dynamics, as shown by Lane and Milesi-Ferretti (2001). We also estimated the incomplete market models by using the quarterly U.S. net foreign asset position as an observed variable. The estimated parameters were broadly similar and in-sample forecast performance did not improve. Results are available upon request.

foreign asset position (Lane and Milesi-Ferreti, 2001) would help clarify the role of these models for policy formulation and analysis.

REFERENCES

Adolfson, M., S. Laseen, J. Lindé and M. Villani, 2004, "Bayesian Estimation of an Open Economy DSGE Model with Incomplete Pass-Through," Sveriges Riskbank Working Paper No. 179.

Altig, D., L. Christiano, M. Eichenbaum and J. Lindé, 2005, "Firm-Specific Capital, Nominal Rigidities and the Business Cycle," NBER Working Paper No. 11034.

Backus, D., P. Kehoe and F. Kydland, 1992, "International Business Cycles," *Journal of Political Economy,* Vol. 100, No. 41, pp. 745-775.

Batini, N., Justiniano, A., Levine, P., and J. Pearlman, 2005, "Model Uncertainty and the Gains from Coordinating Monetary Rules," paper presented at the International Research Forum on Monetary Policy: Third Conference, European Central Bank.

Baxter, M., and M. Crucini, 1993, "Explaining Saving-Investment Correlations," *American Economic Review,* Vol. 83, No. 3, pp. 416-436.

Baxter, M., and M. Crucini, 1995, "Business Cycles and the Asset Structure of Foreign Trade," *International Economic Review,* Vol. 36, No. 4, pp. 821-854.

Benigno, G., 2004, "Real Exchange Rate Persistence and Monetary Policy Rules," *Journal of Monetary Economics*, vol.51, pp. 473-502.

Benigno, P., 2001, "Price Stability with Imperfect Financial Integration," Manuscript, New York University.

Benigno, G. and P. Benigno, 2003, "Price Stability in Open Economies," *Review of Economic Studies*, 2003, vol. 70(4), no 245, pp. 743-65

Benigno, G. and C. Thoenissen, 2004, "Consumption and Real Exchange Rates with Incomplete Markets and Non-Traded Goods," mimeo, LSE

Bergin, P., 2004, "How Well Can the New Open Economy Macroeconomics Explain the Exchange Rate and the Current Account?," Manuscript, University of California at Davis.

Calvo, G., 1983, "Staggered Prices in a Utility Maximizing Framework," *Journal of Monetary Economics,* Vol. 12, No. 3, pp 383–398.

Campa, J. and L. Goldberg, 2004, "Do Distribution Margins Solve the Exchange-Rate Disconnect Puzzle?," working draft, Federal Reserve Bank of New York.

Clarida, R. and J. Galí, 1994, "Sources of Real Exchange Rate Fluctuations," *Carnegie-Rochester Series on Public Policy*, Vol. 41, pp. 1-56.

Clarida, R., J. Galí, and M. Gertler, 2002, "A Simple Framework for International Monetary Policy Analysis," *Journal of Monetary Economics*, Vol. 49, No .5, pp.879-904.

Corsetti, G., L. Dedola and S. Leduc, 2004, "International Risk Sharing and the Transmission of Productivity Shocks," European Central Bank Working Paper No. 368.

Chari, V.V. , P. Kehoe and E. McGrattan, 2002, "Can Sticky Price Models Generate Volatile and Persistent Real Exchange Rates?," *Review of Economic Studies*, Vol. 69, pp. 533-563.

Del Negro, M., F. Schorfheide, F., Smets and R. Wouters, 2004, "On the Fit and Forecasting Performance of New-Keynesian Models," Manuscript, Department of Economics, University of Pennsylvania.

Eichenbaum, M, and J. Fischer, 2004, "Evaluating the Calvo Model of Sticky Prices," NBER Working Paper 10617.

Fagan, G., J. Henry, and R. Mestre, 2001, "An Area-Wide Model (AWM) for the Euro Area," European Central Bank Working Paper No. 42.

Faust, J. and J. H. Rogers, 2003, "Monetary Policy's Role in Exchange Rate Behavior," *Journal of Monetary Economics*, Vol.50/7, pp. 1403-1424.

Fernández-Villaverde, J., and J.F. Rubio-Ramírez, 2004, "Comparing Dynamic Equilibrium Models to Data: A Bayesian Approach," *Journal of Econometrics*, Vol. 123, pp. 153-187.

Galí, J., M. Gertler and J.D. López-Salido, 2001, "European Inflation Dynamics," *European Economic Review*, Vol. 45, No. 7, pp. 1237-1270.

Galí, J., and T. Monacelli, 2005, "Monetary Policy and Exchange Rate Volatility in a Small Open Economy," *Review of Economic Studies*, forthcoming.

Galí, J. and P. Rabanal, 2004, "Technology Shocks and Aggregate Fluctuations: How Well Does the RBC Model Fit Postwar U.S. Data?," in M.Gertler and K. Rogoff (eds.), *NBER Macroeconomics Annual*, Vol. 19, pp. 225-288.

Ghironi, F., and M. J. Melitz, 2005, "International Trade and Macroeconomic Dynamics with Heterogeneous Firms ," forthcoming in the *Quarterly Journal of Economics.*

Heathcote, J., and F. Perri, 2002, "Financial Autarky and International Business Cycles," *Journal of Monetary Economics*, Vol. 49/3, pp. 601-627.

Heathcote, J., and F. Perri, 2004, "Financial Globalization and Real Regionalization," *Journal of Economic Theory*, Vol. 119/1, pp. 207-243.

Ireland, P., 2004, “Technology Shocks in the New Keynesian Model,” *Review of Economics and Statistics,* Vol. 86, Issue 4, pp. 923-936.

Justiniano, A., and B. Preston, 2004, “Small Open Economy DSGE Models-Specification, Estimation and Model Fit”, Manuscript, International Monetary Fund.

Kass, R. and A. Raftery, 1995, “Bayes Factors,” *Journal of the American Statistical Association*, Vol. 90, No. 430, pp. 773–795.

Kollmann, R., 2002, “Monetary Policy Rules in the Open Economy: Effect on Welfare and Business Cycles,” *Journal of Monetary Economics* 49, pp. 989-1015.

Lane, P. and G. Milesi-Ferretti, 2001, “Long-Term Capital Movements,” *NBER Macroeconomics Annual 2001.*

Lubik, T., and F. Schorfheide, 2003, “Do Central Banks Respond to Exchange Rate Fluctuations: A Structural Investigation,” Manuscript, Department of Economics, University of Pennsylvania.

Lubik, T., and F. Schorfheide, 2005, “A Bayesian Look at New Open Economy Macroeconomics,” *NBER Macroeconomics Annual 2005,* in press.

Monacelli, T., 2005, “Monetary Policy in a Low Pass-Through Environment,” forthcoming in the *Journal of Money Credit and Banking* (also ECB Working Paper No. 227).

Obstfeld, M. and K. Rogoff, 1995, “Exchange Rate Dynamics Redux,” *Journal of Political Economy* 103, pp 624-60.

Rabanal, P., 2004a, “The Cost Channel of Monetary Policy: Further Evidence for The United States and the Euro Area,” revised version of IMF Working Paper 03/149

Rabanal, P. 2004b, “Monetary Policy Rules and the U.S. Business Cycle: Evidence and Implications,” IMF Working Paper 04/164.

Rabanal, P. and J.F. Rubio-Ramírez, 2005, “Comparing New Keynesian Models of the Business Cycle: A Bayesian Approach,” *Journal of Monetary Economics,* Vol. 52, No. 6, pp.1151-1166 .

Schmitt-Grohe, S., and M. Uribe ,2003, “Closing Small Open Economy Models,” *Journal of International Economics,* 61, pp 163-185.

Selaive, J. and V. Tuesta, 2003a, “Net Foreign Assets and Imperfect Pass-through: The Consumption-Real Exchange Rate Anomaly” Board of Governors of the Federal Reserve System, International Finance Discussion Paper No. 764

Selaive, J. and V. Tuesta, 2003b, “Net Foreign Assets and Imperfect Financial Integration: An Empirical Approach,” Central Bank of Chile Working Paper No. 252.

Smets, F. and R. Wouters, 2003, “An Estimated Stochastic Dynamic General Equilibrium Model for the Euro Area,” *Journal of the European Economic Association*, Vol. 1, No. 5, pp. 1123-1175.

Stockman, A., and L. Tesar, 1995, “Tastes and Technology in a Two-Country Model of the Business Cycle: Explaining International Comovements,” *American Economic Review,* Vol. 85, No. 1, pp. 168-185.

Taylor, J., 1999, “Staggered Price and Wage-Setting in Macroeconomics," in *Handbook of Macroeconomics*, ed. by John Taylor and Michael Woodford (Amsterdam: Elsevier Science).

Trefler, D., and H. Lai, 1999, “The Gains from Trade: Standard Errors with CES Monopolistic Competition Model,” mimeo University of Toronto.

Yun, T., 1996, “Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles,” *Journal of Monetary Economics*, Vol. 37 (April), pp. 345-370.

APPENDIX: THE LIKELIHOOD FUNCTION AND THE METROPOLIS-HASTINGS ALGORITHM

The law of motion and the likelihood function

Let ψ denote the vector of parameters that describe preferences, technology, the monetary policy rules, and the shocks in the two countries of the model, d_t be the vector of all endogenous variables (state and forward looking), z_t the vector of exogenous variables (i.e. shocks), and ε_t the vector of innovations. x_t is the vector of the nine observable variables that will enter the likelihood function.

The system of equilibrium conditions and the process for the exogenous shocks can be written as a second-order difference equation

$$\begin{aligned} &A(\psi)E_t d_{t+1} = B(\psi)d_t + C(\psi)d_{t-1} + D(\psi)z_t, \\ &z_t = N(\psi)z_{t-1} + \varepsilon_t, \qquad E(\varepsilon_t\varepsilon_t^{'}) = \Sigma(\psi). \end{aligned}$$

We use standard solution methods for linear models with rational expectations to write the law of motion in state-space form. The *transition* and *measurement* equations are:

$$\begin{aligned} &d_t = F(\psi)d_{t-1} + G(\psi)z_t, \\ &z_t = N(\psi)z_{t-1} + \varepsilon_t, \qquad E(\varepsilon_t\varepsilon_t^{'}) = \Sigma(\psi). \end{aligned}$$

and

$$x_t = Hd_t$$

Let $y_t = [d_t^{'}, z_t^{'}]'$ be the vector of all variables, endogenous and exogenous. The evolution of the system can be rewritten as

$$y_t = \tilde{A}y_{t-1} + \tilde{B}\xi_t, \quad \text{where } E(\xi_t\xi_t^{'}) = I, \tilde{B} = \tilde{C}\Sigma^{1/2}, \text{ and } \varepsilon_t = \Sigma^{1/2}\xi_t.$$

and

$$x_t = \tilde{D}y_t$$

The $\tilde{A}, \tilde{B}, \tilde{C}$ and $\tilde{D}$ matrices are functions of *F,G,N,* and *Σ*. The $\tilde{D}$ matrix contains zeros everywhere, and a one in each row to select the variable of interest from the vector of all variables y_t. We can evaluate the likelihood function of the observable data conditional on the parameters $L(\{x_t\}_{t=1}^{T} \mid \psi)$, by applying the Kalman filter recursively as follows.

Define the prediction error as

$$v_t = x_t - x_{t|t-1} = x_t - \tilde{D}y_{t|t-1}.$$

whose mean squared error is

$$K_t = \tilde{D}P_{t|t-1}\tilde{D}',$$

where $x_{t|t-1}$ is the conditional expectation of the vector of observed variables using information up to *t-1*, and $P_{t|t-1} = E[(y_t - y_{t|t-1})(y_t - y_{t|t-1})']$.

The updating equations are:

$$y_t = y_{t|t-1} + P_{t|t-1}\tilde{D}' K_t^{-1} v_t, \text{ and } P_t = P_{t|t-1} - P_{t|t-1}\tilde{D}' K_t^{-1}\tilde{D}P_{t|t-1}.$$

And the prediction equations are:

$$y_{t+1|t} = \tilde{A}y_t, \text{ and } P_{t+1|t} = \tilde{A}P_t\tilde{A}' + \tilde{C}\Sigma\tilde{C}'.$$

Then, the log-likelihood function is equal to

$$L_t = -\frac{1}{2}\sum_{t=1}^{T}\{n\log(2\pi) + \log[\det(K_t)] + v_t' K_t^{-1} v_t\}.$$

where n is the size of the vector of observable variables x.

Note that the log-likelihood function has to be computed recursively. To initialize the filter, we set $y_0 = x_0 = 0$, and we set P_0 as the solution to the nonlinear system of equations $P = \tilde{A}P\tilde{A}' + \tilde{C}\Sigma\tilde{C}'$.

Drawing from the Posterior

To obtain a random draw of size N from the posterior distribution, a random walk Markov Chain using the Metropolis-Hastings algorithm is generated. The algorithm is implemented as follows:

1. Start with an initial value (ψ^0). From that value, evaluate the product $L(\{x_t\}_{t=1}^{T} | \psi^0)\Pi(\psi^0)$.

2. For each i:

$$\begin{cases} \psi^{i} = \psi^{i-1} \text{ with probability } 1\text{-}R \\ \psi^{i} = \psi^{i,*} \text{ with probability } R \end{cases}$$

where $\psi^{i,*} = \psi^{i-1} + v^{i}$, v^{i} follows a multivariate Normal distribution, and $R = \min\{1, \frac{L(\{x_t\}_{t=1}^{T} \mid \psi^{i,*})\Pi(\psi^{i,*})}{L(\{x_t\}_{t=1}^{T} \mid \psi^{i-1})\Pi(\psi^{i-1})}\}$.

The idea for this algorithm is that, regardless of the starting value, more draws will be accepted from the regions of the parameter space where the posterior density is high. At the same time, areas of the posterior support with low density (the tails of the distribution) are less represented, but will eventually be visited. The variance-covariance matrix of v^{i} is proportional to the inverse Hessian of the posterior mode and the constant of proportionality is specified such that the random draw has some desirable time series properties.

In all cases, the acceptance rates were between 20 and 30 percent, and the autocorrelation functions of the parameters decay fairly fast. We used two methods to simulate the posterior that delivered the same result (with very small numerical differences). First, we simulated the posterior 250,000 times taking as initial value the prior mean, and updating the Hessian of the posterior every 25,000 draws. We discarded all the values from that chain, and from the last value, generated a second chain of size 250,000, updating the Hessian each 25,000 draws. The second method involved finding the posterior mode using standard optimization algorithms to be used as initial value. Then, we generated a chain of 250,000 draws, updating the Hessian every 25,000 draws.