

# Limited Asset Markets Participation Inverts the Taylor Principle.

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**ABSTRACT.** This paper develops a simple framework to analyse monetary policy analytically in sticky-price dynamic general equilibrium with limited asset markets participation (LAMP). Some standard results in the monetary policy literature are *inverted*. Firstly, for simple interest rate rules, the usual Taylor principle is *generically* reversed: in order to ensure equilibrium uniqueness the central bank needs to pursue a *passive* rule. Responding to output gap and/or using fiscal policy for redistributive purposes may restore the results dictated by conventional wisdom. An interest rate peg is consistent with a unique rational expectations equilibrium under more restrictive conditions. Secondly, optimal time consistent monetary policy in a LAMP economy also implies passive policy rule. The conditions for these results to hold are relatively mild compared to some existing empirical evidence. We calculate theoretical effects of one-time sunspot and fundamental shocks to technology, policy rule, marginal cost under the two scenarios. Our results may help explain the 'Great Inflation' and give optimism for pre-Volcker FED policy.

**Keywords:** *monetary policy rules; Taylor Principle; real (in)determinacy; non-Ricardian features; limited asset markets participation; the Great Inflation*

**JEL codes:** *E31; E32, E44; E52; E58; E65.*

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## 1. Introduction

A tremendous amount of research has grown recently studying monetary policy in optimizing, dynamic general equilibrium models. The importance of this research, from a normative and positive standpoint and its influence on real-life policymaking need not be stressed here further. An excellent overview of the state-of-the-art in the field is the recent book by Michael Woodford (2003)<sup>1</sup>. Some of this literature's normative prescriptions are robust across a variety of modelling strategies. A brief enumeration follows. First, the central bank needs to adopt an '*active*' policy rule whereby it increases the nominal interest rate by more than inflation (i.e. increases the *real* interest rate), for policy to be consistent with a unique rational expectations equilibrium; this is labeled 'the Taylor Principle' following Woodford 2001<sup>2</sup>. Secondly, optimal and time consistent (discretionary) policy, minimizing inflation and output variability, also requires that the interest rate increase by more than inflation. Thirdly, when there is no trade-off between output and inflation stabilization, full stabilization of both is possible by making the policy instrument equal the '*natural rate of interest*'; however, a commitment to fulfill the Taylor principle is still required to ensure that the resulting equilibrium is unique. Relatedly, an interest rate peg (and any '*passive*' policy rule) is inconsistent with a unique equilibrium<sup>3</sup>, for any such policy would lead to multiple equilibria and stationary sunspot fluctuations (i.e. driven by beliefs and not fundamentals).

Such theoretical developments have been used by an intimately related branch of the literature to interpret various historical episodes. This literature (exemplified by i.a. Taylor 1999 and Clarida, Gali and Gertler 2000) estimates policy rules and tries to explore the link between monetary policy and macroeconomic performance. Estimated policy rules are then appended to general equilibrium models to study the effects of various fundamental shocks on, as well as variability of, macroeconomic variables. These theoretical predictions can then be compared with results of empirical studies. One instance of this is the study of the '*Great Inflation*' in the US in the 1970s. Researchers in the field first identified a change in monetary policymaking with the coming to office of Paul Volcker as a chairman of the FED in the US. Since macroeconomic performance (variability and responses of macro variables to shocks) was also found to have changed, explaining the latter by the former (policy change) became the norm in the profession. Namely, many authors have argued that policy before Volcker was 'badly' conducted along one or several dimensions, which led to worse macroeconomic performance as compared to the Volcker-Greenspan era. One such argument relies upon the estimated pre-Volcker policy rule non fulfilling the 'Taylor principle', hence containing the seeds of macroeconomic instability driven by non-fundamental uncertainty (CGG 2000).

The scope of our paper is *normative*: we show that standard theoretical prescriptions reviewed above are **reversed** when enough agents do not participate in asset markets. Namely, the Taylor Principle is inverted, optimal policy features

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<sup>1</sup>Earlier overviews of these issues comprise, amongst others, Clarida, Gali and Gertler (1999) and Goodfriend and King (1997).

<sup>2</sup>To be rigorous, this conclusion changes under some modelling choices. For example, in continuous time, Dupor (2000) shows that merely introducing capital invalidates the Taylor principle. A non-Ricardian fiscal policy in the sense of Woodford (1996) can also require a passive policy rule for equilibrium determinacy, as noted also by Leeper (1991).

<sup>3</sup>As in the much celebrated paper by Sargent and Wallace (1975).

a passive rule, and effects of some shocks are overturned. We derive our results analytically, and relate them to a particular situation in the labor market. We first exposit a standard dynamic sticky price model without capital incorporating *limited asset markets participation* (hereinafter *LAMP*). We derive the canonical form of this model reducing it to two equations for aggregate demand and supply. This makes our model easily comparable to the workhorse sticky price model (which occurs as a special case)<sup>4</sup>; since the resulting system is very simple, it might be of interest in itself to some researchers. Notably, we manage to capture the influence of LAMP on aggregate dynamics through a unique parameter, the elasticity of aggregate demand to real interest rates. We explain this influence by the impact of LAMP on labor markets: when participation in asset markets is restricted enough, the equilibrium wage-hours locus is upward sloping and cuts the labor supply curve from above (whereas it does so from below in the standard model). This will be at the core of the intuition for all our results. The degree of LAMP necessary and sufficient for our results to hold turns out to be small when compared to empirical estimates of Campbell and Mankiw (1989) or to data on asset market participation e.g. in Vissing-Jorgensen 2002.

In a companion paper (Bilbiie 2003), we take a *positive* standpoint for periods whereby estimated policy rules are passive, such as 1970's in the US. We argue that FED policy might have been better managed than conventional wisdom dictates, if such financial market imperfections as the ones making our theoretical results hold were pervasive during the 'Great Inflation' period. Most importantly, passive policy implies a determinate equilibrium and this allows effects of fundamental shocks to be studied. A change in financial imperfections might help explain the change in macroeconomic performance, aside the change in the policy response. The tremendous financial innovation and deregulation process in the 1979-1982 period and the abnormally high degree of regulation in the 1970's provide some support to this view. Moreover, it can be argued that such a change took place around 1980, same time as coming to office of Paul Volcker. The timing of the policy and structural change may not be a mere coincidence; instead, the abrupt change in the policy rule might be an optimal response to the structural change, an hypothesis which is yet to be tested. Finally, we present theoretical responses to and second moments of a parameterized (to US data) economy to cost-push and technology shocks, incorporating the structural change mentioned above. Since they qualitatively match what is found in some empirical exercises, one might conclude that our explanation cannot be fully dismissed.

Probably the most controversial among our assumptions concerns the structure of asset markets; hence, some further justification is in order. Following an emerging literature reviewed below, we assume that some agents have zero asset holdings, being either unable (constrained) or unwilling (myopic, uninformed) to participate to asset markets. This modelling assumption has been used in at least two strands of the macroeconomic literature: first, some version of it has been proposed by Mankiw (2000) and extended by Gali, Lopez-Salido and Valles (2003) for

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<sup>4</sup>There is very high variance regarding a label for such a framework in the literature. This goes from 'New Keynesian' (Clarida Gali and Gertler 1999 - henceforth CGG) to 'New Neoclassical synthesis' (Goodfriend and King 1997) to 'Neomonetarist' (Kimball 1996) to 'optimizing IS-LM' (McCallum and Nelson 1999). Woodford (2003) refers to such a framework as 'Neo-Wicksellian'.

fiscal policy issues. Second, it is the norm in the monetary policy literature trying to capture the 'liquidity effect' (e.g. Alvarez, Lucas and Weber 2001, Occhino 2003). Our assumption is very close in spirit to this second approach<sup>5</sup>: a subset of households cannot trade assets (this is sometimes called 'market segmentation'). Introducing physical capital as in Mankiw and GLV would merely allow for more heterogeneity: this is not needed for any of our points. Empirical support for our modelling choice comes from a variety of directions, e.g. failure of consumption smoothing as a good description of behaviour, high share of wealth-poor households in the data, low share of households holding assets, etc.. Mankiw argues for such a modelling choice based on evidence in Campbell and Mankiw (1989) suggesting that about half of the US population does not act in a consumption-smoothing manner<sup>6</sup>. Data on wealth distribution presented i.a. in Wolf 2000, Wolf et al 2002 shows that a high fraction of the population (even in the US) is wealth-poor – e.g., 50% of population have less than 5000 USD in liquid assets. It is hard to argue that these households have the means to perfectly smooth consumption. Moreover, data on asset holdings presented i.a. in Mankiw and Zeldes 1991, Vissing-Jorgensen 2002, Guiso, Haliassos and Japelli 2002 shows that few people hold assets in various forms. Vissing-Jorgensen 2002 reports based on the PSID data that of US population 21.75 percent hold stock and 31.40 percent bonds<sup>7</sup>. Data from the Survey of Consumer Finances (see e.g. Mulligan and Sala-i-Martin 2002) shows that in 1989 59 percent of US population had no interest-bearing financial assets, while 25 percent have no checking account either.

While having been already used to explain some puzzles in the finance-asset pricing and in the fiscal policy literature<sup>8</sup>, this modelling choice has only recently been incorporated into the sticky-price monetary policy research. A recent paper by Gali, Lopez-Salido and Valles (2003b, henceforth GLV) indeed argues that such a distinction makes the Taylor principle not a good guide for policy. Namely, GLV argue that if the central bank responds to current inflation via a simple Taylor rule, when the share of 'rule-of-thumb' agents is high enough the response coefficient has to be higher than that suggested by the Taylor principle. On the contrary, for a rule responding either to past or future expected inflation, GLV suggest, based on numerical simulations, that for a high share of non-asset holders the policy rule needs to violate the Taylor principle to ensure equilibrium uniqueness. Our paper's first point is closest (although not identical) to this last point. Instead, we argue that an 'Inverted Taylor principle' holds *in general* when asset market participation is restricted enough, no matter whether the policy rule responds to contemporaneous or future expected inflation. We provide intuition and explain the economic mechanism underlying our result. We then show how the Taylor principle

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<sup>5</sup>Papers in the 'liquidity effect' vein study a completely different question: whether a contractionary monetary policy shock (decrease of money supply) is indeed associated to an increase in interest rates, and has effects consistent with the data.

<sup>6</sup>Other papers (Flavin 1981, Zeldes 1989) find similar results, and support the view that such a failure to smooth consumption comes from liquidity constraints.

<sup>7</sup>She further uses this heterogeneity to obtain better micro estimates of the elasticity of intertemporal substitution and other preference parameters

<sup>8</sup>Gali, Lopez-Salido and Valles (2002) argue that this modelling assumption can help explaining the effects of government spending shocks if this is deficit-financed, taxation is lump-sum and labor is demand-determined. See also Bilbiie and Straub (2003b) for different labor market and budgetary structures.

can be restored by either an appropriate response to output or via distortionary redistributive taxation of dividend income.

The rest of the paper is organized as follows. In Sections 2 and 3 we introduce the LAMP sticky-price model and its reduced log-linear aggregate demand-supply system. A discussion of the labor market useful for further intuition is also presented. Section 4 outlines the 'Inverted Taylor Principle' and provides an intuitive discussion of this requirement. Section 5 discusses ways to restore the Taylor principle with limited participation. Section 6 contains a brief analysis of optimal monetary policy; we show this implies a passive instrument rule under LAMP. Section 7 calculates analytically the responses of the economy to cost-push, technology and sunspot shocks under various scenarios and Section 8 concludes.

## 2. A LAMP Sticky-Price Model

The model we use draws on Galí, López-Salido and Valles (2003), being a standard cashless dynamic general equilibrium sticky price model with Calvo-Yun pricing, augmented for limited asset markets participation. There is a continuum of households, a single perfectly competitive final-good producer and a continuum of monopolistically competitive intermediate-goods producers setting prices on a staggered basis. There is also a monetary authority setting its policy instrument, the nominal interest rate. The model is different from GLV in a few important respects. Most importantly, we abstract from capital accumulation<sup>9</sup>. The difference between the two classes of households comes from their access to complete markets for state-contingent securities, as in most papers on market segmentation and limited participation, e.g. Alvarez, Lucas and Weber (2001). This allows us to obtain analytical solutions and helps understanding the mechanism behind our results<sup>10</sup>. Two other differences are: (i) an additively separable utility function, useful for emphasizing the role of labor supply; and (ii) a fixed cost in the intermediate-goods sector, which when set properly insures there are no long-run profits (and increasing returns).

**2.1. Households.** There is a continuum of households  $[0, 1]$ . A  $1 - \lambda$  share is represented by households who are forward looking and smooth consumption, being able to trade in all markets for state-contingent securities: '*asset holders*' or savers. Each asset holder (subscript  $S$  denotes the representative asset holder) chooses consumption, asset holdings and leisure solving the following standard intertemporal problem:  $\max E_t \sum_{i=0}^{\infty} \beta^i U_S(C_{S,t+i}, 1 - N_{S,t+i})$ , subject to the sequence of constraints:

$$B_{S,t} + \Omega_{S,t+1} V_t \leq Z_{S,t} + \Omega_{S,t} (V_t + P_t D_t) + W_t N_{S,t} - P_t C_{S,t}.$$

Asset holder's momentary felicity function  $U_S(C_{S,t}, 1 - N_{S,t}) = \ln C_{S,t} + \theta_S \left[ (1 - N_{S,t})^{1-\gamma_S} \right] / (1 - \gamma_S)$  takes the form considered here to be consistent with most DSGE studies<sup>11</sup>.  $\beta \in$

<sup>9</sup>We have studied numerically a version of the model with capital accumulation subject to adjustment costs. The conclusions being largely robust, this extension did not justify the increase in complexity and hence loss of analytical solution. For the sake of space and clarity we stick to the version without investment, which is enough to make our point.

<sup>10</sup>Note that capital accumulation in itself may overturn the Taylor principle, at least in continuous time, as emphasized by Dupor (2000). This would obscure our paper's message.

<sup>11</sup>This function is in the King-Plosser-Rebelo class and leads to constant steady-state hours.

$(0, 1)$  is the discount factor,  $\theta_S > 0$  indicates how leisure is valued relative to consumption, and  $\gamma_S > 0$  is the coefficient of relative risk aversion to variations in leisure.  $C_{S,t}, N_{S,t}$  are consumption and hours worked by saver (time endowment is normalized to unity),  $B_{S,t}$  is the nominal value at end of period  $t$  of a portfolio of all state-contingent assets held, except for shares in firms. We distinguish shares from the other assets explicitly since their distribution plays a crucial role in the rest of the analysis.  $Z_{S,t}$  is beginning of period wealth, not including the payoff of shares.  $V_t$  is the market value at time  $t$  of a share,  $D_t$  is its real dividend payoff and  $\Omega_{S,t}$  are share holdings.

Absence of arbitrage implies that there exists a stochastic discount factor  $\Lambda_{t,t+1}$  such that the price at  $t$  of a portfolio with uncertain payoff at  $t + 1$  is (for both shares and rest of assets):

$$(2.1) \quad B_{S,t} = E_t [\Lambda_{t,t+1} Z_{S,t+1}] \text{ and } V_t = E_t [\Lambda_{t,t+1} (V_{t+1} + P_{t+1} D_{t+1})]$$

Note that the Euler equation for shares iterated forward gives the fundamental pricing equation:  $V_t = E_t \sum_{i=t+1}^{\infty} \Lambda_{t,i} P_i D_i$ . The riskless gross short-term nominal interest rate  $R_t$  is a solution to:

$$(2.2) \quad \frac{1}{R_t} = E_t \Lambda_{t,t+1}$$

Substituting the no-arbitrage conditions (2.1) into the wealth dynamics equation gives the flow budget constraint. Together with the usual 'natural' no-borrowing limit for *each* state, this will then imply the usual intertemporal budget constraint:

$$(2.3) \quad E_t \sum_{i=t}^{\infty} \Lambda_{t,i} P_i C_{S,i} \leq Z_{S,t} + V_t + E_t \sum_{i=t}^{\infty} \Lambda_{t,i} W_i N_{S,i}$$

Maximizing utility subject to this constraint gives the first-order necessary and sufficient conditions at each date and in each state:

$$\begin{aligned} \beta \frac{U_C(C_{S,t+1})}{U_C(C_{S,t})} &= \Lambda_{t,t+1} \frac{P_{t+1}}{P_t} \\ \theta_S (1 - N_{S,t})^{-\gamma_S} &= \frac{1}{C_{S,t}} \frac{W_t}{P_t} \end{aligned}$$

along with (2.3) holding with equality (or alternatively flow budget constraint hold with equality and transversality conditions ruling out overaccumulation of assets and Ponzi games be satisfied:  $\lim_{i \rightarrow \infty} E_t [\Lambda_{t,t+i} Z_{S,t+i}] = \lim_{i \rightarrow \infty} E_t [\Lambda_{t,t+i} V_{t+i}] = 0$ ). Using (2.3) and the functional form of the utility function, the short-term nominal interest rate must obey:

$$\frac{1}{R_t} = \beta E_t \left[ \frac{C_{S,t}}{C_{S,t+1}} \frac{P_t}{P_{t+1}} \right].$$

The rest of the households on the  $[0, \lambda]$  interval have no assets<sup>12</sup>: 'non-asset holders'. For a variety of reasons, these households do not smooth consumption. Reasons could include constraints of participation to asset markets, myopia, extreme hyperbolic discounting, limited information (whereby current income is the

<sup>12</sup>These households are labeled 'non-traders' by Alvarez, Lucas and Weber, 'rule-of-thumb' or 'non-Ricardian' by GLV, and 'spenders' by Mankiw 2000.

most salient piece of information), etc. Whether absence of assets is due to constraints or myopia (case in which their optimal asset holdings are zero), the problem of the representative non-asset holder indexed by  $H$  is:

$$(2.4) \quad \max_{C_{H,t}, N_{H,t}} \ln C_{H,t} + \theta_H \frac{(1 - N_{H,t})^{1-\gamma_H}}{1 - \gamma_H} \text{ s.t. } C_{H,t} = \frac{W_t}{P_t} N_{H,t}.$$

The first order condition is:

$$(2.5) \quad \theta_H (1 - N_{H,t})^{-\gamma_H} = \frac{1}{C_{H,t}} \frac{W_t}{P_t},$$

which further allows *reduced-form solutions* for  $C_{H,t}$  and  $N_{H,t}$  (functions only of  $W_t/P_t$  and exogenous processes). There is no need to keep consumption (or marginal utility of income) of  $H$  constant, as this does not depend on saving decisions or any other intertemporal feature. Note that due to the very form of the utility function, hours are constant for these agents: the utility function is chosen to obtain constant hours in steady state, and this agent is 'as if' she were in the steady state always. In this case labour supply of non-asset holders is fixed, no matter  $\gamma_H$ , as income and substitution effects cancel out. While this facilitates algebra, it is in no way necessary for our results (elastic labor supply will be discussed below). Hours are given by:  $(1 - N_{H,t})^{-\gamma_H} N_{H,t} = 1/\theta_H$  and consumption will track the real wage to exhaust the budget constraint.

**2.2. Firms.** The firms' problem is completely standard - see Gali (2002) or Woodford (2003) and can be skipped by some readers without loss of continuity (one generalization is in the production function of intermediate goods).

The **final good** is produced by a representative firm using a CES production function (with elasticity of substitution  $\varepsilon$ ) to aggregate intermediate goods:  $Y_t = \left( \int_0^1 Y_t(i)^{(\varepsilon-1)/\varepsilon} di \right)^{\varepsilon/(\varepsilon-1)}$ . Final good producers behave competitively, maximizing profit  $P_t Y_t - \int_0^1 P_t(i) Y_t(i) di$  each period, where  $P_t$  is the overall price index of the final good and  $P_t(i)$  are the prices of the intermediate goods. The demand for each intermediate input is  $Y_t(i) = (P_t(i)/P_t)^{-\varepsilon} Y_t$  and the price index is  $P_t = \left( \int_0^1 P_t(i)^{1-\varepsilon} di \right)^{1/(1-\varepsilon)}$ .

Intermediate goods are produced by a continuum of monopolistically competitive firms indexed by  $i$  and distributed over the unit interval. Firms in this sector face a technology which is linear in labor:

$$Y_t(i) = A_t N_t(i) - F(i), \text{ if } N_t(i) > F(i) \text{ and } 0 \text{ otherwise.}$$

$F(i)$  is a firm-specific fixed cost: this will be a free parameter that can be chosen such that profits are zero in steady state and there are increasing returns to scale, consistent with evidence by Rotemberg and Woodford (1995). Alternatively, if the fixed cost is zero, there are steady-state profits (which is the case in GLV). We shall encompass both cases. Cost minimization taking the wage as given implies that real marginal cost is:  $MC_t/P_t = W_t/(P_t A_t)$ . The (nominal) profit function

is given by  $P_t(i) D_t(i) = P_t(i) Y_t(i) - MC_t (Y_t(i) + F(i))$ .

Following Calvo (1983) and Yun (1996) we assume that intermediate good firms adjust their prices infrequently. The opportunity to adjust follows a Bernoulli distribution with  $\theta$  being the probability of keeping the price constant. This exogenous

probability is independent of history. Thus each period there is a fraction of firms that keep their prices unchanged. The firm maximizes its value, i.e. the discounted sum of future nominal profits, using the relevant stochastic discount factor  $\Lambda_{t,t+i}$  used as pricing kernel for nominal payoffs by the shareholders:

$$\max_{P_t(i)} E_t \sum_{s=0}^{\infty} (\theta^s \Lambda_{t,t+s} [P_t(i) Y_{t,t+s}(i) - MC_{t+i} Y_{t,t+s}(i)]),$$

subject to the demand equation (at  $t+s$ , conditional upon price set  $s$  periods in advance)  $Y_{t,t+s}(i) = (P_t(i)/P_{t+s})^{-\varepsilon} Y_{t+s}$ . The optimal price of the firm is then found as usually as a markup over a weighted average of expected future nominal marginal costs:

$$(2.6) \quad \begin{aligned} P_t^{opt}(z) &= (1 + \mu) E_t \sum_{s=0}^{\infty} \varpi_{t,t+s} MC_{t+s} \\ \varpi_{t,t+s} &= \frac{\theta^s \Lambda_{t,t+s} (P_{t+s})^{\varepsilon-1} Y_{t+s}}{E_t \sum_{k=0}^{\infty} \theta^k \Lambda_{t,t+k} (P_{t+k})^{\varepsilon-1} Y_{t+k}} \end{aligned}$$

In equilibrium each producer that chooses a new price  $P_t(i)$  in period  $t$  will choose the same price and the same level output. Then the dynamics of the price index given the aggregator above is:  $P_t = \left( (1 - \theta) P_t^{opt}(i)^{1-\varepsilon} + \theta P_{t-1}(i)^{1-\varepsilon} \right)^{1/(1-\varepsilon)}$ . The combination of these two conditions leads in the log-linearized equilibrium to the well known New Keynesian Phillips curve given below. Profits will also be equal across producers,  $D_t = (1 - MC_t/P_t) Y_t - (MC_t/P_t) F$ .

**2.3. Monetary policy.** We consider two policy frameworks prominent in the literature. First, we study *instrument rules* in the sense of a feedback rule for the instrument (short-term nominal interest rate) as a function of macro variables, mainly inflation. We focus on rules within the family (where variables with a star are 'natural' levels of the corresponding variables, defined below):

$$(2.7) \quad R_t = (R_t^*)^{\phi^*} R \left( E_t \frac{P_{t+k}}{P_{t-1+k}} \right)^{\phi_\pi} \left( E_t \frac{Y_{t+k}}{Y_{t+k}^*} \right)^{\phi_y} e^{\varepsilon_t}.$$

We shall also consider targeting rules under discretionary policymaking, whereby the path of the nominal rates is found by optimization by the central bank - this is described in detail in Section 6 below. Such a framework will also imply a behavioral relationship for the instrument rule, but this is only an *implicit instrument rule*.

**2.4. Market clearing, aggregation and accounting.** Labor and goods market clearing imply respectively  $N_t = \lambda N_{H,t} + (1 - \lambda) N_{S,t}$  and  $Y_t = C_t \equiv \lambda C_{H,t} + (1 - \lambda) C_{S,t}$ , where  $C_t$  is aggregate consumption. State-contingent assets are in zero net supply, as is the case since markets are complete and agents trading in them are identical. By Walras' Law, the equity market clears and hence share holdings of each of the asset holders are then given by:

$$\Omega_{S,t+1} = \Omega_{S,t} = \Omega = \frac{1}{1 - \lambda}.$$

### 3. A simplified linear aggregate demand-supply LAMP model

We seek to express the above model in a form similar to models usually employed for monetary policy analysis (see CGG 1999, Woodford 2003); several substitutions of the log-linearized equations, exposted in detail in the Appendix, deliver a model of such a form:

$$(3.1) \quad AS : \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t, \text{ where } \kappa \equiv \psi \chi$$

$$(3.2) \quad IS : x_t = E_t x_{t+1} - \delta^{-1} [r_t - E_t \pi_{t+1} - r_t^*]$$

The reduced-form parameters can be explained in terms of deep parameters as:

$$\begin{aligned} \chi &\equiv 1 + \varphi^s \frac{1}{1 + F_Y} \left( 1 + \frac{\lambda}{1 - \lambda} D_Y \right) \geq 1, \\ \delta &\equiv 1 - \varphi^s \frac{\lambda}{1 - \lambda} \frac{1}{1 + \mu}, \end{aligned}$$

where  $\varphi^s = \frac{\gamma^s N_s}{1 - N_s}$  is the inverse of the Frisch elasticity of labor supply of asset holders,  $F_Y$  the share of the fixed cost in total output in steady state (and the degree of increasing returns to scale),  $D_Y \equiv (\mu - F_Y) / (1 + \mu)$  is the share of profits in steady state output and  $x_t \equiv y_t - y_t^*$  is output gap. *Natural levels*  $y_t^*$  and  $r_t^*$  of output and interest rates are calculated under flexible prices and are denoted with a star. Natural output is a function of technology:  $y_t^* = \left[ 1 + F_Y \left( 1 - \frac{1}{\chi} \right) \right] a_t$ . Note that permanent shocks have permanent effects on natural output. The natural (Wicksellian) rate of interest  $r_t^*$  as the level of interest rates consistent with output being at its natural level (and hence with zero inflation), as in Woodford 2003:  $r_t^* = \left[ 1 + F_Y \left( 1 - \frac{\delta}{\chi} \right) \right] [E_t a_{t+1} - a_t]$ . We assume that technology growth ( $\Delta a_t \equiv a_t - a_{t-1}$ ) is given by an AR(1) process  $\Delta a_t = \rho^a \Delta a_{t-1} + \varepsilon_t^a$ , which implies shocks to technology have permanent effects (see Gali 1999, Gali, Lopez-Salido and Valles 2003a). Note that  $r_t^* = \left[ 1 + F_Y \left( 1 - \frac{\delta}{\chi} \right) \right] \rho^a \Delta a_t$ , such that the natural interest rate increases with technology growth shocks.

Equation (3.1) is a Philips curve relating inflation to expected inflation and output gap variations. Following Clarida, Gali and Gertler (1999) or Gali (2002) we also introduce 'cost-push' shocks  $u_t$ , i.e. variations in marginal cost not due to variations in excess demand. These could come from the existence of sticky wages creating a time-varying wage markup, a time-varying elasticity of substitution among intermediate goods or other sources creating this inefficiency wedge between the efficient and natural levels of output (e.g. distortionary taxation). For details as to what these time-varying wedges could be, see Woodford (2003, Ch. 6). Generally, marginal cost variations will be given by  $mc_t = \chi x_t + \psi^{-1} u_t$ . The aggregate supply side AS differs from the standard framework only insofar as the presence of non-asset holders modifies  $\chi$ , i.e. the elasticity of the marginal cost to movements in the output gap, and hence the response of inflation to aggregate demand variations. However, with increasing returns to scale of a degree making profits zero in steady state, the AS curve does not modify at all<sup>13</sup>. Rotemberg and Woodford (1995) make a strong case for such a degree of increasing returns.

Equation (3.2) is the correspondent of an aggregate demand AD (or IS) function derived from the Euler equation of the asset holders (please find Appendix for a

<sup>13</sup>In this case consumption is equal across groups in steady state.

detailed derivation). The main difference under LAMP is that to obtain a dynamic equation in terms of the aggregate output (gap), we need to express consumption of asset holders as a function of output. In our model these two objects might be related negatively ( $\delta < 0$ ) for reasons explained below. This modifies drastically determinacy properties of the model, its response to shocks, and the optimal design of interest rate rules.

**DEFINITION 1.** *We call an economy in participation in asset markets is limited enough such that  $\delta < 0$  a '**LAMP economy**' for short.*

To end up in such an economy, the share of non-asset holders needs to be larger than a threshold  $\lambda^*$ :

$$(3.3) \quad \lambda > \lambda^* = \frac{1}{1 + \varphi^s \frac{1}{1+\mu}}.$$

This threshold is smaller, the more inelastic is labor supply. The intuition for this result is simple<sup>14</sup>. The IS curve is obtained, as usually, from the Euler equation for consumption. However, only consumption of asset holders obeys such an equation. To find an equation in output, one needs to express consumption of asset holders as a function of total output. The two objects can be related *negatively* due to the negative effect of real wages (real marginal costs) on profits (and hence dividends). This negative link between wage and profits is stronger, the more inelastic labor supply is (the higher is  $\varphi^s$ ), for an inelastic labor supply implies low movements in output and hence in sales. Profit variations, on the other hand, become relatively more important in asset holders' total income as the relative share of non-asset holders increases (higher  $\frac{\lambda}{1-\lambda}$ )<sup>15</sup>. When both these conditions are met (high  $\varphi^s$  and  $\frac{\lambda}{1-\lambda}$ ) as in (3.3), asset holders' consumption decreases when output increases, since the increase in output is in that case driven by a high increase in real wage, which implies a strong fall in profits and a fall in dividends dominating the real wage increase<sup>16</sup>. It is obvious that the only way for  $\delta$  to be independent of  $\lambda$  is for  $\varphi^s$  to be zero, i.e. labor supply of asset holders be infinitely elastic. In this case, consumption of all agents is independent of wealth, making the heterogeneity introduced in this paper irrelevant.

**Parameterization** We shall now have a first glance at the magnitude of  $\lambda$  required for our results to hold, quantitatively. To that end, and for further use in simulations, we parameterize the model at quarterly frequency; the baseline case follows GLV (except for the mentioned differences) and most monetary policy studies. Namely, we set the discount factor  $\beta$  such that  $r = 0.01$ , the steady state markup  $\mu = 0.2$  corresponding to an elasticity of substitution of intermediate goods of 6. The fixed cost parameter (and degree of increasing returns to scale) is set to either 0 (steady-state profits) or  $F_Y = \mu = 0.2$ . The average price duration is one year, implying  $\theta = 0.75$ . As to parameterizing labour, this is somehow more delicate, for there is no data to the best of our knowledge disentangling various preferences for leisure, or equivalently hours worked, as a function of wealth. Here,

<sup>14</sup>For derivations and further intuition see Bilbiie (2003).

<sup>15</sup>In the standard model all agents hold assets, so this mechanism is completely irrelevant. Any increase in wage exactly compensates the decrease in dividends, since all output is consumed by asset holders.

<sup>16</sup>Obviously, the mechanism relies upon real wage flexibility. Introducing wage stickiness would weaken this mechanism and help restore the standard results.

as we have no priors for imposing otherwise, we assume both types work the same number of hours in steady state hence  $N = N_S = N_H = \frac{1}{3}$  as commonly assumed in the literature. Then, for each value of the elasticity of marginal utility of leisure to leisure  $\gamma_j$  we can find a level of  $\theta_j$ . This allows us to have a free parameter for the inverse Frisch elasticity of labor supply  $\varphi^S = \frac{\gamma^S N}{1-N}$ , a parameter for which we shall consider different values. Different values are also considered for the share of non-asset holders  $\lambda$  since this is probably our most controversial parameter - empirical evidence by Campbell and Mankiw 1989 suggests this is around 0.5 for the US economy.

In Figure 1, we plot the inverse slope of IS,  $\delta$  as a function of the share of non-asset holders and for different labor supply elasticities. The thick line illustrates our baseline parameterization  $\varphi^S = 10$ , where we see that  $\delta$  changes sign at around 0.1 share of non-asset holders. The other two cases (presented for illustrative purposes) are the extremes of: infinitely elastic labor supply (horizontal line), when  $\delta$  is independent of the share of asset holders; and of almost completely inelastic labor supply (almost vertical line), where  $\delta$  changes sign for a small measure of non-asset holders.

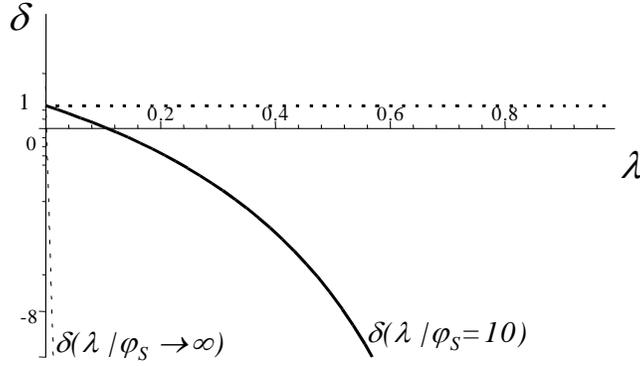


Fig.1: Inverse slope of IS as a function of the share of non-asset holders, for different labor supply elasticities.

**3.1. Further intuition - the labor market.** Key to understanding the results further obtained here is the equilibrium in the labor market. In system (3.4) we outline the labor supply and the equilibrium wage-hours locus. For labor supply, we only keep consumption of asset holders constant, for there is no intertemporal substitution for the non-asset holders. The equilibrium wage-hours locus labeled  $WN$  is derived taking into account all equilibrium conditions, most notably how consumption is related to real wage in equilibrium. This schedule will be fixed in equilibrium (in fact, it will be shifted by technology shocks only) and hence not affected by policy.

$$(3.4) \quad \begin{aligned} WN &: w_t = \left[ (1 + F_Y) \delta + \varphi^s \frac{1}{1 - \lambda} \right] n_t + (1 + F_Y) a_t \\ LS &: w_t = \varphi^s \frac{1}{1 - \lambda} n_t + c_{s,t} \end{aligned}$$

A 'LAMP economy' ( $\delta < 0$ ) has an intuitive interpretation in labor market terms, for it implies that the equilibrium wage-hours locus is less upward sloping than (and hence cuts from above) the labor supply curve. Intuitively, the presence of non-asset holders generates overall a 'negative income effect', which cannot be obtained *ceteris paribus* when  $\lambda = 0$  and  $\delta = 1$ . In the latter, standard case, the wage-hours locus is more upward sloping than LS. The difference between the two is the intertemporal elasticity of substitution in consumption, normalized to 1 in our case (multiplied by returns to scale  $1+F_Y$ ). *Ceteris paribus*, if the labor demand shifts out, labor supply shifts leftward due to the usual income effect, since agents anticipate higher income and higher consumption. If labor supply shifts up due to a positive income effect, same effect makes labor demand shift out. This gives a WN locus more upward sloping than the labor supply curve LS. The threshold value for  $\lambda$  for this insight to change is the same as that making  $\delta < 0$  and given in (3.3) above. When the share of non-asset holders is higher than this threshold (or equivalently for a given share, labor supply of asset holders is inelastic enough), the wage-hours locus becomes less upward sloping than the labor supply. An intuition for that follows, as illustrated in Figure 2; we assume that the real interest rate is constant along the equilibrium path for simplicity<sup>17</sup>.

Take first an outward *shift in labor demand*. Keeping supply fixed, there would be an increase in real wage and an increase in hours (their relative sizes depending on elasticity of labor supply as usual). The increase in the real wage would boost consumption of non-asset holders, henceforth amplifying the initial demand effect. When labor supply is relatively inelastic, this increase in wage is large and the increase in hours is small compared to that necessary to generate the extra output demanded; note that the effect induced on demand is larger, higher the share of non-asset holders. The only way for supply to meet demand is for labor supply to shift right. This is insured in equilibrium by a fall in profits, resulting from: (i) increasing marginal cost (since wage increases) and (ii) the weak increase in hours and hence in output and sales. This is like and indirect negative income effect induced on asset holders by the presence of non-asset holders. Next consider a *shift in labor supply*, for example leftward as would be the case if consumption of asset holders increased. Keeping demand fixed, wage increases and hours fall. The increase in wage (and the increase in consumption of asset holders itself) has a demand effect due to sticky prices. As labor demand shifts right, the real wage would increase by even more; hours would increase, but by little due to the relatively inelastic labor supply (the overall effect would again depend on the relative slopes of the two curves). The increase in the real wage means extra demand through non-asset holders' consumption<sup>18</sup>. To meet this demand, only way for increasing output is an increase in labor supply, which instead obtains only if labor supply shifts right, which is insured as before by the fall in profits. This explains why in a 'LAMP economy' the wage-hours locus cuts the labor supply curve from above. This instead will help our intuition in explaining the further results<sup>19</sup>. Note that such a

<sup>17</sup>How the nominal interest rate reacts to inflation, generated here by variations in demand, will be crucial in the further analysis.

<sup>18</sup>The assumptions on preferences ensuring constant steady-state hours are less crucial than it might seem. Below we consider alternative preference specifications.

<sup>19</sup>Note that the intuition for real indeterminacy to obtain in standard models (see e.g. Benhabib and Farmer 1994) requires the wage hours locus be upward sloping but cut the labor supply curve from *below*. This is also the case in standard sticky-price models, and gives rise to a certain

wage-hours locus implies that the model generates a higher equilibrium elasticity of hours to the real wage, and more so more negative  $\delta$  is. The model still generates a procyclical real wage, but less so than the standard model. Relatedly, it predicts a higher volatility of hours and lower volatility of the wage in response to shocks shifting the two curves.

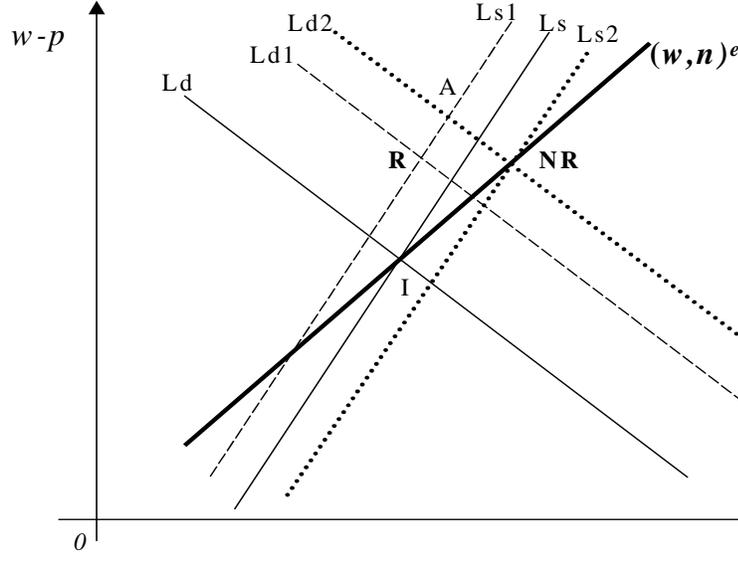


Fig. 2: The equilibrium wage-hours locus and labor supply curve with LAMP.

Having derived the equilibrium wage-hours locus gives us a simple way of thinking intuitively about the effects of shocks and of monetary policy in general; monetary policy, by changing nominal interest rates, modifies real interest rates and hence shifts the labor supply curve (by changing the intertemporal consumption profile of asset holders). But such shocks have no effect on the wage-hours locus by construction, since this describes a relationship that holds in equilibrium always.

**3.2. Robustness.** One might rightly wonder whether the mere theoretical possibility of a change in the sign of  $\delta$  is entirely dependent upon the specification of preferences. It turns out this possibility is robust to two obvious candidates: an elastic labor supply of non-asset holders, and a non-unitary elasticity of intertemporal substitution in consumption. For completion, we briefly study these extensions jointly. Consider preferences given by a general CRRA utility function for both agents  $j$  ( $\gamma_j^C$  is relative risk aversion or inverse of intertemporal elasticity of substitution in consumption):

$$(3.5) \quad U_j(\cdot, \cdot) = \frac{C_{j,t}^{1-\gamma_j^C}}{1-\gamma_j^C} + \theta_j \frac{(1-N_{j,t})^{1-\gamma_j}}{1-\gamma_j}$$

requirement for the monetary policy rule to result into real determinacy - see below. Our intuition will be that having the wage-hours locus cut the labor supply from *above*, changes determinacy properties in a certain way.

Following the same method as before one can show that the solution to non-asset holders' problem will be (in log-linearized terms, where elasticity of hours to wage  $\eta_H \equiv (1 - \gamma_H^C) / (\gamma_H^C + \varphi_H)$  is positive iff  $\gamma_H^C < 1$ ):

$$(3.6) \quad n_{h,t} = \eta_H w_t; \quad c_{h,t} = (1 + \eta_H) w_t$$

For asset holders, the new Euler equation and intratemporal optimality in log-linearized form are:

$$(3.7) \quad E_t [c_{s,t+1}] - c_{s,t} = (\gamma_S^C)^{-1} (r_t - E_t [\pi_{t+1}])$$

$$(3.8) \quad \varphi_s n_{s,t} = w_t - \gamma_S^C c_{s,t}$$

Using the same method as previously, one finds that the new condition to be fulfilled in order for  $\delta$  to become negative and hence end up in a 'LAMP economy':

$$(3.9) \quad \lambda > \frac{1}{1 + \frac{1}{1+\mu} \varphi_S - \eta_H \frac{\mu}{1+\mu} \varphi_S}$$

Two results stand out: (1) the threshold level is *independent* of the elasticity of intertemporal substitution in consumption  $\gamma_S^{C20}$ ; (2) the elasticity of labor supply of non-asset holders influences the magnitude of the threshold but does not cancel the result. Consider for example equal labour supply elasticities,  $\eta_H = (\varphi_S)^{-1} = \eta$ . If  $\eta$  is in the 0.1-0.5 range as suggested by empirical evidence, the threshold is in the 0.109-0.4 range. The intuition is that while making aggregate labor supply more elastic, a positive  $\eta_H$  also makes *equilibrium* hours more elastic to wage changes since it makes consumption of non-asset holders more responsive to the wage. In view of the relative innocuousness of these assumptions, we shall continue using the log-CRRA utility function in the remainder, since it preserves constant steady-state hours and hence allows analyzing permanent technology shocks.

#### 4. The Inverted Taylor Principle: Determinacy properties of interest rate rules

In this Section we study determinacy properties of simple interest rate rules. We shall consider for analytical simplicity only rules whereby the interest rate does not respond to the output gap, and there is no inertia (interest rate smoothing) - such extensions should be straightforward. We first consider rules involving a response to expected inflation, as done for example by CGG (2000). This specification provides simpler (sharper) determinacy conditions, and captures the idea that the central bank responds to a larger set of information than merely the current inflation rate:

$$(4.1) \quad r_t = \phi_\pi E_t \pi_{t+1} + \varepsilon_t$$

where  $\varepsilon_t$  is the non-systematic part of policy-induced variations in the nominal rate. The dynamic system for the  $z_t \equiv (y_t, \pi_t)'$  vector of endogenous variables and the  $\nu_t \equiv (\varepsilon_t - r_t^*, u_t)'$  vector of disturbances is:

$$E_t z_{t+1} = \mathbf{\Gamma} z_t + \mathbf{\Psi} \nu_t$$

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<sup>20</sup>The *magnitude* of the elasticity of aggregate demand to interest rate will of course depend on  $\gamma_S^C$ , but not its sign.

The coefficient matrices are given by:

$$(4.2) \quad \begin{aligned} \mathbf{\Gamma} &= \begin{bmatrix} 1 - \beta^{-1}\delta^{-1}\kappa(\phi_\pi - 1) & \delta^{-1}\beta^{-1}(\phi_\pi - 1) \\ -\beta^{-1}\kappa & \beta^{-1} \end{bmatrix} \\ \mathbf{\Psi} &= \begin{bmatrix} \delta^{-1} & -\beta^{-1}\delta^{-1}(\phi_\pi - 1) \\ 0 & -\beta^{-1} \end{bmatrix} \end{aligned}$$

Determinacy again requires that both eigenvalues be outside the unit circle. The determinacy properties of such a rule are emphasized in Proposition 2.

**PROPOSITION 1. *The Inverted Taylor Principle:*** *An interest rate rule such as (4.1) delivers a unique rational expectations equilibrium (i.e. the equilibrium is determinate) if and only if:*

$$\begin{aligned} \text{Case I:} & \text{ If } \delta > 0, \phi_\pi \in \left(1, 1 + \delta \frac{2(1+\beta)}{\kappa}\right); \\ \text{Case II:} & \text{ If } \delta < 0, \phi_\pi \in \left(1 + \delta \frac{2(1+\beta)}{\kappa}, 1\right) \cap [0, \infty). \end{aligned}$$

A proof is in the Appendix. Case I can be viewed as the standard case: the Taylor principle (Woodford 2001) is at work, and as noted in the previous literature the central bank should respond more than one-to-one to increases in inflation. It should also not respond 'too much', which is a well-established result first noted by Bernake and Woodford (1997).

Case II is the 'LAMP economy'. In this case, the Central Bank should follow an *Inverted Taylor Principle*: only passive policy is consistent with a unique rational expectations equilibrium. A result such as our Proposition 2 has first been noted (relying upon numerical simulations and not as a general result) by Gali et al 2003. The message of our paper, however, is different. For we provide analytical conditions for an inverted Taylor principle to hold generically, independently on the policy rule followed; while in Gali et al, it is only for a forward-looking rule, and only for a high share of non-asset holders that this result applies. Further down we show that a version of the *Inverted Taylor Principle* holds for a contemporary rule also. Obviously, the condition for the *Inverted Taylor Principle* to hold is the same as the one causing a change in the sign of  $\delta$ , as in (3.3). In the figure below, we plot the threshold  $\lambda$  and  $\varphi^s$ , such that values under the curve give the  $\delta > 0$  case, whereas above the curve we have the LAMP economy with  $\delta < 0$ .

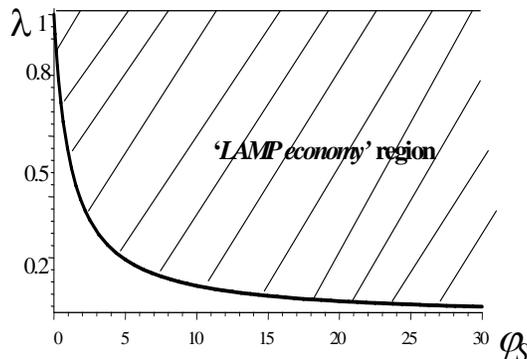


Fig. 3: Threshold share of non-asset holders as a function of inverse labor supply elasticity. Above the threshold we have the 'LAMP economy' where the Inverted Taylor Principle applies.

This means that for the Taylor principle to work, the Frisch elasticity of labor supply (and of intertemporal substitution in labor supply), should be high, and higher, the higher the share of non-asset holders  $\lambda$  is. For a range of  $\varphi^s$  between 1 (unit elasticity) and 10 (0.1 elasticity) the threshold share of non-asset holders should be lower than 0.5 to as low as around 0.1 respectively. This shows once more that the required share of non-asset holders for the standard results to be overturned is not that large. I6PENM02.wmf

**4.1. A simple Taylor rule.** For completion we also study determinacy properties of a simple Taylor rule. This is done to further illustrate the differences of our results to Gali et al (2003), where there is a dramatic distinction between forward-looking and contemporaneous rules. For a contemporaneous rule to be compatible with a unique equilibrium, they note that the central bank should respond to increases in inflation more strongly (and indeed very strongly under some parameter constellations). Our results have the same flavor as for a forward-looking rule: an inverted Taylor principle holds generically, i.e. if we exclude some extreme values for some of the parameters. We consider rules of the form:

$$(4.3) \quad r_t = \phi_\pi \pi_t + \varepsilon_t$$

Replacing this in the IS equation (B.10) and using the same method as previously (described at length in the Appendix) we obtain the following Proposition.

**PROPOSITION 2.** *An interest rate rule such as 4.3 delivers a unique rational expectations equilibrium (i.e. the equilibrium is determinate) if and only if:*

*Case I: If  $\delta > 0$ ,  $\phi_\pi > 1$  (the 'Taylor Principle')*

*Case II: If  $\delta < 0$ ,*

$$\phi_\pi \in \left[ 0, \min \left\{ 1, \delta \frac{\beta - 1}{\kappa}, \delta \frac{-2(1 + \beta)}{\kappa} - 1 \right\} \right) \cup \left( \max \left\{ 1, \delta \frac{-2(1 + \beta)}{\kappa} - 1 \right\}, \infty \right)$$

In the Appendix we prove this and distinguish a few cases for the implied condition on the policy rule coefficient as a function of deep parameters of the model. It turns out that the 'inverted Taylor principle' holds in Case II for a somewhat larger share of non-asset holders than was the case under a forward-looking rule. It is also the case that a policy rule responding to current inflation very strongly would insure equilibrium uniqueness<sup>21</sup>. But we also argue that the implied response ( $\phi_\pi = 35$  under the baseline parameterization): (i) is much larger than any plausible empirical estimate; (ii) would imply that zero bound on nominal interest rates be violated for even small deflations; (iii) would have little credibility. This is in clear contrast with Gali et al (2003), who do not look at a possible inversion of the Taylor principle in their numerical analysis of such rules, but instead argue that for a large share of non-asset holders making the required policy response too strong under a Taylor rule, the central bank should switch to a passive forward-looking rule.

What is missing is an intuitive explanation as to why is it that a passive rule is compatible with a unique equilibrium in a 'LAMP economy', whereas an active rule is generally not. This is what we try to study next.

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<sup>21</sup>This is not the case under a forward looking rule, since there, even in a standard full-participation economy too strong a response leads to indeterminacy - see Bernake and Wodford 1997.

**4.2. Intuition: sunspot equilibria with LAMP.** It is useful to conduct a mental experiment trying to construct a sunspot equilibrium - in the last section, we will compute sunspot equilibria formally. In an economy whereby  $\delta > 0$ , this is possible if monetary policy is passive, and not if it is active, which is a well-established result in the literature in a model like our Case I. There, a non-fundamental increase in expectations about inflation and/or output matched by too weak a policy response (a fall in the real rate) causes an increase in consumption of asset holders today; since total demand is equal to consumption of asset holders, this boosts aggregate demand and increases inflation, which in turn makes the initial inflationary expectation self-fulfilling.

In a 'LAMP economy' (Case II), this is reversed. First, we cannot construct sunspot equilibria with a *passive* policy rule  $\phi_\pi < 1$ . The crucial difference in this case is that aggregate demand is no longer completely forward-looking, i.e. linked to demand of asset holders. Suppose for simplicity and without losing generality that the sunspot is located in inflationary expectations. A non-fundamental increase in expected inflation causes a fall in the real interest rate. This leads to an increase in consumption of asset holders, and an increase in the demand for goods; but note these are now partial effects. Indeed, to work out the overall effects one needs to look at the component of aggregate demand coming from non-asset holders and hence at the labor market. The partial effects identified above would cause an increase in the real wage (and a further boost to consumption of non-asset holders) and a fall in hours. Increased demand, however, means that (i) some firms adjust prices upwards, bringing about a further fall in the real rate (as policy is passive); (ii) the rest of firms increase labor demand, due to sticky prices. Note that the real rate will be falling along the entire adjustment path, amplifying these effects. But since this would translate into a high increase in the real wage (and marginal cost) and a low increase in hours, it would lead to a fall in profits, and hence a negative income effect on labor supply. The latter will then not move, and no inflation will result, ruling out the effects of sunspots. This happens when asset markets participation is limited 'enough' in a way made explicit by (3.3).

Next, consider 'LAMP economy' with an *active* interest rate rule  $\phi_\pi > 1$ . We suggest that a sunspot equilibrium is always possible to construct in this case. Consider the same thought experiment as above, which now leads to a fall in the consumption of asset holders (real rate increases). This implies a rightward shift of labor supply, and hence a fall in wage and increase in hours. Consumption of non-asset holders also falls one-to-one with the wage, and hence aggregate demand falls by more than it would in a full-participation economy. Firms who can adjust prices will adjust them downwards, causing deflation, and a further fall in the real rate. Firms who cannot adjust prices will cut demand, causing a further fall in the real wage and a small fall in hours (since labor supply is inelastic). But this will mean higher profits (since marginal cost is falling), and eventually a positive income effect on labor supply of asset holders. As labor supply starts moving leftward, demand starts increasing, its increase being amplified by the sensitivity of non-asset holders to wage increases. The economy will establish at a point on the wage-hours locus consistent with the overall negative income effect on labor

supply of asset holders, i.e. with higher inflation and real activity. Hence, the initial inflationary expectations become self-fulfilling<sup>22</sup>.

### 5. Restoring the Taylor Principle

This section studies briefly two policy regimes that may make the Taylor principle a good policy prescription even in a 'LAMP economy'.

**5.1. Output stabilization.** Consider a rule that incorporates an output stabilization motive (closer to the rule estimated by CGG (2000)) of the form:

$$(5.1) \quad r_t = \phi_\pi E_t \pi_{t+1} + \phi_x x_t$$

$\phi_x$  is the response to output gap. Replacing this in the IS-AS system, the  $\mathbf{\Gamma}$  matrix becomes:

$$\mathbf{\Gamma} = \begin{bmatrix} 1 - \delta^{-1} [\beta^{-1} \kappa (\phi_\pi - 1) - \phi_x] & \delta^{-1} \beta^{-1} (\phi_\pi - 1) \\ -\beta^{-1} \kappa & \beta^{-1} \kappa \end{bmatrix}$$

Applying exactly the same method as in the proof of Proposition 1 it can be shown that the determinacy conditions are as follows.

PROPOSITION 3. (a) An interest rate rule such as (5.1) delivers a unique rational expectations equilibrium (i.e. the equilibrium is determinate) if and only if:

Case I: If  $\delta > 0$ ,  $\phi_\pi + \frac{1-\beta}{\kappa} \phi_x > 1$  and  $\phi_\pi < 1 + \frac{1+\beta}{\kappa} (\phi_x + 2\delta)$  (the 'Taylor Principle')

Case II: If  $\delta < 0$ , EITHER

$$II.A: \phi_x < -\delta(1 - \beta) \text{ and } \phi_\pi + \frac{1-\beta}{\kappa} \phi_x < 1 \text{ and } \phi_\pi > 1 + \frac{1+\beta}{\kappa} (\phi_x + 2\delta)$$

OR

$$II.B: \phi_x > -\delta(1 + \beta) \text{ and } \phi_\pi + \frac{1-\beta}{\kappa} \phi_x > 1 \text{ and } \phi_\pi < 1 + \frac{1+\beta}{\kappa} (\phi_x + 2\delta)$$

(b) If  $\delta < 0$ , equilibrium is indeterminate regardless of  $\phi_\pi$  if  $\phi_x \in (-\delta(1 - \beta); -\delta(1 + \beta))$ .

Part (a) studies equilibrium uniqueness. Case I is the standard Taylor principle for an economy where  $\delta > 0$ . In contrast to Proposition 1, in Case II the inversion of the Taylor Principle is now *not* granted. If either  $\delta$  is very large in absolute value (a high degree of limited participation  $\lambda$ ) or the response to output is low, we end up in case II.A and an instance of the Inverted Taylor Principle is observed. However, for moderate values of  $\lambda$  and/or a high enough response to the output gap, the Taylor Principle is restored. Another way to put this is that for a given share of non-asset holders, the Taylor Principle is a good guide for policy only insofar as the response to output is high enough. The response to output, however, can generate perverse effects if it is not high enough and participation to asset markets is very limited. As part (b) of the Proposition shows, the equilibrium is indeterminate if  $\phi_x$  is in a certain range, regardless of the magnitude of the inflation response. This region is sharply increasing with the share of non-asset holders.

To assess the magnitude of the policy coefficients needed for restoring the Taylor principle, consider the otherwise baseline parameterization for a 'LAMP economy' with  $\lambda = 0.4$  and  $\varphi^s = 2$  giving  $\delta = -0.11$  and  $\kappa = 0.228$ . The conditions for Case

<sup>22</sup>Benhabib and Farmer (2000) obtain a 'mirroring' result: the Taylor principle is inverted by a supply-side modification to the standard framework making labor supply downward-sloping. Namely, introducing money in the production function results in a liquidity effect on supply.

IIB are  $\phi_x > 0.21$ ;  $\phi_\pi > 1 - 0.043\phi_x$ ;  $\phi_\pi < 8.728\phi_x - 0.92016$ . The figure below shows that as soon as the Central Bank responds to output, the Taylor principle is restored under the baseline parameterization for a large parameter region. However, this result should be taken with care, for the very dangers associated with responding to output might outweigh potential benefits. As soon as the share of non-asset holders increases or labor supply becomes more inelastic, equilibrium is more likely to become indeterminate for any inflation response. In that case, it seems advisable not to respond to the output gap.

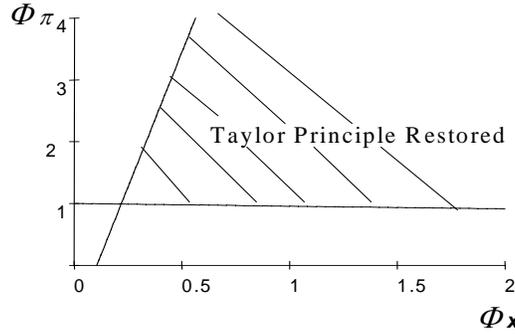


Fig. 4 : Policy parameter region whereby Taylor Principle is restored in a 'LAMP economy' by responding to output under baseline parameterization.

**5.2. Distortionary redistributive taxation restores the Taylor Principle.** The mechanism of the previous results relies on income effects on labor supply from the return on shares. This hints to an obvious possible way to restore the Taylor Principle relying on a specific fiscal policy rule: tax dividend income and distribute proceedings as transfers to non-asset holders. Since we focus on ways to restore the Taylor principle in cases where it does not hold, we consider only 'LAMP economies', i.e.  $\delta < 0$ . To make this point, consider the following simplified fiscal rule: profits are taxed at rate  $\tau_t$  and the budget is balanced period-by-period, with total tax income  $\tau_t D_t$  being distributed lump-sum to all non-asset holders. We focus on the case where profits are zero in steady-state. The balanced-budget rule then is  $\tau_t D_t = \lambda L_{H,t}$  which in log-linearized form (both profits and transfers are shares of steady-state GDP) is:

$$(5.2) \quad \lambda l_{H,t} = \tau o_t$$

In the Appendix we derive the following expressions for the wage hours equilibrium locus, and consumption of asset holders as a function of total output (setting shocks to zero as they are irrelevant for this point):

$$\begin{aligned} WN & : w_t = \frac{1}{1-\tau} [1 + \varphi^S + \mu(1-\tau)] n_t \\ c_{s,t} & = \delta_\tau y_t \\ \delta_\tau & = \frac{1}{1-\tau} \left[ 1 - \tau \frac{\mu}{1+\mu} + \frac{\tau-\lambda}{1-\lambda} \frac{\varphi^S}{1+\mu} \right] \end{aligned}$$

Consider a simple forward looking-rule for monetary policy. In this case, the crucial parameter is again the elasticity of asset holders' consumption with respect

to output  $\delta_\tau$ . Its sign dictates whether the Taylor Principle or the Inverted Taylor Principle apply. Now for any  $\lambda$  there will exist a minimum threshold for the tax rate such that  $\delta_\tau > 0$  when in the absence of such fiscal policy  $\delta < 0$ . This threshold is (note that  $\varphi^S / (1 - \lambda) - \mu > 0$  where  $\delta < 0$ ):

$$\tau > 1 - \frac{1 + \varphi^S}{\frac{1}{1-\lambda}\varphi^S - \mu} < 1$$

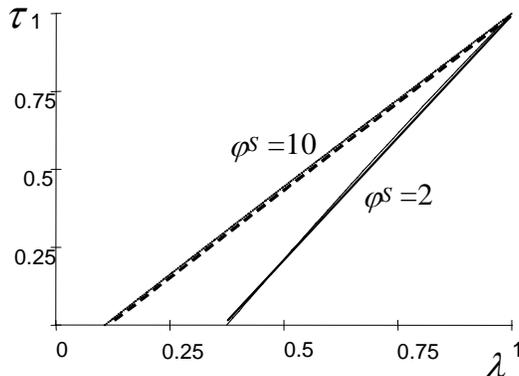


Fig. 5 : Threshold tax rate restoring Taylor Principle as a function of share of non-asset holders, for different labor supply elasticities.

The necessary tax rate is higher, the more inelastic is labor supply (dashed versus solid line in the figure) and the higher the share of agents with no assets. The intuition for this result is straightforward: a higher tax rate on profits makes the wage-hours locus WN more upward sloping, since it eliminates some of the income effect of profit income variation on asset holders' labor supply.

## 6. Optimal time consistent monetary policy

The above analysis suggests that in an economy with binding borrowing constraints, underdeveloped financial markets, low shareholding and/or a high share of myopic agents, the central bank following an active rule would leave room for sunspot-driven real fluctuations. The size of these fluctuations would depend upon the size of the sunspot shocks (something impossible to quantify in practice), but this would unambiguously increase the variances of real variables. If such variance is welfare-damaging, as is almost uniformly believed to be the case and assumed in the literature, it is clear that such policies would be suboptimal since sunspot fluctuations themselves would be welfare-reducing. In contrast, in the same 'LAMP economy', a passive rule would rule out such fluctuations and would be closer to optimal policy. Our next task is to characterize some form of 'optimal' policy rules, when variability of inflation and output gap are costly by assumption. We seek to establish whether and how does the presence of non-asset holders alter the *optimal* design of monetary policy rules in the simple LAMP IS-AS model introduced above. To keep things simple, we shall only focus on the discretionary, and not fully optimal (commitment) solution to the central banker's problem. This case can be argued to be more realistic in practice, as do CGG (1999). There is another sense in which we cannot treat our solution as an 'optimal' rule. The objective function we use is a quadratic loss function in inflation and the output gap. While in the

full-participation case this can be derived as a second-order approximation to the representative agent's utility (as is done in Woodford 2003 Ch. 6), this welfare metric would modify in our case, for there is no representative agent in the first place<sup>23</sup>. But our approach could be justified if one sees relative price distortions as dominating any other distortions from a welfare standpoint<sup>24</sup>. We shall henceforth assume that the central bank has the following intertemporal objective function, standard in the literature:

$$(6.1) \quad -\frac{1}{2}E_t \left\{ \sum_{i=0}^{\infty} \beta^i [\alpha x_{t+i}^2 + \pi_{t+i}^2] \right\}$$

The optimal discretionary rule  $\{r_t^o\}_0^{\infty}$  is found by maximizing this objective function taking as a constraint the IS-AS system, and re-optimizing every period. Note that by usual arguments this equilibrium will be time-consistent. This is, up to interpretation of the solution, isomorphic to the standard problem in CGG (1999). Hence, for brevity, we skip solution details available elsewhere and go to the result:

$$(6.2) \quad x_t = -\frac{\kappa}{\alpha} \pi_t$$

When inflation increases the central bank has to act in order to contract demand, and expand it in case of deflation. Assuming an AR(1) process for the cost-push shock  $E_t u_{t+1} = \rho_u u_t$  for simplicity, we obtain the following reduced forms for inflation and output from the aggregate supply curve:

$$(6.3) \quad \begin{aligned} \pi_t &= \alpha \frac{1}{\kappa^2 + \alpha(1 - \beta\rho_u)} u_t \\ x_t &= -\kappa \frac{1}{\kappa^2 + \alpha(1 - \beta\rho_u)} u_t \end{aligned}$$

Substituting these expressions into the aggregate demand curve, we obtain the *implicit instrument rule* consistent with the optimal time consistent equilibrium, written in terms of expected inflation for comparison with our previous instrument rule<sup>25</sup>:

$$(6.4) \quad \begin{aligned} r_t^o &= r_t^* + \phi_\pi^o E_t \pi_{t+1} \\ \phi_\pi^o &= \left[ 1 + \frac{\delta\kappa}{\alpha} \frac{1 - \rho_u}{\rho_u} \right] \end{aligned}$$

We can see that some of the (by now) classical results of CGG (1999) obtained in a full-participation economy carry over: from the existence of a trade-off between inflation and output stabilization, to convergence of inflation to its target under the optimal policy. Shocks affecting only the natural rate of output (to the extent they would exist, which is not the case in our model) should not cause a policy reaction. Also, real disturbances affect nominal rates only insofar as they affect

<sup>23</sup>Amato and Laubach (2003) do calculate the proper welfare function in a somehow related model with 'rule-of-thumb' households; however, their non-standard consumers' rule merely equates present to last period's consumption, which is not the case in our model.

<sup>24</sup>This is indeed found to be the case by Woodford (2003, Ch 6), who introduces a series of other distortions in a welfare-maximizing framework. Schmitt-Grohe and Uribe (2003) reach a similar conclusion via a different method of analyzing optimality.

<sup>25</sup>A positive policy response to inflation requires  $\alpha \geq -\delta\kappa \frac{1 - \rho_u}{\rho_u}$ . This is obviously satisfied in the Ricardian case, but not necessarily in the non-Ricardian one, unless the central bank aims for output stabilization enough.

the Wicksellian interest rate, as discussed in detail for example by Woodford 2003 p.250. There is one important exception however, emphasized in the following Proposition.

PROPOSITION 4. *In a LAMP economy ( $\delta < 0$ ) the implied instrument rule for optimal policy is passive  $\phi_\pi^o < 1$ . The optimal response to inflation is decreasing in the share of non-asset holders  $\frac{\partial \phi_\pi^o}{\partial \lambda} < 0$  and changes sign when  $\delta$  changes sign.*

The above Proposition shows the exact way in which the central bank has to change its instrument in order to meet the targeting rule 6.2: contract demand when inflation increases, but move nominal rates such that the real rate *decreases* when  $\delta$  changes sign. This happens because part of aggregate demand (given by the non-asset holders) is insensitive to interest rate changes, and the intuition is the same as provided before when ruling out sunspot equilibria with a passive rule. As consumption of non-asset holders moves one-to one with (and hence overreacts to increases in) the wage, the other part of aggregate demand becomes oversensitive to interest rate changes through the channel emphasized repeatedly above. A decrease in the real rate is optimal, since otherwise (if the real rate increased) there would be too strong a fall in consumption of asset holders, violating the optimality condition 6.2. It can be expected that the presence of agents who do not smooth consumption alters the objective function of the central bank, and not only the constraints. However, note that the change from passive to active in the optimal rule would still correspond to the change in sign of  $\delta$ , if two conditions are met: (i) the welfare function can still be represented by the quadratic form  $\alpha(\lambda) x_{t+i}^2 + \pi_{t+i}^2$ ; (ii)  $\alpha(\lambda) > 0$  for any value of  $\lambda$ .

Note that responses to shocks are independent of the share of non-asset holders under optimal policy 6.3. This is again merely an implication of our ad-hoc objective function - it is likely that an utility-based welfare objective would at least have  $\alpha$  depend on the share of non-asset holders. This is a natural next tackle but is beyond this paper's scope.

**6.1. Optimal policy without trade-off.** It is clear from the above analysis that another insight of the monetary policy literature (see again i.a. Woodford's Ch 4 and CGG 1999) carries over in our setup - when cost-push shocks are absent (and so is the inflation-output stabilization trade-off), the flexible-price allocation can be achieved. This is done by having  $r_t^o = r_t^*$ , i.e. the nominal rate equal the Wicksellian rate at all times. However, there is one major difference with the usual literature: aside from being optimal, tracking the Wicksellian rate may also imply a unique rational expectations equilibrium under some conditions.

PROPOSITION 5. *A policy rule whereby the nominal interest rate tracks the Wicksellian natural rate:*

(i) *implements the flexible-price allocation in the LAMP economy (when cost-push shocks are absent);*

(ii) *supports the optimum with stable prices as a unique rational expectations equilibrium if the parameters satisfy the condition:*

$$(6.5) \quad \lambda \geq \frac{1 + \frac{1}{1+F_Y} \varphi^S \frac{(1-\theta)(1-\beta\theta)}{(1+\theta)(1+\beta\theta)}}{1 + \frac{1}{1+\mu} \varphi^S}$$

Part (i) follows directly by inspecting the IS-AS system. Part (ii) is just a corollary of Proposition 1, Case II: just consider  $\phi_\pi = 0$  therein. The condition for this to be consistent with an unique equilibrium is then:  $1 + \delta \frac{2(1+\beta)}{\kappa} \leq 0$  which gives (6.5) above. Note also that  $r_t^o = r_t^*$  can also be optimal under the existence of the cost-push shocks, but this implies restrictions on the preferences of the central bank. Namely, such happens if and only if  $\alpha = -\delta\kappa \frac{1-\rho_u}{\rho_u}$  (which is around 0.65 under the baseline parameterization).

Point (ii) above means that price stability can be achieved *in principle* by having the Central Bank follow variations in the Wicksellian rate, and that would result in an unique rational expectations equilibrium, with no need for committing to react to inflation. This is in clear contrast with the standard case studied in detail by Woodford (2003 Ch.4). There, the bank needs to commit to respond to inflation by fulfilling the Taylor principle  $r_t^o = r_t^* + \phi_\pi \pi_t$ ,  $\phi_\pi > 1$  in order to pin down a unique equilibrium. But since such inflation would never occur in equilibrium, one then wonders whether one can estimate a Taylor rule of this type.

One further implication is that an interest rate peg, or any exogenous path for the interest rates, will result in a determinate equilibrium if the same condition is satisfied ((6.5) above). But is this parameter condition unrealistically restrictive<sup>26</sup>? Not quite: assuming usual numbers for the price stickiness and inverse Frisch elasticity of labor supply, namely  $\theta = 0.75$ ,  $\varphi^S = 10$ , and zero steady-state profits  $F_Y = \mu$ , the threshold value of the share of non-asset holders is 0.126; compared to the empirical estimates of Campbell and Mankiw of around 0.5, this is a quite small number. Note that this threshold level is decreasing with price stickiness as illustrated in Figure 6 for two labor supply elasticities:  $\varphi^S = 10$  and 2. In the first case (thick line), while for flexible prices this condition cannot be fulfilled, for a high degree of price stickiness a very small share of non-asset holders is enough to render the equilibrium determinate under an exogenous interest rate. Under a 1/2 elasticity (which is can be seen as an upper bound empirically - see discussion and references in King and Rebelo 2000), the required share of non-asset holders is higher as can be seen from the dashed thin line.

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<sup>26</sup>In fact, the parameter subspace where this condition is fulfilled is non-empty if and only if  $\left(1 + \frac{1}{1+F_Y} \varphi^S \frac{(1-\theta)(1-\beta\theta)}{(1+\theta)(1+\beta\theta)}\right) / \left(1 + \frac{1}{1+\mu} \varphi^S\right) < 1$ , implying  $\frac{2\theta(1+\beta)}{(1+\theta)(1+\beta\theta)} > D_Y = (\mu - F_Y) / (1 + \mu)$ , which is always satisfied as long as prices are sticky to some (no matter how small) extent and the steady-state share of profits  $D_Y$  is zero.

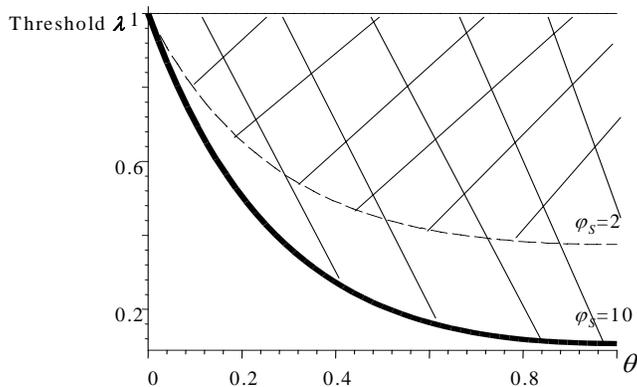


Fig. 6: Threshold share of non-asset holders for Proposition 4 to hold, as a function of price stickiness, for different labor supply elasticities.

However, the ability of the central bank to achieve full price stability as the *unique* equilibrium by tracking the Wicksellian interest rate applies to the simple model assumed here. Furthermore, it relies upon the ability/willingness of the bank to monitor the natural rate of interest and match its movement one-to-one by movements in the nominal rate. So usual caveats of such a policy proposal emphasized by Woodford apply, where one can add that the natural interest rate can sometimes be negative. In practice moreover, central banks do respond to movements in macroeconomic aggregates, as a huge and important literature emphasized, by following interest rate rules, whether resulting from optimization or not, as the ones we studied previously.

## 7. The effects of shocks

In this section we go back to the simple instrument rule and try to compute analytically the effects of fundamental and sunspot shocks under determinacy and indeterminacy, allowing for the change in sign of  $\delta$  due to LAMP. Our interest in this exercise is twofold. First, it might be of interest in itself to understand the effects of shocks in a *determinate* LAMP economy. One obvious historical candidate for such a case could be found in the pre-Volcker era; it is fairly well established (see e.g. CGG 2000, Taylor 1999, Lubik and Schorfheide 2003b) that the response of monetary policy in that period implied a (long-run) response to inflation of less than one. But if we allow for the possibility that participation to asset markets was so limited that  $\delta < 0$ , this would **not** imply that policy was inconsistent with a unique equilibrium. Hence, we will be able to assess the effects of fundamental shocks, an impossible task under indeterminacy (more below). Indeed, in a companion paper (Bilbiie (2004)) we argue that in this was the case, and fundamental shocks can explain stylized facts of the pre-Volcker period (impulse responses to shocks and moments) quite well. Secondly, there is the mirror image of the above argument. Estimates of policy rule coefficients in the Volcker-Greenspan era for the US (and similarly, for most other industrialized countries) such as e.g. CGG 2000 indicate a response of nominal rates to inflation larger than one. Coupled with the possibility of a LAMP economy ( $\delta < 0$ ), this would instead imply indeterminacy. Hence, it might be of interest to assess the effects of various (fundamental and sunspot) shocks in an indeterminate equilibrium.

We follow the new method proposed by Lubik and Schorfheide (2003a) to compute sunspot equilibria by decomposing expectational errors, building upon the approach of Sims (2000). The IS-AS system can be written, in terms of the defined variables  $\xi_t^z \equiv E_t z_{t+1}$ , so  $\xi_t \equiv (\xi_t^y, \xi_t^\pi)$  and define the expectational errors  $\eta_t^z \equiv z_t - E_{t-1} z_t$ .

$$\xi_t = \mathbf{\Gamma} \xi_{t-1} + \mathbf{\Psi} \nu_t + \mathbf{\Gamma} \eta_t$$

The coefficient matrices  $\mathbf{\Gamma}$ ,  $\mathbf{\Psi}$  are given in (4.2). We replace  $\mathbf{\Gamma}$  by its Jordan decomposition  $\mathbf{\Gamma} = JQJ^{-1}$  and define the auxiliary variables  $z_t = J^{-1} \xi_t$  and rewrite the above model as

$$(7.1) \quad z_t = Qz_{t-1} + J^{-1} \mathbf{\Psi} \varepsilon_t + J^{-1} \mathbf{\Gamma} \eta_t$$

The eigenvalues of  $\mathbf{\Gamma}$  are found to be:

$$(7.2) \quad q_{\pm} = \frac{1}{2} \left[ \text{tr} \mathbf{\Gamma} \pm \sqrt{(\text{tr} \mathbf{\Gamma})^2 - 4 \det \mathbf{\Gamma}} \right]$$

(where the determinant and trace are  $\det \mathbf{\Gamma} = \beta^{-1} > 1$ ,  $\text{tr} \mathbf{\Gamma} = 1 + \beta^{-1} - \beta^{-1} \delta^{-1} \kappa (\phi_\pi - 1)$ ). The corresponding eigenvectors are stacked in the  $J$  matrix:

$$J = \begin{bmatrix} \frac{1}{\kappa} (1 - \beta q_-) & \frac{1}{\kappa} (1 - \beta q_+) \\ 1 & 1 \end{bmatrix}$$

**7.1. Determinacy.** The equilibrium under determinacy is particularly easy to calculate when shocks have zero persistence, since the only stable solution is  $\xi_t = 0$ , obtained for:

$$\mathbf{\Psi} \nu_t + \mathbf{\Gamma} \eta_t = 0$$

Hence, the expectation errors are determined exclusively by fundamental shocks (and sunspot shocks would have no effect on dynamics) by  $\eta_t = -\mathbf{\Gamma}^{-1} \mathbf{\Psi} \nu_t$ , namely:

$$(7.3) \quad \eta_t = -\delta^{-1} \begin{bmatrix} 1 \\ \kappa \end{bmatrix} (\varepsilon_t - r_t^*) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_t$$

The initial impact on output and inflation is also given by the same expression. Since both roots are eliminated under determinacy, there is no persistence. Note the sharp differences for the two subcases identified above, showing asymmetric effects of some shocks depending on the sign of  $\delta$ .

**Case I:**  $\delta > 0$ ,  $\phi_\pi \in \left(1, 1 + \delta \frac{2(1+\beta)}{\kappa}\right)$ : The effects at work are as usual and we should not insist upon their interpretation. A policy-induced interest rate cut or an increase in the natural rate of interest (coming here only from shocks to technology growth) increase both the output gap and inflation. One-time cost-push shocks have no effect on the output gap, and increase inflation one-to-one; this is only because the interest rate rule responds to expected future inflation, whereas a one-time shock increases only inflation today.

**Case II: LAMP economy,**  $\delta < 0$ ,  $\phi_\pi \in \left(1 + \delta \frac{2(1+\beta)}{\kappa}, 1\right)$ . In contrast to the standard case, a monetary contraction (positive  $\varepsilon_t$ ) has expansionary effects, and causes inflation. This follows directly from our intuition above regarding the labor market equilibrium. A monetary contraction shifts the labor supply curve right, making agents want to work more at the same wage, which instead leads to an increase in hours and the real wage, and in output and inflation thereby (we do not insist upon the whole mechanism making this happen, as this was emphasized

above). An increase in the natural rate of interest driven by technology results in a recession and deflation - this shall be addressed in some detail below. It is clear then that a policy response increasing the nominal rate by more than the natural rate  $\varepsilon_t > r_t^*$  increases both output and inflation, whereas when it falls short of doing so, it has deflationary effects, and causes a fall in output. Cost-push shocks have the same effects regardless of the sign of  $\delta$  due to the zero-persistence assumption. However, in the presence of persistence the magnitude of the responses to these shocks will depend on  $\delta$ , since in that case the roots of the system matter for dynamics.

**7.2. Indeterminacy.** In this case one of the roots  $q_{\pm}$  will be inside the unit circle. In this case sunspot shocks can have real effects, and the responses to fundamental shocks change too, in a way made explicit below. We confine ourselves to the case whereby the smaller root is inside the unit circle and the larger one is greater than one, i.e.  $q_- \in (-1, 1)$  and  $q_+ > 1$ . This can be shown to be the case if either (i)  $\delta > 0, \phi_{\pi} < 1$  or (ii)  $\delta < 0, \phi_{\pi} > 1$ <sup>27</sup> Since in this case there is one-dimensional indeterminacy, the stability condition for 7.1 modifies: expectation errors are not spanned by fundamental shocks, but by both fundamental and sunspot shocks.

We can apply the results in Proposition 1 in Lubik and Schorfheide to solve for the full solution set for the expectational errors. This is described in some detail in the Appendix, and the solution is:

$$(7.4) \quad \eta_t = -\frac{\kappa\delta^{-1}}{d^2} \begin{bmatrix} \kappa q_+ \\ 1 - q_+ \end{bmatrix} (\varepsilon_t - r_t^*) + \frac{1}{d^2} \begin{bmatrix} \kappa\beta^{-1}(1 - q_+) \\ (1 - q_+)(q_- - \beta^{-1}) \end{bmatrix} u_t + \frac{1}{d} \begin{bmatrix} q_+ - 1 \\ \kappa q_+ \end{bmatrix} (M_1 \nu_t + \varsigma_t^*)$$

where  $M_1$  is an arbitrary  $2 \times 2$  matrix and  $\varsigma_t^*$  is a reduced-form sunspot shock, which can be interpreted as a belief-induced increase in output and/or inflation of undetermined size. First thing to note is that a positive realization of this shock will increase output and inflation no matter whether  $\delta \leq 0$  since  $q_+ > 1$  as established above. This conforms our intuitive construction of sunspot equilibria when discussing determinacy properties of interest-rate rules.

On the other hand, the effects of fundamental shocks become ambiguous, and depend crucially upon the choice of  $M_1$ . Unfortunately, there is nothing to pin down a choice for this matrix, which captures the well-known problem of indeterminate equilibria - the effects of fundamental shocks cannot be studied without further restrictions. Two leading possibilities to restrict the  $M_1$  matrix are suggested by Lubik and Schorfheide:

**7.2.1. Orthogonality.** The two sets of shocks are orthogonal in their contribution to the forecast error, and hence  $M_1 = 0$  in 7.4. The effect of a cost-push shock is of the same sign under either scenario, as is independent of  $\delta$ . A positive realization of this shock would increase inflation (since  $(1 - q_+)(q_- - \beta^{-1}) > 0$ ) and decrease output ( $q_+ > 1$ ). The effects of policy shocks, and of shocks to the natural rate of interest, are again different depending on which case we consider:

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<sup>27</sup>For the rest of the parameter regions where there is indeterminacy we would have  $q_+ \in (-1, 1)$  and  $q_- < -1$ , but this can be shown to imply very restrictive conditions on the deep parameters and the policy rule coefficient.

Standard case,  $\delta > 0$ : An interest rate increase keeping constant the natural rate decreases output under its natural level but causes inflation as  $1 - q_+ < 0$  (this is also found by Lubik and Schorfheide for a contemporaneous rule). An increase in the natural rate without a discretionary policy response increases the output gap and causes deflation.

'LAMP economy'  $\delta < 0$ : A policy-induced interest rate increase increases output and causes deflation. An increase in the natural rate not matched by policy depresses output and causes inflation. In either case, the overall effect on inflation and the output gap depends on whether the policy response is stronger or weaker than the variation in the natural rate.

7.2.2. *Continuity*. In order to preserve continuity of the impulse responses to the fundamental shock when passing from determinacy to indeterminacy,  $M_1$  can be chosen such that it implies that the response to the fundamental shock is the same, i.e.:

$$\eta_t = -\delta^{-1} \begin{bmatrix} 1 \\ \kappa \end{bmatrix} (\varepsilon_t - r_t^*) + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u_t + \frac{1}{d} \begin{bmatrix} q_+ - 1 \\ \kappa q_+ \end{bmatrix} \varsigma_t^*$$

This happens for a very particular  $M_1$  matrix and implies that the effects of fundamental shocks are as under determinacy, namely in the 'LAMP economy' case a contractionary policy shock increases both output and inflation. While continuity is an attractive feature, there is nothing to insure that the  $M_1$  takes exactly the form necessary to get this result.

## 8. Conclusions

Interest rate changes modify the intertemporal consumption and labor supply profile of *asset holders*, agents who have access to complete asset markets and can smooth consumption. This affects the real wage, and the demand thereby of agents who have no asset holdings, are oversensitive to real wage changes, and insensitive directly to interest rate changes. If the share of non-asset holders is high enough and/or and the elasticity of labor supply is low enough, this last effect works to offset the interest rate effects on demand of asset holders. For in such a case, variations in the real wage mean variations in marginal costs, which instead lead to variations in profits (and hence dividend income). These can offset (and indeed overturn) the initial impact of interest rates on aggregate demand. In labor market terminology, the equilibrium wage-hours locus becomes upward sloping but cuts the labor supply curve from above. This is the main mechanism identified by this paper to change drastically the effects of monetary policy as compared to a standard full-participation case whereby aggregate demand is completely driven by asset holders. The required share of non-asset holders for these results to hold is relatively mild, far below empirical estimates of Campbell and Mankiw 1989, for parameterizations usually employed in the literature.

This paper analyzes monetary policy implications of the foregoing insight and thereby challenges some conventional wisdom in the field. Its scope is very limited: to make a small contribution to the literature emphasizing the role of LAMP in shaping macroeconomic policy and helping towards a better understanding of the economy. In that respect, we just seek to add to a new developing literature analyzing the role of non-asset holders in dynamic general equilibrium models, initiated by Mankiw (200) and Gali, Lopez-Salido and Valles (2002) for fiscal policy and Gali, Lopez-Salido and Valles (2003) for monetary policy.

Our results have clear normative implications. In a nutshell, central bank policy should be pursued with an eye to the aggregate demand side of the economy. Notably, the extent to which agents participate to asset markets and hence smooth consumption would become an important part of the policy input. While the degree of development of financial markets might make this not a concern in present times in the developed economies, central banks in developing countries with low participation in financial markets might find this of some practical interest. The theoretical results hinting to such policy prescriptions are that in a 'LAMP economy': (i) an 'Inverted Taylor Principle' holds generically<sup>28</sup>; the central bank needs to adopt a passive policy rule to ensure equilibrium uniqueness and rule out the possibility of self-fulfilling sunspot-driven fluctuations; (ii) Optimal and time-consistent (discretionary) monetary policy also implies that the central bank should move nominal rates such that real rates decline in a LAMP economy. For it is by decreasing the real rate that aggregate demand is contracted when inflation increases, as required by the targeting rule; this is different from the standard case as explained in text.

The model presented has the advantage of simplicity: we were able to show how to analyze LAMP economies analytically in the same type of framework used in standard analyses. However, this simplicity can potentially also be a shortcoming, for it implies many realistic features have been left out. More important still, in our view, is to relax the way we modelled for asset markets participation. This is a shortcoming we share with the rest of the literature, as this very literature emphasizes (see GLV 2002, 2003). Our approach is only justified for tractability, but potentially important insights can be gained by an explicit modelling of the decision to participate to asset markets. Crucially, where this could help is in deriving a proper welfare metric in order to analyze optimal policy choices meaningfully. This is potentially a cumbersome exercise, but to our mind worth of all further investigation. Lastly, assessing empirically the extent to which some agents do not smooth consumption, the evolution of this over time, and its implications at aggregate level, is in our view a necessary step for understanding business cycle dynamics, which we intend to pursue further.

### Appendix A. Loglinearized equilibrium

For asset holders, we have the Euler equation, intratemporal and budget constraint ( $d_t$  are profits as a share of steady-state GDP,  $d_t = \frac{D_t - D}{Y}$  and we already imposed that the equilibrium value of share holdings is  $\frac{1}{1-\lambda}$ ):

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<sup>28</sup>This result depends only to a small extent on whether the rule is specified in terms of current or expected future inflation. As discussed in text in more detail, this is in contrast to Gali, Lopez-Salido and Valles (2003) who, while having noted the possibility to violate the Taylor principle for a forward-looking rule, also argue that a strengthening of the Taylor principle is required for a contemporaneous rule to result in equilibrium uniqueness. A very strong response to current inflation would also insure determinacy in our model, but we find the implied coefficient is higher than any plausible estimates, makes policy non-credible and could lead to violation of the zero lower bound in case of small deflations.

$$(A.1) \quad E_t [c_{s,t+1}] - c_{s,t} = r_t - E_t [\pi_{t+1}]$$

$$(A.2) \quad \varphi_s n_{s,t} = w_t - c_{s,t} \text{ where } \varphi_s = \left[ \frac{\gamma_S N_s}{1 - N_s} \right]$$

$$(A.3) \quad \frac{C_S}{Y} c_{s,t} = \frac{W}{P} \frac{N_s}{Y} (w_t + n_{s,t}) + \frac{1}{1 - \lambda} d_t$$

For non-asset holders, we have the intratemporal optimality and budget constraint:

$$(A.4) \quad n_{h,t} = 0$$

$$(A.5) \quad c_{h,t} = w_t$$

For firms:

$$(A.6) \quad y_t = (1 + F_Y) n_t + (1 + F_Y) a_t$$

$$(A.7) \quad mc_t = w_t - a_t$$

$$(A.8) \quad d_t = -\frac{1 + F_Y}{1 + \mu} mc_t + \frac{\mu}{1 + \mu} y_t$$

$$(A.9) \quad \pi_t = \beta E_t \pi_{t+1} + \psi mc_t, \quad \psi = \frac{(1 - \theta)(1 - \theta\beta)}{\theta}$$

Labour market clearing implies ( $n$  =labour demand by firms)

$$(A.10) \quad n_t = \frac{(1 - \lambda) N_s}{N} n_{s,t}$$

Aggregate consumption is:

$$(A.11) \quad c_t = \frac{\lambda C_H}{C} c_{h,t} + \frac{(1 - \lambda) C_s}{C} c_{s,t}$$

Equilibrium in goods market, holds by Walras' law, and is redundant once equilibrium in all other markets has been imposed.

$$(A.12) \quad y_t = c_t$$

Monetary policy rule:

$$(A.13) \quad r_t = \phi_\pi E_t \pi_{t+k} + \phi_x E_t x_{t+q}$$

### A.1. Steady state.

$$R = \frac{1}{\beta} \text{ where } R \equiv 1 + r$$

$$\frac{W}{P} = \frac{Y + F}{N} \frac{MC}{P} = \frac{Y}{N} \frac{1 + \frac{F}{Y}}{1 + \mu}$$

$$\frac{D}{\bar{Y}} = \frac{\mu - F_Y}{1 + \mu}$$

We assume hours are the same for the two groups in steady state only,  $N_H = N_S = N$ . Then, for the loglinear budget constraints of both agents the coefficients are fully

determined:

$$\begin{aligned}\frac{W N_s}{P Y} &= \frac{1 + F_Y}{1 + \mu}; \quad \frac{C_S}{Y} = \frac{1 + F_Y}{1 + \mu} + \frac{\mu - F_Y}{1 + \mu} \frac{1}{1 - \lambda} = \frac{1}{1 - \lambda} \left( 1 - \lambda \frac{1 + F_Y}{1 + \mu} \right) \\ \frac{W N_H}{P Y} &= \frac{C_H}{Y} = \frac{1 + F_Y}{1 + \mu}\end{aligned}$$

### Appendix B. Deriving the IS-AS system

Since Walras' law holds I will use the economy resource constraint instead of the budget constraint of asset holders in the derivation. We seek to express everything in terms of aggregate variables, and then use the two dynamic equations to get dynamics only in terms of output, inflation and interest rate. First, try to express consumption of asset holders as function of aggregate variables, from A.5, A.10, A.11 using the steady state coefficients just calculated:

$$(B.1) \quad c_{s,t} = \frac{1}{1 - \lambda} \frac{C}{C_s} y_t - \frac{\lambda}{1 - \lambda} \frac{C_h}{C_s} w_t$$

Substituting this, together with A.10 into A.2 and using the production function we get:

$$(B.2) \quad w_t = \chi y_t - (1 + F_Y)(\chi - 1) a_t = \chi(1 + F_Y) n_t + (1 + F_Y) a_t$$

$$(B.3) \quad \chi \equiv \left[ 1 + \varphi^s \frac{C_s}{C} \frac{1}{1 + F_Y} \right] = \left[ 1 + \varphi^s \frac{1}{1 - \lambda} \frac{1}{1 + F_Y} \left( 1 - \lambda \frac{1 + F_Y}{1 + \mu} \right) \right] \geq 1$$

Note that we have always  $\chi \geq 1$ . Substituting back into B.1 and using the steady state consumption shares we get consumption of saver as a function of output:

$$\begin{aligned}c_{s,t} &= \delta y_t + (1 + F_Y)(1 - \delta) a_t = \frac{\delta}{\chi} w_t + (1 + F_Y) \left( 1 - \delta + \frac{\chi - 1}{\chi} \right) a_t \\ \delta &\equiv 1 - \varphi^s \frac{\lambda}{1 - \lambda} \frac{1}{1 + \mu}\end{aligned}$$

Note  $\chi = \delta + \varphi^s \frac{1}{1 - \lambda} \frac{1}{1 + F_Y}$

The elasticity (share)  $\delta$  will turn out to play an important role for determinacy properties and dynamics. Having done this, we just need to replace these last two equations in the Euler and New Keynesian Philips curve to obtain a system in output and inflation. As we have technology shocks, it is easier to write the whole system in terms of the output gap (difference of actual output from output under flexible prices) as is usually done in the literature. Real marginal cost is given by

$$(B.4) \quad mc_t = \chi y_t - [(1 + F_Y)(\chi - 1) + 1] a_t$$

Since in the flexible-price equilibrium the markup is constant (and so is the real marginal cost) we see directly from B.4 that natural output is:

$$y_t^* = \left[ 1 + F_Y \left( 1 - \frac{1}{\chi} \right) \right] a_t$$

So marginal cost is related to the output gap  $x_t \equiv y_t - y_t^*$  by:

$$(B.5) \quad mc_t = \chi (y_t - y_t^*) = \chi x_t$$

Following Clarida, Gali and Gertler (1999) or Gali (2002) we also introduce cost-push shocks  $u_t$ , i.e. variations in marginal cost not due to variations in excess

demand. These could come from the existence of sticky wages creating a time-varying wage markup, or other sources creating this inefficiency wedge although we do not model this explicitly here. Hence, marginal cost variations are given by

$$(B.6) \quad mc_t = \chi x_t + u_t$$

Substituting consumption of asset holders in the Euler equation, we can write

$$(B.7) \quad \delta E_t x_{t+1} = \delta x_t + [r_t - E_t \pi_{t+1}] + (1 + F_Y)(1 - \delta)[a_t - E_t a_{t+1}] - \delta [E_t y_{t+1}^* - y_t^*]$$

We can define the natural rate of interest (Wicksellian interest rate)  $r_t^*$  as the level of the interest rate consistent with output being at its natural level (and hence with zero inflation), as in Woodford 2003. Solving from B.7 we obtain:

$$(B.8) \quad r_t^* = \left[ 1 + F_Y \left( 1 - \frac{\delta}{\chi} \right) \right] [E_t a_{t+1} - a_t]$$

Assuming  $a_t$  is given by an AR(1) process such that  $E_t a_{t+1} = \rho^a a_t$ , we note that  $r_t^* = - \left[ 1 + F_Y \left( 1 - \frac{\delta}{\chi} \right) \right] (1 - \rho^a) a_t$ , such that the natural interest rate varies negatively with technology.

Using B.5 and B.8 into the New Keynesian Philips curve and the B.7 we get the reduced system:

$$(B.9) \quad \pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t, \text{ where } \kappa \equiv \psi \chi$$

$$(B.10) \quad E_t x_{t+1} = x_t + \delta^{-1} [r_t - E_t \pi_{t+1} - r_t^*]$$

For simplicity, we consider only monetary policy rules involving inflation stabilization and no inertia, of the form:

$$r_t = \phi_\pi E_t \pi_{t+k} + \varepsilon_t$$

where  $\varepsilon_t$  are policy shocks, i.e. movements in nominal rates coming from anything else than systematic response to inflation.

### Appendix C. Proof of Proposition 1

Necessary and sufficient conditions for determinacy in such systems are (given in Woodford Appendix to Chapter 4) as follows. Either **Case A**:  $(Aa) \det \mathbf{\Gamma} > 1$ ;  $(Ab) \det \mathbf{\Gamma} - tr \mathbf{\Gamma} > -1$  and  $(Ac) \det \mathbf{\Gamma} + tr \mathbf{\Gamma} > -1$  or **Case B**:  $(Ba) \det \mathbf{\Gamma} - tr \mathbf{\Gamma} < -1$  and  $(Bb) \det \mathbf{\Gamma} + tr \mathbf{\Gamma} < -1$ . For our forward-looking rule case, the determinant and trace are:

$$(C.1) \quad \begin{aligned} \det \mathbf{\Gamma} &= \beta^{-1} > 1 \\ tr \mathbf{\Gamma} &= 1 + \beta^{-1} - \beta^{-1} \delta^{-1} \kappa (\phi_\pi - 1) \end{aligned}$$

Imposing the determinacy conditions in Case A above (where Case B can be ruled out due to sign restrictions), we obtain the requirement for equilibrium uniqueness:

$$\delta^{-1} (\phi_\pi - 1) \in \left( 0, \frac{2(1+\beta)}{\kappa} \right)$$

This implies the two cases in Proposition 1: Case I:  $\delta > 0, \phi_\pi \in \left( 1, 1 + \delta \frac{2(1+\beta)}{\kappa} \right)$ , which is a non-empty interval; Case II:  $\delta < 0, \phi_\pi \in \left( 1 + \delta \frac{2(1+\beta)}{\kappa}, 1 \right)$ . Notice that (i)  $1 + \delta \frac{2(1+\beta)}{\kappa} < 1$  so the interval is non-empty; (ii)  $1 + \delta \frac{2(1+\beta)}{\kappa} > 0$  implies instead that we can rule out an interest rate peg, whereas a peg is consistent with

a unique REE for  $1 + \delta \frac{2(1+\beta)}{\kappa} < 0$ . The last condition instead holds if and only if  $\lambda \geq \frac{1 + \frac{1}{1+F_Y} \varphi^S \frac{(1-\theta)(1-\beta\theta)}{(1+\theta)(1+\beta\theta)}}{1 + \frac{1}{1+\mu} \varphi^S} \geq \frac{1}{1 + \frac{1}{1+\mu} \varphi^S}$  which is the condition in proposition 5 (imposing  $\phi_\pi = 0$  here).

When this condition is not fulfilled, we have  $0 < 1 + \delta \frac{2(1+\beta)}{\kappa} < 1$ , so there still exist policy rules  $\phi_\pi \in \left(1 + \delta \frac{2(1+\beta)}{\kappa}, 1\right)$  bringing about a unique rational expectations equilibrium. But in this case an interest rate peg, and any policy rule with too weak a response  $\phi_\pi \in \left[0, 1 + \delta \frac{2(1+\beta)}{\kappa}\right)$  is not compatible with a unique equilibrium.

#### Appendix D. Determinacy properties of a simple Taylor rule

##### Proof of Proposition 2

Substituting the Taylor rule in the IS equation and writing the dynamic system in the usual way for the  $z_t \equiv (y_t, \pi_t)'$  vector of endogenous variables and the  $\nu_t \equiv (\varepsilon_t - r_t^*, u_t)'$  vector of disturbances :

$$E_t z_{t+1} = \mathbf{\Gamma} z_t + \Psi \nu_t$$

The coefficient matrices are given by:

$$\mathbf{\Gamma} = \begin{bmatrix} 1 + \beta^{-1} \delta^{-1} \kappa & \delta^{-1} (\phi_\pi - \beta^{-1}) \\ -\beta^{-1} \kappa & \beta^{-1} \end{bmatrix}$$

$$\Psi = \begin{bmatrix} \delta^{-1} & 0 \\ 0 & -\beta^{-1} \end{bmatrix}$$

Determinacy requires that both eigenvalues of  $\mathbf{\Gamma}$  be outside the unit circle. Note that:

$$\det \mathbf{\Gamma} = \beta^{-1} (1 + \delta^{-1} \kappa \phi_\pi)$$

$$\text{tr} \mathbf{\Gamma} = 1 + \beta^{-1} (1 + \delta^{-1} \kappa)$$

For Case A we have: (Aa) implies:

$$\delta^{-1} \phi_\pi > \frac{\beta - 1}{\kappa}$$

(Ab) implies

$$\delta^{-1} (\phi_\pi - 1) > 0$$

(Ac) implies

$$\delta^{-1} (1 + \phi_\pi) > \frac{-2(1 + \beta)}{\kappa}$$

A comparison with the standard model could help. Notice that we implicitly assumed unit elasticity of substitution in consumption. The standard determinacy condition (labeled by Woodford 2001 'Taylor principle') is  $\phi_\pi > 1$ , which also holds here, of course, for  $\lambda = 0$  (this can be seen by direct substitution in the expressions for  $\delta$  and  $\gamma$ , obtaining 1, respectively  $1 + \frac{\varphi^s}{1+F_Y}$ ). The 'Taylor principle' comes from the second requirement above, since for the standard case requirements 1 and 3 are automatically satisfied (quantities on the right-hand side are negative, and those on the left-hand side positive in this case). In the 'LAMP economy' case, however, this is no longer true. Instead, the determinacy requirements are as follows. First, note that (2) merely requires that  $\delta^{-1}$  and  $(\phi_\pi - 1)$  have the same sign. Hence, we can distinguish two cases:

Case I:  $\delta^{-1} > 0, \phi_\pi > 1$ . As we shall see, the standard case is encompassed here and the Taylor principle is at work as one would expect. The other conditions are automatically satisfied, since both  $\delta^{-1}\phi_\pi$  and  $\delta^{-1}(1 + \phi_\pi)$  are positive, and  $\frac{\beta-1}{\kappa}, \frac{-2(1+\beta)}{\kappa} < 0$ . In terms of deep parameters, the requirement for the sufficiency of the Taylor principle is:

$$\varphi^s < \frac{1-\lambda}{\lambda} (1 + \mu)$$

Case II:  $\delta^{-1} < 0, \phi_\pi < 1$ . Hence, we are looking at the parameter sub-space whereby:

$$\varphi^s > \frac{1-\lambda}{\lambda} (1 + \mu)$$

Condition 1 implies (note that since  $\delta < 0$  the right-hand quantity will be positive):

$$\phi_\pi < \delta \frac{\beta-1}{\kappa}$$

The third requirement for uniqueness implies:

$$\phi_\pi < \delta \frac{-2(1+\beta)}{\kappa} - 1$$

Since  $\phi_\pi \geq 0$ , this last requirement implies a further condition on the parameter space, namely  $\delta \frac{-2(1+\beta)}{\kappa} - 1 \geq 0$ . Overall, the requirement for determinacy when  $\delta^{-1} < 0$  is hence:

$$(D.1) \quad 0 \leq \phi_\pi < \min \left\{ 1, \delta \frac{\beta-1}{\kappa}, \delta \frac{-2(1+\beta)}{\kappa} - 1 \right\}$$

Case B, instead, involves fulfilment of the following conditions: (Ba) implies

$$\delta^{-1} (\phi_\pi - 1) < 0$$

(Bb) implies

$$\delta^{-1} (1 + \phi_\pi) < \frac{-2(1+\beta)}{\kappa}$$

Note that in Case I, whereby  $\delta^{-1} > 0$ , these conditions cannot be fulfilled due to sign restrictions (this is the case in a standard economy as in Woodford 2003, e.g.). In Case II however, the two conditions imply:

$$(D.2) \quad \phi_\pi > \max \left\{ 1, \delta \frac{-2(1+\beta)}{\kappa} - 1 \right\}$$

D.1 and D.2 together imply the following overall determinacy condition for the policy parameter:

$$\phi_\pi \in \left[ 0, \min \left\{ 1, \delta \frac{\beta-1}{\kappa}, \delta \frac{-2(1+\beta)}{\kappa} - 1 \right\} \right) \cup \left( \max \left\{ 1, \delta \frac{-2(1+\beta)}{\kappa} - 1 \right\}, \infty \right)$$

Note that  $\delta$  and  $\kappa$  are functions of the deep parameters.

To assess the magnitude of policy responses needed for determinacy as a function of deep parameters, we can distinguish a few cases for different parameter regions (note that we are always looking at the subspace whereby  $\delta^{-1} < 0$ ):

Share of non-asset holders	Determinacy condition
$\lambda < \bar{\lambda}_1$	$\phi_\pi > 1$
$\lambda \in [\bar{\lambda}_1, \bar{\lambda}_2)$	$\phi_\pi \in \left[0, \delta \frac{-2(1+\beta)}{\kappa} - 1\right) \cup (1, \infty)$
$\lambda \in [\bar{\lambda}_2, \bar{\lambda}_3)$	$\phi_\pi \in \left[0, \delta \frac{\beta-1}{\kappa}\right) \cup (1, \infty)$
$\lambda \in [\bar{\lambda}_3, \bar{\lambda}_4)$	$\phi_\pi \in \left[0, \delta \frac{\beta-1}{\kappa}\right) \cup \left(\delta \frac{-2(1+\beta)}{\kappa} - 1, \infty\right)$
$\lambda \in [\bar{\lambda}_4, 1)$	$\phi_\pi \in [0, 1) \cup \left(\delta \frac{-2(1+\beta)}{\kappa} - 1, \infty\right)$

where

$$\bar{\lambda}_i = \frac{1 + \frac{1}{1+F_Y} \varphi^S \frac{(1-\theta)(1-\beta\theta)}{h_i(\theta)}}{1 + \frac{1}{1+\mu} \varphi^S}$$

$$h_1(\theta) = (1 + \theta)(1 + \beta\theta); h_2(\theta) = 1 + \beta\theta^2 + 2\beta\theta; h_3(\theta) = 1 + \beta\theta^2; h_4(\theta) = 1 - \beta\theta^2$$

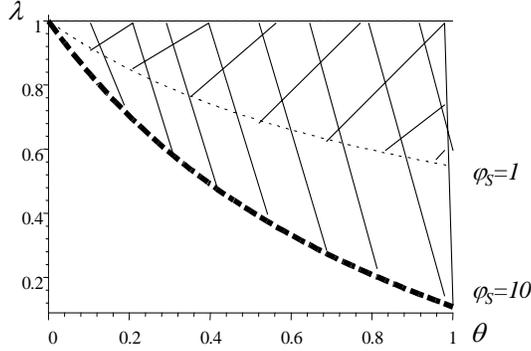


Fig.7: Threshold value for Share of non-asset holders making determinacy conditions closest to Inverted Taylor Principle.

We plot the last case  $\lambda \in [\bar{\lambda}_4, 1)$ ;  $\phi_\pi \in [0, 1) \cup \left(\delta \frac{-2(1+\beta)}{\kappa} - 1, \infty\right)$  in Figure 7 above, where the region above the curve and below the horizontal line gives parameter combinations compatible with the above condition. The different curves correspond to different labor supply elasticities ( $\varphi^S = 1$  dotted line and  $\varphi^S = 10$  thick solid line). Note that for parameters most often assumed in the literature (e.g.  $\theta = 0.75$ ;  $\varphi^S = 10$ ), a share of non-asset holders as low as 0.25 would bring us in this region. Hence, in view of usual estimates of lambda in the literature (see e.g. Campbell and Mankiw) we shall consider this case as the most plausible. Whenever these parameter restrictions are met, determinacy is insured by either a violation of the Taylor principle, or for a strong response to inflation. However, note that the lower bound on the inflation coefficient then becomes very large (35.433 under the baseline calibration), which is far from any empirical estimates. Indeed, the threshold inflation coefficient is sharply increasing in the share of non-asset holders, elasticity of labor supply, as can be seen by merely differentiating  $\delta \frac{-2(1+\beta)}{\kappa} - 1$  with respect to all these parameters.

### Appendix E. Restoring the Taylor Principle

Redistributive distortionary fiscal policy: Replacing the fiscal rule  $\lambda l_{H,t} = \tau o_t$  into the new budget constraint of non-asset holders we get  $c_{h,t} = w_t + \frac{\tau}{\lambda} d_t$ .

Asset holders' consumption will then be given by (substituting the expression for profits)

$$c_{s,t} = \frac{1}{1-\lambda} \left( 1 - \tau \frac{\mu}{1+\mu} \right) y_t + \frac{\tau-\lambda}{1-\lambda} w_t$$

The wage-hours locus WN is then

$$w_t = \frac{1}{1-\tau} [1 + \varphi^S + \mu(1-\tau)] n_t$$

Substituting back, consumption of asset holders is related to output according to:

$$c_{s,t} = \frac{1}{1-\tau} \left[ 1 - \tau \frac{\mu}{1+\mu} + \frac{\tau-\lambda}{1-\lambda} \frac{\varphi^S}{1+\mu} \right] y_t$$

### Appendix F. Computing sunspot equilibria

The stability condition in the case of indeterminacy is - see Lubik and Schorfheide 2003, p. 278 (where  $[A]_2$  denotes the second row of the  $A$  matrix, attached to the explosive component):

$$[J^{-1}\Psi]_2 \nu_t + [J^{-1}\Gamma]_2 \eta_t = 0$$

Straightforward algebra to calculate

$$\begin{aligned} [J^{-1}\Psi]_2 &= \frac{1}{q_+ - q_-} \begin{bmatrix} -\kappa\delta^{-1} & \beta^{-1} - q_- \end{bmatrix} \\ [J^{-1}\Gamma]_2 &= \frac{1}{q_+ - q_-} \begin{bmatrix} -\kappa q_+ & q_+ - 1 \end{bmatrix} \end{aligned}$$

delivers the stability condition as:

$$-\kappa\delta^{-1} (\varepsilon_t - r_t^*) + (\beta^{-1} - q_-) u_t - \kappa q_+ \eta_t^y + (q_+ - 1) \eta_t^\pi = 0$$

Since only one root is suppressed, there is endogenous persistency of the effects of shock (which was not the case under determinacy).

Following Lubik and Schorfheide (2003) we compute a singular value decomposition of  $(q_+ - q_-) [J^{-1}\Gamma]_2$  :

$$\begin{aligned} [J^{-1}\Gamma]_2 &= 1 \cdot \begin{bmatrix} d & 0 \end{bmatrix} \cdot \begin{bmatrix} \frac{-\kappa q_+}{d} & \frac{q_+ - 1}{d} \\ \frac{q_+ - 1}{d} & \frac{\kappa q_+}{d} \end{bmatrix} \\ d &= \sqrt{(\kappa q_+)^2 + (q_+ - 1)^2} \end{aligned}$$

Using also  $\beta(q_+ - q_-) [J^{-1}\Psi]_2 = \begin{bmatrix} -\kappa\delta^{-1} & \beta^{-1} - q_- \end{bmatrix}$  we get the full set of stable solutions as described in text.

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