

Aggregation and Design of Information in Asset Markets with Adverse Selection

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Abstract

How effectively does a decentralized marketplace aggregate information that is dispersed throughout the economy? We study this question in a dynamic setting where sellers have private information that is correlated with an unobservable aggregate state. In any equilibrium, each seller's trading behavior provides an informative and conditionally independent signal about the aggregate state. We ask whether the state is revealed as the number of informed traders grows large. Surprisingly, the answer is no; we provide conditions under which information aggregation necessarily fails. In another region of the parameter space, aggregating and non-aggregating equilibria co-exist. We solve for the optimal information policy of a social planner who observes trading behavior. We show that non-aggregating equilibria are always constrained inefficient. The optimal information policy Pareto improves upon the laissez-faire outcome by concealing information about trading volume when it is sufficiently high.

JEL: G14, G18, D47, D53, D82, D83.

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1 Introduction

Since the seminal work of Hayek (1945), the question of whether markets effectively aggregate dispersed information has been a central one in economics. Formal investigations of this question are plentiful. Yet, they are typically conducted in a setting with a single asset about which traders have dispersed information. Whether information is aggregated then usually boils down to whether the equilibrium price reveals the value of the asset conditional on the union of traders' information. This broad class of models is natural for many applications from static common-value auctions to dynamic trading in financial markets. For other applications (e.g., real estate, OTC markets), information dispersion arises due to dispersion in ownership, and one is interested in the extent to which aggregate trading behavior across heterogeneous assets reveals information about the underlying state of the economy. In this paper, we explore such a setting.

Building on the framework of Asriyan, Fuchs, and Green (2017), we investigate the question of information aggregation in a dynamic setting with many assets, whose values are independently and identically drawn from a distribution that depends on an underlying aggregate state. The value of each asset is privately observed by its seller, who receives offers each period from competitive buyers. We ask whether the history of all transactions reveals the aggregate state as the number of informed sellers in the economy (denoted by N) grows large.

To answer this question, we begin by characterizing the set of equilibria for arbitrary N . Due to a complementarity between the amount of information collectively revealed by others and the optimal strategy of an individual seller, multiple equilibria can exist. A feature common to all equilibria is that each individual seller's trading behavior provides an informative and conditionally independent signal about the aggregate state. Therefore, one might intuitively expect that, by the law of large numbers, the state would be revealed as the number of sellers tends to infinity.

Our first main result highlights that this intuition is incomplete because it ignores the sellers' endogenous response to the expectation of information arrival, which in turn changes the information content of each individual trade. We provide necessary and sufficient conditions under which aggregation necessarily fails, i.e., there does not exist a sequence of equilibria that reveal the state as $N \rightarrow \infty$. The reason for aggregation failure is that the information content of each individual seller's behavior tends to zero at a rate of $1/N$, just fast enough to offset the additional number of observations. As a result, some information is revealed by the limiting trading behavior, but not enough to precisely determine the underlying state. Roughly speaking, the conditions ensuring aggregation failure require that the correlation

of asset values is sufficiently high and that agents are sufficiently patient. Intuitively, these conditions guarantee that if the aggregate state were to be revealed with certainty tomorrow, then the option value of delaying trade today is relatively high. An immediate corollary is that information aggregation always obtains in a static model.

When the conditions for aggregation failure do not hold, there exists a sequence of equilibria such that information about the state is aggregated as $N \rightarrow \infty$. However, even in this case, aggregation is not guaranteed. Our second main result shows that there is a region of the parameter space in which there is coexistence of equilibria that reveal the state with equilibria that do not. The key difference across the two types of equilibria is the rate at which trade declines as the number of informed sellers grows. In the non-aggregating equilibria, trade declines at rate $1/N$ whereas in aggregating equilibria, the rate of trade declines slower than $1/N$.

Whether information aggregates has important implications for welfare, prices, and trading behavior. To understand them, it is useful to draw comparison to a fictitious economy in which the state is exogenously revealed after the first trading period. When information aggregates, both trading volume and welfare converge to their levels in the fictitious economy and the volatility of prices conditional on the true state goes to zero. In contrast, along a sequence of non-aggregating equilibria, trading volume and welfare are strictly lower than in the fictitious economy and the conditional price volatility remains strictly positive even as $N \rightarrow \infty$.

Two immediate implications follow. First, from a social welfare perspective, aggregating equilibria are always preferable to non-aggregating equilibria when they co-exist. Thus, among laissez-faire outcomes, aggregation is optimal. Second, if all equilibria are non-aggregating, then a planner could improve overall welfare by learning and revealing the true state. Of course, it is not obvious how the planner could learn the true state. It is more natural to think that the planner is uninformed, but can learn about the true state by observing the trading behavior of market participants. The problem facing the planner is then how best to reveal this information to other agents in the economy. The question we ask is whether an uninformed planner can improve on the laissez-faire outcomes.

We address this question in Section 4. Doing so involves formulating and solving an information design problem related to the literature on Bayesian persuasion (Kamenica and Gentzkow, 2011; Rayo and Segal, 2010). One key difference is that the planner in our model must take into account the fact that her policy influences the information content of trading behavior and thus the information content of whatever is revealed. In other words, the statistical properties of the information that is revealed by the planner endogenously depend on the planner's policy.

Our solution technique involves several steps. First, employing standard techniques, we solve the information design problem for an “informed” planner who (exogenously) learns the aggregate state. Under an additional incentive constraint, we show that the solution to the constrained informed planner’s problem is an upper bound for the surplus that can be attained by an uninformed planner. We then construct a reporting policy for the uninformed planner such that the payoffs converge to the upper bound as $N \rightarrow \infty$.

We demonstrate that when information aggregation fails, the laissez-faire outcome (where all trades are observed) is not constrained efficient. Perhaps counterintuitively, the planner can improve upon such outcomes by concealing some information about trading behavior in order to promote more gains from trade being realized. Moreover, information aggregation does not necessarily imply constrained efficiency. Even when information aggregates, the optimal information policy can improve overall efficiency when the discount factor is sufficiently high. Importantly, welfare under the optimal information policy Pareto dominates any laissez-faire outcome, i.e., all agents would be willing to delegate the information dissemination about past trades to the social planner. These findings have obvious implications for policies aimed at promoting market transparency.

Recently, there has been a strong regulatory push towards making financial markets more transparent (i.e., disclosing more information about trading activity to market participants). For example, one of the stated goals of the Dodd-Frank Act of 2010 is to increase transparency and information dissemination in the financial system. The European Commission is considering revisions to the Markets in Financial Instruments Directive (MiFID), in part to improve the transparency of European financial markets. Our results highlight a potential trade-off for such policies and provide a potential justification for limiting the amount of information available to market participants.

Relatedly, the introduction of benchmarks that reveal some aggregate trading information has also received recent attention by policymakers and academics. Duffie, Dworczak, and Zhu (2017) analyze the role of benchmarks (e.g., LIBOR) in revealing information about fundamentals and suggest that the introduction of benchmarks is welfare enhancing. Our analysis highlights an important consideration that is absent in their setting. Namely, that the informational content of the benchmark may change once it is published due to endogenous responses by market participants.

While our model is stylized and abstracts from institutional details of specific markets, we believe that it allows us to identify mechanisms which can be at play in a broad range of market settings. The two ingredients at the core of our analysis – dynamic adverse selection and asset correlation – are plausible in a variety of both real and financial asset markets. For instance, the assets in our model could correspond to securities issued by firms in the

same industry, houses in the same neighborhood, or mortgage-backed securities exposed to the same regional shocks. Recent empirical work on real-estate markets shows that sellers of nearby houses are privately informed about their (common) neighborhood characteristics (Kurlat and Stroebel, 2015), and that trades of some houses release information that is relevant for the valuation of others (Alti, Engelberg, and Parsons, 2011). In the secondary market for residential mortgages, Adelino, Gerardi, and Hartman-Glaser (2019) provide evidence consistent with predictions of dynamic models with adverse selection.

1.1 Related Literature

This paper builds upon our previous work, Asriyan et al. (2017) (henceforth AFG), which demonstrates that multiple equilibria can exist in a model with two informed sellers. In the current paper, we focus on the information aggregation properties of equilibria. In order to do so, we first extend the two-seller model of AFG to a model with an arbitrary number of sellers and characterize the set of equilibria. This extension, which is carried out in Section 2, follows closely from AFG. The primary contribution of this paper is along two different dimensions. First, we identify the information aggregation properties of equilibria as the market grows large, and their implications for trade and welfare (Section 3). Second, we analyze the normative implications by considering the information design problem of a social planner who observes trading behavior and chooses what information to communicate to the traders (Section 4). Here, we also contribute to the recently growing literature on information design (Bergemann and Morris, 2013, 2016). In addition to the information policy affecting the ex-ante incentives as in Boleslavsky and Kim (2018), we show how the endogeneity of the planner’s information set constrains the optimal policy.

Within static environments, there is a large literature that studies questions regarding information aggregation. Seminal works on this topic include Grossman (1976), Wilson (1977), Milgrom (1979), Hellwig (1980), and Kyle (1989). More recent progress on this question has been made by Pesendorfer and Swinkels (1997), Kremer (2002), Reny and Perry (2006), Rostek and Weretka (2012), Bodoh-Creed (2013), Albagli, Tsyvinski, and Hellwig (2015), Lauermaun and Wolinsky (2017), Axelson and Makarov (2017), Babus and Kondor (2016), and Siga and Mihm (2018), among others.¹ By and large, this literature is largely defined by a centralized trading environment in which there is a single asset about which agents have dispersed information. The question of information aggregation is whether the price is a sufficient statistic for the union of this dispersed information. In contrast, we explore a decentralized trading environment with heterogeneous assets and ask whether

¹Palfrey (1985) and Vives (1988) explore this question within a Cournot setting.

the history of trading behavior is sufficient to infer the underlying state. Moreover, our results pertaining to non-aggregation crucially rely on dynamic considerations—with a single opportunity to trade, information is *always* aggregated in our setting.²

Kyle (1985) studies a dynamic insider trading model and shows that the insider fully reveals his information as time approaches the end of the trading interval. Foster and Viswanathan (1996) and Back, Cao, and Willard (2000) extend this finding to a model with multiple strategic insiders with different information. Ostrovsky (2012) further generalizes these findings to a broader class of securities and information structures. He considers a dynamic trading model with finitely many partially informed traders and provides necessary and sufficient conditions on security payoffs for information aggregation to obtain. Our paper differs from these works in that we study a setting with heterogeneous but correlated assets owned by privately informed sellers. We ask whether information aggregates as the number of sellers becomes arbitrarily large. Despite the fact that we look at the limit as $N \rightarrow \infty$, the strategic considerations do not vanish in our model since there is an idiosyncratic component to the value of each asset.

Golosov, Lorenzoni, and Tsyvinski (2014) consider an environment in which some agents have private information about an asset while the rest are uninformed. Agents trade in a decentralized anonymous market through bilateral matches, i.e., signaling with trading histories is not possible. They find that information aggregation obtains in the long run. In contrast, in our setting observing trading histories plays a crucial role: signaling through delay diminishes the amount of trade, thus reducing the information content of the market, leading to the possibility that information aggregation fails.

Lauermaun and Wolinsky (2016) study information aggregation in a search market, in which an informed buyer sequentially solicits offers from sellers who have noisy information about the buyer’s value. They provide conditions under which information aggregation fails, and they trace this failure to a strong form of winner’s curse that arises in a search environment. Although our model is quite different, we share the common feature that the fear of adverse selection hinders trade and thus reduces information generation in markets.

Also within a search framework, Lester, Shourideh, Venkateswaran, and Zetlin-Jones (2018) study trade, bid-ask spreads and learning in a model of intermediation with privately informed traders. They show that lower search frictions reduce the difference in reserva-

²To illustrate the role of dynamics, we can contrast our results with Kyle (1989). Theorem 8.2 in his paper shows that, as the number of informed traders increases, there is always more information revealed by the market, leading to information aggregation. This is because (i) there are more informed traders and, thus, more information available in the market, and; (ii) each informed trader trades more aggressively. In our setting, while the first effect also leads to more information, because seller’s continuation value depends on market informativeness, they may actually trade less aggressively as the number of traders increases. This countervailing force is what drives our aggregation failure result.

tion values between traders with high and low value assets, thereby causing them to trade more similarly, which slows the rate at which uninformed dealers learn. In their setting, the difference in reservation values is also decreasing with respect to the discount factor, which is related to our finding that information aggregation is more likely to fail for higher discount factors. Maestri, Gerardi, and Camargo (2016) study a setting with a continuum of privately informed traders where asset values are perfectly correlated and show that full efficiency is attained as the discount factor goes to one. We focus primarily on a setting with imperfect asset correlation and non-trivial costs of delay. However, our setting also differs from theirs in several other respects: we have a countable number of traders and buyers face intra-temporal competition. These differences explain why trade can remain inefficient in our model even as both asset correlation and the discount factor go to one.

In addition to the welfare implications studied in this paper, there are a variety of other reasons for why information aggregation may be a desirable property of markets. For instance, such information may be useful for informing firms' investment decisions (Fishman and Hagerty, 1992; Leland, 1992; Dow and Gorton, 1997; Camargo, Kim, and Lester, 2015), government interventions (Bond, Goldstein, and Prescott, 2009; Bond and Goldstein, 2015; Boleslavsky, Kelly, and Taylor, 2017), and monetary policy (Bernanke and Woodford, 1997). Markets that convey more information can also be more useful for providing better incentives to managers (Baumol, 1965; Fishman and Hagerty, 1989) and mitigating the winner's curse in common-value auctions (Milgrom and Weber, 1982). As documented by a number of papers in this literature, the feedback loop between real decisions and price informativeness may undermine the ability of markets to aggregate information and lead to aggregation failures.³ To highlight how our mechanism differs from this literature, we abstract from any such considerations here.

Our paper is related to a growing literature that studies dynamic markets with adverse selection (e.g., Janssen and Roy (2002), Hörner and Vieille (2009), Fuchs and Skrzypacz (2012), Guerrieri and Shimer (2014), Fuchs, Öery, and Skrzypacz (2016), Daley and Green (2012, 2016), Asriyan, Fuchs, and Green (2019)). Our innovation is the introduction of asset correlation, which allows us to study the information aggregation properties of these markets. The optimal information policy in our model is similar in spirit to the use of transaction taxes, as explored in Fuchs and Skrzypacz (2015). Both instruments can lower continuation values and stimulate more trade. However, as we discuss in Section 4.4, taxes are not a perfect substitute for information design. In addition, as we highlight in Section 3.3, though non-aggregation of information can arise in other settings as well, the possibility of coexistence

³See Bond, Edmans, and Goldstein (2012) for a survey of both the theoretical and empirical literature on the real effects of information conveyed through markets.

of aggregating and non-aggregating equilibria hinges critically on adverse selection.

Finally, our paper is also related to a long tradition in macroeconomics of studying the aggregate implications of dispersed information (e.g., Phelps (1970), Lucas (1972), Angeletos and La’o (2010)). More recently, Gaballo (2018) considers an “islands” setting à la Phelps-Lucas, and he shows that when the heterogeneity across islands is positive, but sufficiently small, multiple equilibria emerge: one in which information aggregates and another in which aggregation fails. Though our settings are quite different, the conditions for multiplicity are similar in that both endow the possibility of complementarities across agents.

2 The Model

In this section, we extend the model and equilibrium characterization of AFG to a setting with an arbitrary number of sellers. Our main results are located in Sections 3 and 4. There are $N + 1$ sellers indexed by $i \in \{1, \dots, N + 1\}$, with $N \geq 1$. Each seller is endowed with an indivisible asset and is privately informed of her asset’s type, denoted by $\theta_i \in \{L, H\}$. Seller i has a value c_{θ_i} for her asset, where $c_L < c_H$. The value of a type- θ asset to a buyer is v_θ and there is common knowledge of gains from trade, $v_\theta > c_\theta$.

We start by considering a model in which there are two trading periods: $t \in \{1, 2\}$. We generalize our main results to an infinite-horizon model in Section 5.1.⁴ In each period, each seller is matched with two competing buyers who make private offers to the seller. Each buyer can make one offer; a buyer whose offer is rejected gets a payoff of zero and exits the game. The payoff to a buyer who purchases an asset of type θ at price p is $v_\theta - p$.

Sellers discount future payoffs by a factor $\delta \in (0, 1)$. The payoff to a seller with an asset of type θ , who trades at price p in period t is

$$(1 - \delta^{t-1})c_\theta + \delta^{t-1}p. \tag{1}$$

If the seller does not trade at either date, his payoff is c_θ . One can interpret c_θ and v_θ as the present value of the flow payoffs from owning the asset to the seller and buyer respectively.⁵ All players are risk neutral.

Asset values are correlated with an unobservable underlying state, S , that takes values in $\{\ell, h\}$. The unconditional distribution of θ_i is $\mathbb{P}(\theta_i = H) = \pi_0 \in (0, 1)$. Assets are mutually independent conditional on the state, but their conditional distributions are given

⁴The two-period model facilitates a more precise characterization of the set of equilibria and thus a sharper intuition for our main results.

⁵Alternatively, we could specify the seller’s payoff as $\delta^{t-1}(v_\theta - c_\theta)$ and interpret c_θ as the seller’s production cost.

by $\mathbb{P}(\theta_i = L|S = \ell) = \lambda \in (1 - \pi_0, 1)$. To allow for arbitrarily high level of correlation, we set $\mathbb{P}(S = h) = \pi_0$. Our correlation structure introduces the possibility that trades of one asset convey relevant information about the aggregate state and therefore the value of other assets. To capture this possibility, we assume that all transactions are observable. Therefore, prior to making offers in the second period, each buyer observes the set of assets that traded in the first period.

Notice that by virtue of knowing her asset quality, each seller has a private and conditionally independent signal about the aggregate state of nature. Thus, if each seller were to report her information truthfully to a central planner, the planner would learn the aggregate state with probability one as $N \rightarrow \infty$. Our interest is to explore under what conditions the same information can be gleaned from the transaction data of a decentralized market. To fix ideas, we assume that the aggregate state is publicly realized at the end of period 2 and ask whether the trading behavior in period 1 reveals the state.⁶ To ensure that strategic interactions remain relevant, we focus on primitives which satisfy the following assumptions.

Assumption 1 $\pi_0 v_H + (1 - \pi_0)v_L < c_H$.

Assumption 2 $v_L < (1 - \delta)c_L + \delta c_H$.

The first assumption, which we refer to as the “lemons” condition, asserts that the adverse selection problem is severe enough to rule out the efficient equilibrium in which all sellers trade immediately. In this equilibrium, trade is uninformative about the underlying state (regardless of N). The second assumption implies a lower bound on the discount factor and ensures that dynamic considerations remain relevant. Our main results do not rely on this assumption but it simplifies exposition and rules out fully separating equilibria, which are also independent of N .

2.1 Remarks on Modeling Assumptions

To illustrate the key mechanism for our findings as clearly as possible, we have made several rather stark assumptions regarding the buyers’ side of the market. In particular: buyers are short-lived in that they can only make one offer, they have identical values and information, and they compete in Bertrand fashion. This ensures that (1) a buyer makes zero (expected) profits, and (2) there do not exist mutually agreeable unrealized gains from trades. The primary purpose of these assumptions is to isolate the reason by which trade is delayed (i.e.,

⁶In Section 5.1, we demonstrate that our main results extend to an infinite horizon model in which the state is never (publicly) realized.

it is only due to the seller's strategic considerations) and to ensure that prices (i.e., buyers' offers) respond to new information.

While these features seem reasonable and are not strictly necessary for most of our results, it is worth discussing them in a bit more detail. First, that buyers make offers in only one period is a fairly standard assumption in this literature (e.g., Swinkels, 1999; Kremer and Skrzypacz, 2007; Hörner and Vieille, 2009). The set of equilibrium outcomes we identify remain equilibrium outcomes in a model where buyers make offers over multiple periods provided those offers are publicly observable. Complications arise when buyers are long-lived and offers are private, as then a buyer may have incentive to experiment in the first period by making an offer that loses money if it is accepted in order to make profits conditional on a rejection (Deneckere and Liang, 2006; Daley and Green, 2018; Lee, 2020).

If buyers' values or information is not identical, then a seller may have incentive to delay trade in the first period in order to meet a more favorable buyer in the second period. The assumption that each buyer is matched to a single seller in a given period is purely for convenience and can easily be relaxed. In what follows, we will also assume that buyers in the second period observe first-period transactions but not prices. This too is simply for convenience. Because buyers are uninformed and make the offers using pure strategies in the first period, no additional information (beyond whether a transaction occurred) is revealed by the price.

2.2 Strategies and Equilibrium Concept

A strategy of a buyer is a mapping from his information set to a probability distribution over offers. In the first period (i.e., at $t = 1$), a buyer's information set is empty. In the second period, buyers' information set is a vector in $\{0, 1\}^{N+1}$ which indicates whether each asset traded in the first period. If asset i trades in the first period, then it is efficiently allocated and it is without loss of generality to assume that buyers do not make offers for it in the second period (Milgrom and Stokey, 1982). The strategy of each seller is a mapping from her information set to a probability of acceptance. Seller i 's information includes her type, the set of previous and current offers as well as the information set of buyers.

We use Perfect Bayesian Equilibria (PBE) as our solution concept. This has three implications. First, each seller's acceptance rule must maximize her expected payoff at every information set taking buyers' strategies and the other sellers' acceptance rules as given (*Seller Optimality*). Second, any offer in the support of the buyer's strategy must maximize his expected payoff given his beliefs, other buyers' strategies and the seller's strategy (*Buyer Optimality*). Third, given their information set, buyers' beliefs are updated according to

Bayes' rule whenever possible (*Belief Consistency*).

2.3 Updating

There are two ways by which a buyer's belief about seller i is updated between the first and second period. First, they update their beliefs based on whether each asset traded. Let σ_i^θ denote the probability that buyers assign to seller i trading in the first period if her asset is type θ . Conditional on rejecting the offer in the first period, the buyers' interim belief is given by

$$\pi_i^{Int} \equiv \mathbb{P}(\theta_i = H | \text{reject at } t = 1) = \frac{\pi_0(1 - \sigma_i^H)}{\pi_0(1 - \sigma_i^H) + (1 - \pi_0)(1 - \sigma_i^L)}. \quad (2)$$

Second, before making offers in the second period, buyers learn about any other trades that took place in the first period. How this information is incorporated into the posterior depends on buyer beliefs about the trading strategy of the other sellers (i.e., σ_j^θ , $j \neq i$). Let $z^j \in \{0, 1\}$ denote the indicator for whether seller j trades in the first period, and let $\mathbf{z} = (z^j)_{j=1}^{N+1}$ and $\mathbf{z}_{-i} = (z^j)_{j \neq i}$. Denote the probability of \mathbf{z}_{-i} conditional on seller i being of type θ by $\rho_\theta^i(\mathbf{z}_{-i})$, which can be written as

$$\rho_\theta^i(\mathbf{z}_{-i}) \equiv \sum_{s \in \{l, h\}} \mathbb{P}(S = s | \theta_i = \theta) \cdot \prod_{j \neq i} \mathbb{P}(z^j | S = s), \quad (3)$$

where $\mathbb{P}(z^j = 1 | S = s) = \sum_{\theta \in \{L, H\}} \sigma_j^\theta \cdot \mathbb{P}(\theta_j = \theta | S = s)$ is the probability that buyers assign to seller j trading in state s . Provided there is positive probability that i rejects the bid at $t = 1$ and \mathbf{z}_{-i} is realized, we can use equations (2) and (3) to express the posterior probability of seller i being type H conditional on these two events:

$$\pi_i(\mathbf{z}_{-i}) \equiv \mathbb{P}(\theta_i = H | z^i = 0, \mathbf{z}_{-i}) = \frac{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}_{-i})}{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}_{-i}) + (1 - \pi_i^{Int}) \cdot \rho_L^i(\mathbf{z}_{-i})}. \quad (4)$$

2.4 Equilibrium Properties

AFG establish several features that must hold in any equilibrium of the two-seller model. It is straightforward to show that these properties extend to the model studied here with an arbitrary number of sellers. However, developing an intuition for them will be useful for understanding our main results in Sections 3 and 4, so we provide some explanation of them here. Because they are simply generalizations of prior work and not the contribution of this paper, we will refer to them here as “properties” rather than lemmas or propositions.

In order to introduce them, we will use the following definitions and notation. We refer to the *bid* for asset i at time t as the maximal offer made across all buyers for asset i at time t . Let $V(\tilde{\pi}) \equiv \tilde{\pi}v_H + (1 - \tilde{\pi})v_L$ denote buyers' expected value for an asset given an arbitrary belief $\tilde{\pi}$. Let $\bar{\pi} \in (\pi_0, 1)$ be such that $V(\bar{\pi}) = c_H$, and recall that π_i denotes the probability that buyers assign to $\theta_i = H$ prior to making offers in the second period.

Property 1 (Second period) *If seller i does not trade in the first period, then in the second period:*

- (i) *If $\pi_i > \bar{\pi}$ then the bid is $V(\pi_i)$, which the seller accepts w.p.1.*
- (ii) *If $\pi_i < \bar{\pi}$ then the bid is v_L , which the high type rejects and the low type accepts w.p.1.*
- (iii) *If $\pi_i = \bar{\pi}$, then with some probability $\phi_i \in [0, 1]$, the bid is $c_H = V(\pi_i)$ and the seller accepts w.p.1. With probability $1 - \phi_i$, the offer is v_L , the high type rejects, and the low type accepts w.p.1.*

Note that a high type will only accept a bid higher than c_H . When the expected value of the asset is above c_H (as in (i)), competition forces the equilibrium offer to be the expected value. When the expected value of the asset is below c_H (as in (ii)), buyers cannot attract both types without making a loss. Thus, only the low type will trade and competition pushes the bid to v_L . Finally, when the expected value of the asset is exactly c_H (as in (iii)), buyers are indifferent between offering c_H and trading with both types or offering v_L and only trading with the low type.

Notice that Property 1 implies a second-period payoff to a type- θ seller i as a function of (π_i, ϕ_i) , which we denote by $F_\theta(\pi_i, \phi_i)$, where

$$F_H(\pi_i, \phi_i) \equiv \max \{c_H, V(\pi_i)\}, \quad (5)$$

and

$$F_L(\pi_i, \phi_i) \equiv \begin{cases} v_L & \text{if } \pi_i < \bar{\pi} \\ \phi_i c_H + (1 - \phi_i)v_L & \text{if } \pi_i = \bar{\pi} \\ V(\pi_i) & \text{if } \pi_i > \bar{\pi}. \end{cases} \quad (6)$$

Figure 1 plots the equilibrium price in the second period as a function of the posterior belief. Due to the local convexity of the price near $\pi_i = \bar{\pi}$, the low type has a strong incentive to reject the first-period bid at beliefs just below $\bar{\pi}$ and wait for noisy information to be

revealed—even if she expects the posterior to decrease.⁷ Thus, from seller i 's perspective, the strategy of seller $j \neq i$ in the first period is relevant because it influences the distribution of news and therefore the distribution of π_i . In particular, the (expected) continuation value of a seller from rejecting an offer in the first period can be written as

$$Q_\theta^i \equiv (1 - \delta)c_\theta + \delta \sum_{\mathbf{z}_{-i}} \rho_\theta^i(\mathbf{z}_{-i}) F_\theta(\pi_i(\mathbf{z}_{-i}), \phi_i). \quad (7)$$

It depends on seller i 's own trading strategy σ_i^θ through the interim belief. But, importantly, it also depends on (i) the correlation of types with the state, and (ii) the strategies of other sellers, since both influence the distribution of “news” that the buyers receive about seller i in the second period.

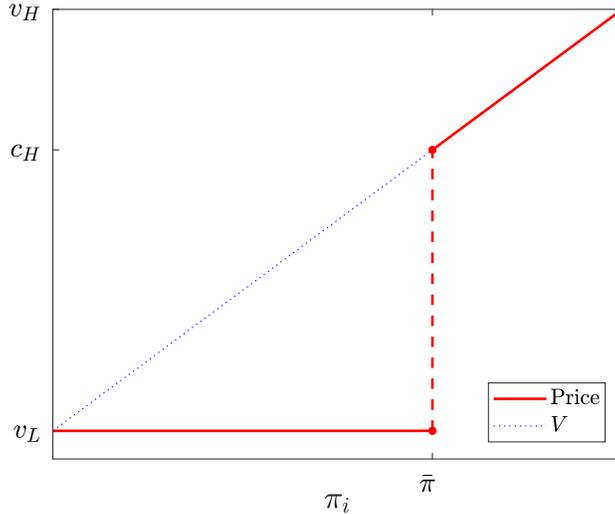


Figure 1: The equilibrium price in the second period as it depends on buyers' posterior belief.

Property 2 (Skimming) *In any equilibrium, the expected continuation value of the high type is strictly greater than that for the low type: $Q_H^i > Q_L^i$.*

This result, often referred to in the literature as a “skimming” property, is due to the fact that both the flow payoff c_θ and the continuation payoff F_θ are higher for the high type, and because the high type rationally expects a (weakly) more favorable distribution of buyer posteriors (and thus prices) in the second period.

⁷Because the low type assigns a higher probability to state l than the buyers, she expects information arrival will reduce the buyers' posterior belief.

Property 3 (First period) *In the first period, the bid for each asset is v_L . The high-type seller rejects this bid with probability 1. The low-type seller accepts this bid with probability less than 1.*

By Property 2, any offer that is acceptable to a high type in the first period is accepted by the low type w.p.1. But Assumption 1 implies that any such offer yields negative profits for the buyers. Hence, in equilibrium only low types trade in the first period and competition pushes the bid to v_L . Finally, if the low-type seller i trades with probability 1, then the bid in the second period must be v_H (Property 1). But then the low-type seller i would strictly prefer to delay trade to the second period (Assumption 2), a contradiction.

Henceforth, we drop the θ superscript from the trading probability in the first period, and we let σ_i denote the probability that the low-type seller i trades in the first period.

Property 4 (Symmetry) *In any equilibrium, $\sigma_i = \sigma > 0$ for all i . If buyer mixing is part of the equilibrium then $\phi_i = \phi$ for all i .*

The key step to prove symmetry is to show that if $\sigma_i > \sigma_j \geq 0$, then $Q_L^i > Q_L^j$. This follows from the fact that, due to imperfect correlation, π_i (and therefore Q_L^i) is more sensitive to i 's own trading probability than it is to that of the other sellers. Note that, by Property 3, $Q_L^j \geq v_L$. Hence, if $Q_L^i > Q_L^j$, then the low-type seller i strictly prefers to wait, which contradicts $\sigma_i > 0$ being consistent with an equilibrium. Next, note that it cannot be that $\sigma_i = 0$ for all i . If that were the case, then no news arrives (since observing no trade contains no information) and buyers in the second period would have the same beliefs as buyers in the first period. This would imply that the second period bid is v_L (Property 1), but in that case the low-type sellers would be strictly better off by accepting v_L in the first period, which contradicts $\sigma_i = 0$.

2.5 Equilibria

Given Properties 1–4, we will henceforth drop the i subscripts wherever possible and denote a candidate equilibrium by the pair (σ, ϕ) . Because all equilibria are symmetric, any information about seller i that is contained in news \mathbf{z}_{-i} does not depend on the identity of those who sold but only on the number (or fraction) of other sellers that traded. For example, suppose that $\mathbf{z}_{-i} = \mathbf{z}(K)$ where $\mathbf{z}(K)$ is such that $\sum_{j \neq i} z^j = K \leq N$. Then

$$\rho_\theta^i(\mathbf{z}(K)) = \sum_{s \in \{l, h\}} p_s^K \cdot (1 - p_s)^{N-K} \cdot \mathbb{P}(S = s | \theta_i = \theta),$$

where $p_s \equiv \sigma \cdot \mathbb{P}(\theta_i = L | S = s)$ is the probability that any given seller trades in state s . Naturally, the probability of observing K trades among sellers $j \neq i$ is $\binom{N}{K} \cdot \rho_\theta^i(\mathbf{z}(K))$.

Furthermore, since any equilibrium involves $\sigma \in (0, 1)$, a low-type seller must be indifferent between accepting v_L in the first period and waiting until the second period. The set of equilibria can thus be characterized by the solutions to

$$Q_L(\sigma, \phi) = v_L, \tag{8}$$

where we now make explicit the dependence of the continuation value on the strategy (σ, ϕ) .

As we show in the next proposition, there can be multiple solutions to (8) and hence multiple equilibria.

Proposition 1 (Existence and Multiplicity) *An equilibrium always exists. If λ and δ are sufficiently large, there exist multiple equilibria.*

Intuitively, a higher σ has two opposing effects on the seller's continuation value. On the one hand, the posterior beliefs and thus prices in the second period are increasing in σ , which increases the expected continuation value Q_L . On the other hand, as other low types trade more aggressively, the distribution over buyers' posteriors shifts towards lower posteriors, thus decreasing Q_L . The latter force generates complementarities in sellers' trading strategies, which results in multiple equilibria when the correlation between assets is high and traders care sufficiently about the future.

3 Information Aggregation

We now turn to our first main question, specifically, whether information about the underlying state is aggregated as the number of informed participants grows large. To understand the essence of this question, first notice that the trading behavior of each seller provides an informative signal about the aggregate state. If the seller trades in the first period, then she reveals her asset's type is L , which is more likely when the aggregate state is ℓ than when it is h . Conversely, if the seller does not trade, then buyers update their beliefs about the asset toward H and their belief about the aggregate state toward h . Moreover, the amount of information revealed by each seller is increasing in the low-type's trading probability, which we now denote by σ_N .

If the information content of each individual trade were to converge to some positive level (i.e., $\lim_{N \rightarrow \infty} \sigma_N = \bar{\sigma} > 0$), then information about the state would aggregate. The reason is that by the law of large numbers the fraction of assets traded would concentrate around

its population mean $\bar{\sigma} \cdot \mathbb{P}(\theta_i = L|S = s)$, which is strictly greater when the aggregate state is ℓ than when it is h . If, on the other hand, σ_N decreases to zero at a rate weakly faster than $1/N$ (i.e., $\lim_{N \rightarrow \infty} N \cdot \sigma_N < \infty$), then information would not aggregate. In this case, despite having arbitrarily many signals about the state, the informativeness of each signal goes to zero fast enough that the overall amount of information does not reveal the true state.

Of course, the equilibrium trading behavior of each individual seller is determined endogenously. Therefore, in order to establish information aggregation properties of equilibria, we need to understand how the set of equilibrium values of σ_N changes with N . Moreover, since different equilibria have different σ_N , the limiting information aggregation properties could be different for different sequences of equilibria.

To formally analyze the question of aggregation, consider a sequence of economies indexed by N (standing for $N + 1$ assets), and let σ_N denote an equilibrium trading probability in the first period and π_N^{State} be the buyers' posterior belief that the aggregate state is h , conditional on having observed the outcome of trade in the first period. That is, given a trading history $\mathbf{z} = (z^j)_{j=1}^{N+1}$, $\pi_N^{State}(\mathbf{z}) \equiv \mathbb{P}(S = h|\mathbf{z})$. We say that:

Definition 1 *There is **information aggregation** along a given sequence of equilibria if $\pi_N^{State} \rightarrow^p 1_{\{S=h\}}$ as $N \rightarrow \infty$, where \rightarrow^p denotes convergence in probability.*

Our notion of information aggregation requires that, asymptotically, agents' beliefs about the aggregate state become degenerate at the truth. That our definition involves convergence in probability is standard in the literature (see e.g., Kremer (2002)). We focus on whether information aggregates based on trading in the first period (rather than the second period), because only information revealed in the first period has the potential to influence welfare. We generalize our notion of information aggregation when analyzing the infinite horizon model in Section 5.1.

3.1 A 'Fictitious' Economy

Before presenting our main results, it will be useful to consider a 'fictitious' economy in which buyers observe the true state before making second-period offers. This benchmark economy is useful because it approximates the information revealed in the true economy if there is information aggregation. We proceed by deriving a necessary and sufficient condition under which the fictitious economy supports an equilibrium with trade in the first period (Lemma 1). We then show that the same condition is necessary, though not sufficient, for information aggregation (Theorem 1). Intuitively, information aggregation requires trade. But if the fictitious economy does not support an equilibrium with trade, then (by continuity) there cannot exist a sequence of equilibria along which information aggregates.

First, note that Properties 1, 2, and 3 trivially extend to the fictitious economy. Second, observe that conditional on knowing the true state, when forming beliefs about seller i the information revealed by sellers $j \neq i$ is irrelevant. That is, buyers' posterior belief about seller i following a rejection in the first period and observing the true state is s is given by

$$\pi_i^{fict}(s) = \frac{\pi_i^{Int} \cdot \mathbb{P}(S = s | \theta_i = H)}{\pi_i^{Int} \cdot \mathbb{P}(S = s | \theta_i = H) + (1 - \pi_i^{Int}) \cdot \mathbb{P}(S = s | \theta_i = L)}, \quad (9)$$

where π_i^{Int} is the interim belief given in (2). This implies that a seller's continuation value in the fictitious economy is independent of the trading strategies of the other sellers. Since there are no complementarities between sellers' trading strategies, the fictitious economy has a unique equilibrium, which must be symmetric (we will again drop the i subscripts whenever possible). Analogous to (7), the continuation value is given by

$$Q_\theta^{fict}(\sigma, \phi) = (1 - \delta)c_\theta + \delta \sum_s \mathbb{P}(S = s | \theta) F_\theta(\pi_i^{fict}(s), \phi). \quad (10)$$

As in Daley and Green (2012), due to the exogenous arrival of information, it is possible that the equilibrium of the fictitious economy involves zero probability of trade in the first period.

Lemma 1 *The unique equilibrium of the fictitious economy involves zero probability of trade in the first period (i.e., $\sigma^{fict} = 0$) if and only if*

$$Q_L^{fict}(0, 0) \geq v_L. \quad (\star)$$

Furthermore, (\star) holds if and only if the parameters satisfy the following:

$$\lambda \geq \bar{\lambda} \equiv 1 - \frac{\pi_0(1 - \bar{\pi})}{1 - \pi_0}$$

and

$$\delta \geq \bar{\delta} \equiv \frac{v_L - c_L}{v_L - c_L + (1 - \lambda) \cdot \left(1 - \frac{(1 - \lambda)(1 - \pi_0)}{\pi_0}\right) \cdot (v_H - v_L)}.$$

This result is intuitive. The equilibrium of the fictitious economy features no trade whenever the low type's option value from delaying trade to the second period is high. The first condition (i.e., $\lambda \geq \bar{\lambda}$) guarantees that, whenever $\sigma = 0$, then the buyers' posteriors satisfy $\pi_i^{fict}(h) > \bar{\pi}$, which implies that the prospect of the state being revealed increases the expected prices (see Figure 1 and recall that $\pi_0 < \bar{\pi}$). The second condition (i.e., $\delta \geq \bar{\delta}$)

ensures that the cost of delay does not overwhelm the prospect of a higher price in the next period. Observe that $\bar{\delta}$ is increasing in λ with $\lim_{\lambda \rightarrow 1} \bar{\delta} = 1$.

3.2 When Does Information Aggregate?

We now establish our first main result, which shows that (\star) is also the crucial determinant of the information aggregation properties of equilibria.

Theorem 1 (Aggregation Properties)

- (i) *If (\star) holds with strict inequality, then information aggregation fails along any sequence of equilibria.*
- (ii) *If (\star) does not hold, then there exists a sequence of equilibria along which information aggregates.*

The proof of the first statement uses the observation that if information were to aggregate, then for N large enough the continuation payoffs of the sellers are close to the continuation payoffs in the fictitious economy. Thus, when (\star) holds strictly, delay is also uniquely optimal when there are a large but finite number of assets. But this contradicts Property 4, which states that $\sigma_N = 0$ cannot be part of an equilibrium for any finite N . In fact, when (\star) holds strictly, σ_N must go to zero at a rate proportional to $1/N$ (see Proposition 3), which is fast enough to prevent information from aggregating, but also slow enough to ensure that the transaction data does not become completely uninformative in the limit. If it did, the bid for any asset in the second period would be v_L with probability arbitrarily close to one; hence low types would strictly prefer to trade in the first period (implying $\sigma_N = 1$), which would also contradict Property 4.

On the other hand, when the fictitious economy has an equilibrium with positive trade in the first period (i.e., if (\star) does not hold), we can explicitly construct a sequence of equilibria in which the trading probability σ_N is bounded away from zero. Clearly, information is aggregated along such a sequence. Nevertheless, even when aggregating equilibria exist, it is not the case that information will necessarily aggregate along *every* sequence of equilibria.

Theorem 2 (Coexistence) *There exists a $\underline{\delta} < 1$ such that whenever $\delta \in (\underline{\delta}, \bar{\delta})$ and λ is sufficiently large, there coexists a sequence of equilibria along which information aggregates with a sequence of equilibria along which aggregation fails. If either $\lambda < \bar{\lambda}$ or δ is sufficiently small, then information aggregates along any sequence of equilibria.*

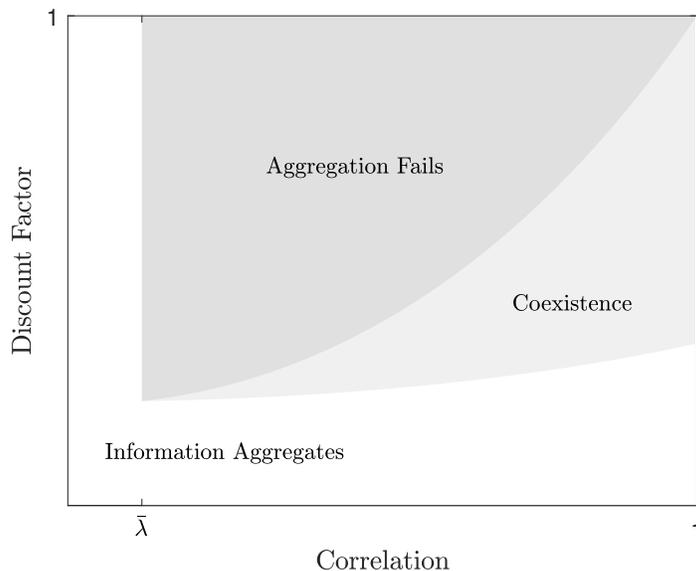


Figure 2: *When does Information Aggregate?* This figure illustrates the regions of the parameter space in which aggregating equilibria exist, fail to exist, or coexists with non-aggregating equilibria. In the top (darkly shaded) region, (\star) holds and hence there do not exist sequences of equilibria that aggregate information. Otherwise, aggregating equilibria exist (Theorem 1). In the bottom-left (unshaded) region, all sequences of equilibria aggregate information and in the middle-right (lightly shaded) region, sequences in which information aggregates coexist with sequences in which information aggregation fails (Theorem 2).

To prove the first statement, we first note that for a given $\delta < 1$, if λ is sufficiently large, then we must have $\delta < \bar{\delta}$ (since $\lim_{\lambda \rightarrow 1} \bar{\delta} = 1$) and thus by Theorem 1 aggregating equilibria must exist. We then show that if we fix δ above a certain threshold, then for a sufficiently large λ , non-aggregating equilibria also exist. In particular, we explicitly construct a sequence of equilibria in which the second-period bid is v_L for all histories except the one in which no seller has traded in the first period. In these equilibria, the probability that no seller trades in the first period remains bounded away from zero, in *both* states of nature. Thus, even as $N \rightarrow \infty$, uncertainty about the state of nature does not vanish.

The second part of Theorem 2 provides sufficient conditions under which information necessarily aggregates. It is instructive to observe that the possibility of aggregation failure requires the two key ingredients of the model: (1) sufficient correlation across assets (i.e., $\lambda > \bar{\lambda}$), and (2) strategic delay is relevant (i.e., δ is large enough). As we discussed in the fictitious economy setting of Section 3.1, (1) ensures that information about the state is relevant – thus there is an option value component; and (2) ensures that players are sufficiently patient for this option value to affect trading incentives. Figure 2 illustrates and summarizes the findings in Theorems 1 and 2.

3.3 Mechanism for Aggregation Failure and Coexistence

In order for information to aggregate, agents with different private signals must take different actions. Aggregation failures emerge when there is sufficient pooling on the same action, thereby making it impossible to identify the underlying distribution of types. In our model, pooling emerges in equilibrium due to adverse selection: low types pool with high types by delaying trade because there is an option value of getting higher price in the future.

Adverse selection is not the only setting in which an option value could lead to pooling and thus an aggregation failure. Nevertheless, there are some unique properties of our mechanism that lead to our co-existence results, which do not naturally obtain in other settings.

To illustrate these points, it is useful to discuss an alternative setting without adverse selection. Consider a model in which agents must decide whether to invest early (i.e., in the first period) or wait (and potentially invest in the second period). There is an aggregate state of nature that determines the profitability of the investment $S \in \{\ell, h\}$. Each agent privately observes a binary signal $\theta_i \in \{L, H\}$ about the profitability of investing, which is correlated with the aggregate state: $P(s = h | \theta_i = H) = \lambda > P(\theta_i = H) = \pi_0$. Therefore, a higher fraction of agents receives high signals in the good state. The expected payoff from investing in the high and low states are $v_h > 0 > v_\ell$, and payoffs are discounted by a factor δ . The payoff to not investing in either period is normalized to zero.

In this model, H types are more optimistic about the investment opportunity and therefore will be more likely to invest early. Analogous to our model, suppose that $E\{v_S | \theta_i = L\} < 0 < E\{v_S | \theta_i = H\}$, which implies that L types will not invest early but H types may. The decision to invest early is observable and thereby a positive signal about the state.

The condition analogous to (\star) is

$$\delta(\lambda v_h + (1 - \lambda)0) \geq \lambda v_h + (1 - \lambda)v_\ell, \quad (\star')$$

which says that if the state will be revealed in the second period then an H type prefers to wait.

Within this setting it is straightforward to prove the analog of Theorem 1.

- If (\star') holds with strict inequality, then information aggregation fails along any sequence of equilibria.
- If (\star') does not hold, then there exist a sequence of equilibria along which information aggregates.

However, the analog of the coexistence result in Theorem 2 does not hold in this setting. Instead, there is a unique symmetric equilibrium. The intuition for uniqueness is that invest-

ing early is purely a strategic substitute: the higher the probability that an H -type player j invests early, the higher is the payoff to player $i \neq j$ from waiting. Therefore, it is not possible to have co-existence of aggregating and non-aggregating equilibria. Whereas, in our model, more aggressive trading by other sellers may increase or decrease the option value from delay. Thus, a feature of a model with adverse selection that does not naturally arise in other settings is that “more” trading can be either a strategic substitute or a strategic complement, depending on the region of the parameter space and on the trading intensity.

3.4 Trading Behavior and Welfare

We now consider the implications of information aggregation for prices, trade volume, and welfare. Recall that in equilibrium the seller weakly prefers to delay trade until the second period. Hence, a type- θ seller’s equilibrium payoff is equal to $Q_{\theta,N}$, as given by (7). As a result, the ex-ante expected surplus of a seller can be expressed as:

$$W_N = (1 - \pi_0)(Q_{L,N} - c_L) + \pi_0(Q_{H,N} - c_H).$$

Because buyers are competitive and thus break even, W_N is also the per trader surplus in our economy. As a benchmark for comparison, the per trader surplus in the unique equilibrium of the fictitious economy is:

$$W^{fict} = (1 - \pi_0)(Q_L^{fict} - c_H) + \pi_0(Q_H^{fict} - c_H),$$

where Q_θ^{fict} is given by (10). The following proposition shows that aggregating equilibria behave very much like the fictitious economy.

Proposition 2 (Aggregating Equilibria) *Consider a sequence of equilibria along which information aggregates. Then, along this sequence:*

- (i) $\lim_{N \rightarrow \infty} W_N = W^{fict}$,
- (ii) $\lim_{N \rightarrow \infty} \sigma_N = \sigma^{fict}$,
- (iii) *Conditional on the true state, the aggregate volatility of prices and volatility of trading volume goes to zero.*

Figure 3 illustrates this result graphically by plotting the equilibrium trading surplus W_N and the trading probability σ_N against the market size, $N + 1$. For small N , multiple equilibria exist due to strategic complementarities among different sellers, and W_N and σ_N can

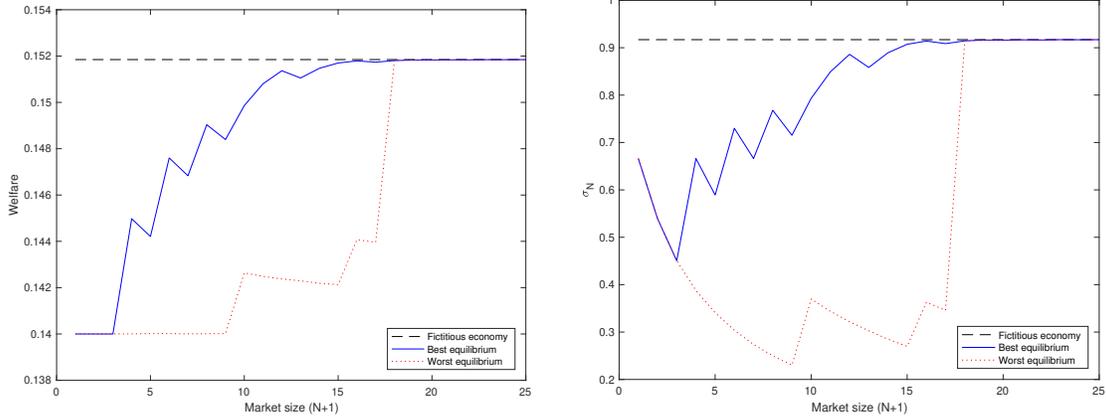


Figure 3: The left panel illustrates how the welfare per trader depends on the number of traders. The right panel shows the corresponding strategy of a low-type seller in the first period. The parameters are such that only aggregating equilibria exist. The jagged shape in both panels is due to the discreteness of the variable on the horizontal axis.

either increase or decrease with N due to the two opposing effects discussed after Proposition 1. As N grows large, however, the aggregate state gets learned, the complementarities vanish, and both welfare and trading behavior converge to those of the fictitious economy. The implication is that, in this economy, conditional on the aggregate state, the volatility in asset prices and trading volume (in both periods) goes to zero. As we show next, however, the findings change dramatically when information fails to aggregate.

Proposition 3 (Non-Aggregating Equilibria) *Consider a sequence of equilibria such that information aggregation fails along any of its subsequences. Then, along this sequence:*

- (i) $\limsup_{N \rightarrow \infty} W_N < W^{fict}$,
- (ii) $N\sigma_N \in (\underline{\kappa}, \bar{\kappa})$ for some constants $\underline{\kappa}, \bar{\kappa} > 0$,
- (iii) *Conditional on the true state, the aggregate volatility of prices and of trading volume remains strictly positive.*

In non-aggregating equilibria, strategic considerations do not vanish as the market grows large, which leads to (excess) volatility in prices conditional on the state and welfare that is below the fictitious benchmark. Figure 4 illustrates this result graphically.

The contrast between Propositions 2 and 3 demonstrates that aggregating equilibria have several nice properties that are not shared by their non-aggregating counterparts. Two immediate implications follow. First, from a social welfare perspective, aggregating equilibria

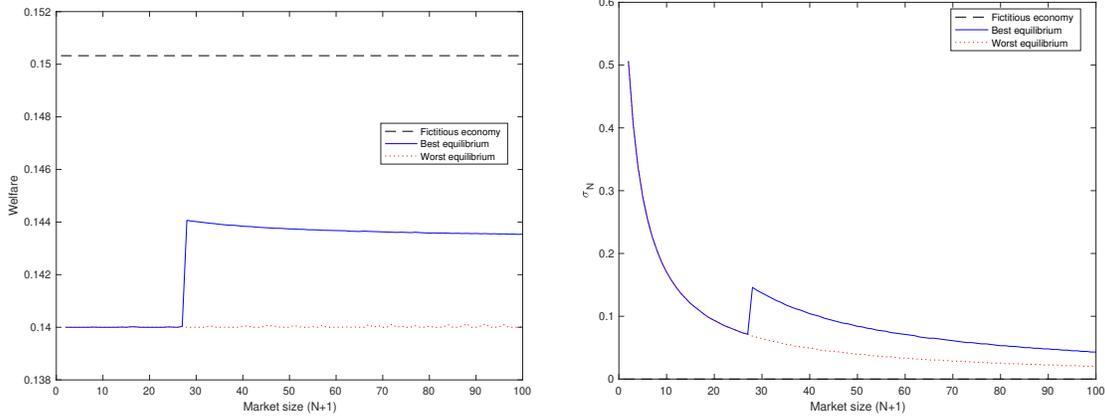


Figure 4: The left panel illustrates how the welfare per trader depends on the number of traders. The right panel shows the corresponding strategy of the seller in the first period. The parameters are such that (\star) holds and, hence, aggregating equilibria do not exist.

are always preferable to non-aggregating equilibria when they co-exist. Thus, among laissez-faire outcomes, aggregation is optimal. Second, if the only laissez-faire outcomes are non-aggregating and a social planner could manage to learn the true state in the first period, then she could improve welfare by revealing her information to market participants.

Of course, it is not obvious how a planner would be able to acquire such information. It is more natural to think that the planner is ex-ante uninformed, but can learn about the true state by observing the trading behavior of market participants. The problem facing the planner is then how best to reveal this information to other agents in the economy. In the next section, we tackle precisely this problem.⁸

4 Optimal Information Policy

In recent years, there has been a dramatic increase in the degree of post-trade transparency in a number of asset markets (e.g., real estate, corporate bonds). Some of these changes were market driven due to new platforms such as Zillow and RedFin, while others were driven by regulation. Most notably, the Securities and Exchange Commission (SEC) introduced TRACE in 2001, which requires dealers to report over-the-counter transactions on fixed-

⁸An alternative way to facilitate aggregation is through speculation; by introducing an Arrow Debreu security that is traded on a centralized market at $t = 1$, whose payoff depends on the true state, which (by assumption) is publicly realized at $t = 2$. As $N \rightarrow \infty$, the market clearing price for the security will reveal the true state and welfare will converge to that in the fictitious economy. Speculation weakly improves welfare compared to the laissez-faire outcome. However, as we will see in Section 4, introducing speculation does not necessarily lead to the constrained efficient outcome.

income securities. Transaction-level data is then publicly disclosed to market participants.⁹ In an effort to increase transparency, these reporting requirements were extended to include various derivatives in the aftermath of the 2008-09 financial crisis (e.g., Title VII of Dodd-Frank). Our framework allows us to analyze the effects of such changes. In particular, we can use it to answer the normative question: how should an (uninformed) planner disclose trading behavior to maximize social welfare?

Before analyzing the planner’s problem, it is useful to compare the problem we consider to the literature on Bayesian persuasion (Kamenica and Gentzkow, 2011; Rayo and Segal, 2010) and “information design” problems more generally (Bergemann and Morris, 2013, 2016).¹⁰ On one hand, the problems are quite similar. Both involve designing an information revelation policy to induce other players to take certain actions. On the other hand, the planner’s problem in our setting must take into account a novel feedback effect. Namely, the planner’s policy influences the information content of trading behavior, and therefore the information content of whatever is revealed. In short, the statistical properties of the information the planner can reveal, which is typically exogenous in a Bayesian persuasion setting, depend on the policy itself.

Our solution method for answering this question will proceed in two steps. First, we consider the information design problem of a planner who (exogenously) learns the aggregate state in the first period. We refer to this as the *Informed Planner’s Problem*. We characterize the solution to this problem, which for δ large enough involves partially concealing the aggregate state in order to increase the trading surplus compared to the laissez-faire outcome. We then return to the problem of interest and provide necessary and sufficient conditions under which the (uninformed) planner can achieve the same welfare as the informed planner. When these conditions do not hold, welfare is strictly lower than when the planner is informed. We relate our normative findings to the information aggregation properties of equilibria. Finally, we discuss the relation between information policy and fiscal policy.

4.1 Informed Planner’s Problem

In this section, we set up the informed planner’s problem and characterize its solution. In doing so, we will assume that the planner (exogenously) learns the aggregate state at

⁹See Gordon (2012) and Bessembinder and Maxwell (2008), respectively, for an overview of these developments in real-estate and corporate bond markets.

¹⁰Bergemann and Morris (2017) provide a more general treatment of information design problems drawing a distinction between whether the designer has an informational advantage (as in Bayesian persuasion) or not (as in communication games). In our model, the planner has no informational advantage ex-ante but has a technology for acquiring one in the interim. Another important distinction of our setting is that the planner has only limited means by which she can elicit information.

$t = 1$ and can design and commit to an information policy ex-ante, the results of which are publicly revealed after trading at $t = 1$. Therefore, buyers of asset i at date $t = 2$ can observe (i) whether asset i traded in the first period, and (ii) any additional information revealed by the planner. Although this problem is not of independent interest, its solution (under an additional constraint) will be useful for deriving the optimal policy for the uninformed planner's problem (Section 4.2).

The planner's objective is to maximize the expected discounted gains from trade. Because we focus on a public information policy and all assets are ex-ante identical, it is sufficient to consider the problem of maximizing the expected discounted gains from trade for a single asset. The planner's objective can be written as

$$W = (1 - \pi_0)(\sigma + \delta(1 - \sigma))(v_L - c_L) + \pi_0(\mathbb{P}(\pi_i = \bar{\pi}|H)\phi + \mathbb{P}(\pi_i > \bar{\pi}|H))\delta(v_H - c_H), \quad (11)$$

where π_i is the random variable representing the buyers' posterior belief at $t = 2$. The first term in equation (11) captures the expected discounted gains from trade with the low-type, which in equilibrium are realized w.p. σ in the first period and w.p. $1 - \sigma$ in the second period. The second term captures the expected discounted gains from trade with the high-type, which in equilibrium are realized only in the second period and only if the buyers' posterior belief is weakly above $\bar{\pi}$. This formulation of expected trading surplus is equivalent to the formulation in Section 3.4, which utilized continuation values, but more convenient for the purposes of deriving the optimal information policy.

From Kamenica and Gentzkow (2011), the problem of choosing state-dependent distributions over signals is equivalent to choosing a distribution of posteriors about the state that is Bayes plausible. Let \tilde{p} denote the random variable representing the buyers' posterior about the state conditional on observing the information revealed by the planner and let G denote the cumulative distribution of \tilde{p} . Bayes plausibility requires that the expected posterior be equal to the prior

$$E_G\{\tilde{p}\} = \pi_0. \quad (12)$$

Of course, the planner's choice of G will influence both the behavior of the seller and the buyers as captured by (σ, ϕ) . We refer to G as the *information policy* of the informed planner.

Definition 2 (Informed Planner's Problem) *The informed planner's problem is to choose a triple (G, σ, ϕ) to maximize (11) subject to two constraints:*

- (1) *Bayes Plausibility (i.e., (12)), and*
- (2) *Given G , (σ, ϕ) must be an equilibrium of the game.*

We say that (G, σ, ϕ) is *feasible* if it satisfies (1) and (2).¹¹ We let $Q_\theta^G(\sigma, \phi)$ denote the continuation value to a type- θ seller.¹² For any information policy, Properties 1-3 must hold in any equilibrium. Moreover, as in Section 2.5, if $\sigma \in (0, 1)$ then constraint (2) requires that the low type's continuation value must equal v_L . However, with an informed planner, it is no longer true that low type sellers must trade with strictly positive probability in the first period (as in Property 4). More specifically, it is possible to design G such that there exist equilibria in which $\sigma = 0$ and $Q_L^G \geq v_L$. Instead of (8), equilibria are characterized by (G, σ, ϕ) such that

$$Q_L^G(\sigma, \phi) \geq v_L \tag{13}$$

$$\sigma (Q_L^G(\sigma, \phi) - v_L) = 0 \tag{14}$$

To characterize the solution to the informed planner's problem, two simple lemmas are useful (formally stated in the Appendix). First, any feasible σ is bounded above by $\bar{\sigma}$, which is defined implicitly by $\pi_i(0; \bar{\sigma}) = \bar{\pi}$ (Lemma C.1), where $\pi_i(\tilde{p}; \sigma)$ denotes the buyers' posterior following a rejection in the first period, given the trading probability σ and conditional on the buyers' posterior about the state being \tilde{p} . Second, for any feasible σ , the solution can be achieved with an information policy that has at most three points in the support, $\{0, \bar{p}(\sigma), 1\}$, where $\bar{p}(\sigma)$ is buyer's belief about the state such that the posterior about the seller following rejection is equal to $\bar{\pi}$ (Lemma C.2).

Thus, for any feasible σ , the information policy (i.e., G) of the informed planner is reduced to choosing a pair $(\mu_0, \mu_1) \in [0, 1]^2$, where $\mu_k = \mathbb{P}^G(\tilde{p} = k)$, and the Bayes plausibility constraint reduces to

$$\mu_1 + (1 - \mu_0 - \mu_1)\bar{p}(\sigma) = \pi_0.$$

Definition 3 *We say that the planner's policy is **fully revealing** if $\mu_0 = 1 - \pi_0$ and $\mu_1 = \pi_0$.*

If the policy attaches a strictly positive weight to $\bar{p}(\sigma)$ (i.e., if $\mu_0 + \mu_1 < 1$) then some information is concealed. To further characterize the solution, it is useful to first consider a modified version of the problem, in which (13) is required to hold with equality.

Lemma 2 *Consider a variant of the informed planner's problem in which (13) is required to hold with equality. Recalling that $Q_L^{fict}(\sigma, \phi)$ is defined in (10), the solution to this modified problem is as follows:*

¹¹Our formulation of the problem implicitly assumes that agents will play the planner's preferred equilibrium if multiple equilibria exist.

¹²See Appendix C for an explicit construction of the continuation values.

- (i) If $Q_L^{fict}(\bar{\sigma}, 0) \leq v_L$, then the optimal information policy is fully revealing, $\sigma^* = \bar{\sigma}$, and ϕ^* is such that $Q_L^{fict}(\bar{\sigma}, \phi^*) = v_L$.
- (ii) If $Q_L^{fict}(\bar{\sigma}, 0) > v_L$, then the optimal information policy conceals some information: $\mu_0^* = 0, \mu_1^* = \frac{\pi_0 - \bar{p}(\sigma^*)}{1 - \bar{p}(\sigma^*)}$, and σ^* is such that $Q_L^{G^*}(\sigma^*, 0) = v_L$.

Intuitively, the reason why the planner conceals information is closely related to the option value effect of information that we identified in the fictitious economy of Section 3.1: the prospect of the high state being revealed more frequently can generate an increase in the low type's expected future prices and thus reduce her incentive to trade in the first period.

Lemma 2 will be useful in the study of the uninformed planner's problem. Before moving to that problem, however, we complete the characterization of the informed planner's true problem.

Proposition 4 *The solution to the informed planner's problem is as follows.*

- (i) *The constraint (13) is slack and the optimal policy involves $\sigma^* = \mu_1^* = 0$ if and only if δ is sufficiently close to 1.*
- (ii) *Otherwise, the constraint (13) holds with equality and the optimal policy is characterized by Lemma 2.*

If (13) is slack then it must be that the low-type seller trades with probability zero in the first period (see (14)). In this case, the planner's problem reduces to maximizing the probability of trade with the high type in the second period. Proposition 4 says that this is accomplished by never fully revealing the high state. However, as we will see in the next section, such a policy is not feasible for the uninformed planner.

4.2 Uninformed Planner's Problem

We are now ready to tackle our problem of interest where, rather than being endowed exogenously with knowledge of the state, the planner must learn it from transaction data. As a result, we will need to keep track of the market size, $N+1$, since it will affect the information the planner observes. That the planner's information is endogenously determined also makes it cumbersome to employ the typical Bayesian persuasion approach (i.e., choose a distribution over posterior beliefs subject to Bayes plausibility) that has become standard and which we adopted with the informed planner.¹³ Instead, we will work directly with the planner's

¹³To employ the Bayesian persuasion approach for the uninformed planner's problem, Bayesian plausibility would need to be supplemented with an additional constraint on the support of the distribution (i.e., the planner cannot reveal information that she does not have), where the additional constraint depends on both N and σ_N .

reporting policy, which is defined as a mapping \mathcal{M}_N from the trading histories she observes, which are elements of $\{0, 1\}^{N+1}$, to distributions over signals that are publicly observed by agents in the economy.

As before, the planner's objective is to maximize the expected discounted gains from trade,

$$W^{\mathcal{M}_N} = (1-\pi_0)(\sigma_N + \delta(1-\sigma_N))(v_L - c_L) + \pi_0 \left(\mathbb{P}^{\mathcal{M}_N}(\pi_i = \bar{\pi} | H) \phi_N + \mathbb{P}^{\mathcal{M}_N}(\pi_i > \bar{\pi} | H) \right) \delta(v_H - c_H), \quad (15)$$

where $\mathbb{P}^{\mathcal{M}_N}(\cdot | \theta)$ denotes the conditional probability distribution over the buyers' posteriors induced by \mathcal{M}_N . We let $Q_\theta^{\mathcal{M}_N}(\sigma_N, \phi_N)$ denote the continuation value to a type- θ seller.

Definition 4 (Uninformed Planner's Problem) *The uninformed planner's problem is to choose a triple $(\mathcal{M}_N, \sigma_N, \phi_N)$ to maximize (15) subject to (σ_N, ϕ_N) being an equilibrium of the game given the reporting policy \mathcal{M}_N .*

The difference from the informed planner's problem is that the information content of the reporting policy is endogenous to the equilibrium trading probability σ_N . This has the following important implication.

Lemma 3 *The solution to the uninformed planner's problem must involve $\sigma_N > 0$ and*

$$Q_L^{\mathcal{M}_N}(\sigma_N, \phi_N) = v_L. \quad (16)$$

Intuitively, if σ_N is equal to zero, the planner has no relevant information and any signals she reveals are completely uninformative. But, by the same argument used to establish Property 4, "no trade" cannot be part of an equilibrium if no information is revealed at $t = 1$.

Lemma 3 implies that the solution to the modified informed planner's problem characterized in Lemma 2 provides an upper bound on the level of surplus that the uninformed planner can achieve. Clearly, a policy for the uninformed planner that achieves this upper bound must also be optimal.

Proposition 5 *There exists a sequence $\{\mathcal{M}_N, \sigma_N, \phi_N\}$ such that $\lim_{N \rightarrow \infty} W^{\mathcal{M}_N} = W^*$, where W^* is the trading surplus under the information policy of the modified informed planner's problem as characterized in Lemma 2.*

To prove this result, we construct an information policy that is characterized by a threshold, τ_N , and the following reporting strategy. When trading volume is below τ_N , the planner reports the number of trades to market participants. When trading volume is above the

threshold, the planner simply reports that “volume was above τ_N .” We show that, when σ_N is close to σ^* and τ_N is chosen appropriately, the planner asymptotically learns the state and the information content of her policy converges to that of the information policy of the (constrained) informed planner as characterized in Lemma 2, which may or may not be fully revealing.¹⁴ Finally, we use continuity arguments to find a sequence of trading probabilities, σ_N , which both converges to σ^* and is consistent with equilibrium under this information policy, for all N .

The optimal information policy achieves a Pareto improvement over the laissez-faire outcomes. The reason is that the buyers break even, the low type’s payoff is v_L (see Lemma 3) and, thus, all the additional surplus generated by the planner’s policy is captured by the high types. The implication of this observation is that all agents would be happy to delegate the information dissemination about past trades to the social planner.

Finally, we observe that by combining Proposition 4 and Lemma 3, it is apparent that for δ large enough, the uninformed planner cannot achieve the same level of surplus as the informed planner, even as $N \rightarrow \infty$.

4.3 Is Information Aggregation Constrained Efficient?

Having characterized the solution to the uninformed planner’s problem, we now relate our findings on optimal policy to the results on Information Aggregation in Section 3. In order to do so, we will refer to any equilibrium that coincides with the solution to the uninformed planner’s problem as *constrained efficient*, and we will refer to any equilibrium in which welfare is strictly below the solution to the uninformed planner’s problem as *constrained inefficient*.

Corollary 1 *Non-aggregating equilibria are constrained inefficient.*

Not surprisingly, there is always scope for intervention if the laissez-faire outcome does not aggregate information. The reason for this is that, since the equilibrium is non-aggregating, there is a vanishing amount of trade in the first period, which is inefficient. Moreover, because the equilibrium is non-aggregating, there is little information revealed in the second period, and thus the adverse selection problem remains severe. However, even when an aggregating equilibrium exists and agents coordinate on playing it, there may also be scope for intervention.

¹⁴When the constrained informed planner’s policy is not fully revealing, as in Lemma 2, part (ii), the sequence of thresholds, $\{\tau_N\}$, is chosen such that upon learning that volume is above the threshold, traders’ posterior about the state converges to $\bar{p}(\sigma^*)$, whereas when traders observe volume below the threshold, their posterior about the state converges to one.

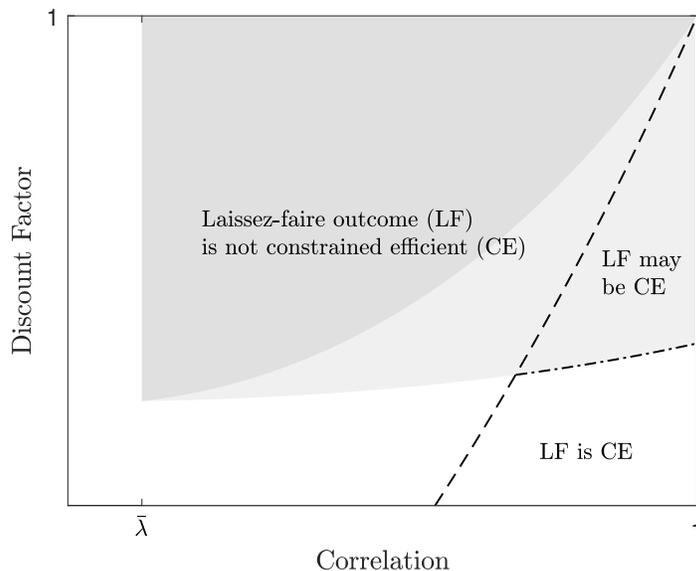


Figure 5: *When is the Laissez-Faire Outcome Constrained Efficient?* This figure relates the constrained efficiency of the laissez-faire outcome to whether information aggregates. The shaded regions are analogous to those in Figure 2 (i.e., aggregation fails in the darkest shaded region, there is coexistence in the lighter shaded region, and information necessarily aggregates in the unshaded region). The black dashed line is $\hat{\delta}$. By Corollary 2, aggregating equilibria are constrained efficient only to the right of this line.

Corollary 2 *Aggregating equilibria are constrained inefficient if and only if*

$$Q_L^{fict}(\bar{\sigma}, 0) \geq v_L, \quad (17)$$

which holds if and only if the parameters satisfy the following:

$$\delta \geq \hat{\delta} \equiv \frac{v_L - c_L}{v_L - c_L + (1 - \lambda)\pi_i(1; \bar{\sigma})(v_H - v_L)}. \quad (18)$$

It is worth noting that $\hat{\delta}$ lies strictly below $\bar{\delta}$ defined in Lemma 1, since $Q_L^{fict}(\bar{\sigma}, 0) > Q_L^{fict}(0, 0)$. Furthermore, $\hat{\delta}$ is increasing in λ with $\lim_{\lambda \rightarrow 1} \hat{\delta} = 1$. Therefore, the region where aggregating and non-aggregating equilibria coexist is bisected by $\hat{\delta}$ as shown in Figure 5.

4.4 Information vs Fiscal Policy

Fuchs and Skrzypacz (2015) have shown that fiscal policy (in the form of taxes) can be used to increase efficiency by reducing the incentive to delay trade, thereby inducing more trade to occur in the first period. We have shown that information policy can be used to achieve a similar goal. A natural question is to what extent are these policy instruments substitutes. More specifically, when taxes are at the policymaker's disposal, is there still a

role for information policy?

The answer is yes. Fiscal and information policy are not perfect substitutes. In fact, given our assumptions on primitives (particularly Assumption 2), there is no scope for fiscal policy to improve the outcome when there is a single seller. If, following Fuchs and Skrzypacz (2015), we used taxes to close the market in the second period, then only low types would trade and their payoff would equal v_L , while that of the high types would be c_H . Instead, under the optimal information policy, low types continue to get a payoff of v_L , we can increase the payoff of the high types strictly above c_H . The important difference between the two instruments is that taxes have a uniform effect on continuation values, whereas information policy can have a differential effect because high and low types have different expectations about the information that will be revealed.

5 Robustness

In this section, we extend our analysis to an infinite horizon model and show that the aggregation properties of equilibria are virtually identical; we also argue that the main results are robust to a setting with a richer type space.

5.1 Infinite horizon

We now extend our main results to a setting with an infinite number of trading opportunities $t \in \{1, 2, \dots\}$. Intuitively, one might expect that with more trading periods there are more opportunities to learn from trade and hence more information will be revealed. However, there is a countervailing force; there are more opportunities for (strategic) sellers to signal through delay. It turns out that two factors essentially cancel each other out. It is precisely the same condition as in the two-period model (i.e., (\star)) that ensures aggregation failure.

Besides allowing for an infinite number of trading opportunities, the model and the information structure is identical to the one presented in Section 2. The only additional notation we will require is the public history at (the end of) date t , which we denote by $\mathbf{z}^t = \{\mathbf{z}_1, \dots, \mathbf{z}_t\}$, consisting of the history of all the trades that have taken place at dates prior to and including t . Note that \mathbf{z}^t also corresponds to buyers' information set prior to making offers at $t + 1$.

Characterizing the set of all possible equilibria in the infinite horizon model is more difficult because the space of relevant histories is a complex object. In principle, the path of play can depend on sellers' beliefs about the quality of other sellers' assets, the distribution of buyers' beliefs about the quality of each seller's asset, the buyers' and the sellers' beliefs about the

aggregate state, as well as the number of assets remaining on the market. Nevertheless, we are able to obtain sharp predictions regarding the information aggregation properties of the set of equilibria.

In order to illustrate these findings, we must generalize our notion of information aggregation. Let $\pi_{t,N}^{State}$ denote the buyers' posterior belief that the state is high, conditional on having observed the trading history, \mathbf{z}^t , in an economy with $N + 1$ sellers.

Definition 5 *There is **information aggregation at date t** along a given sequence of equilibria if $\pi_{t,N}^{State} \xrightarrow{p} 1_{\{S=h\}}$ as $N \rightarrow \infty$.*

We say that *information aggregates* along a given sequence if there exists a $t < \infty$ such that information aggregates at date t . Otherwise, we say that *information aggregation fails*.

The following theorem shows that, with an infinite trading horizon, (\star) is indeed necessary and sufficient to rule out aggregating equilibria.

Theorem 3 *Consider the infinite horizon model.*

- (i) *If (\star) holds with strict inequality, then information aggregation fails along any sequence of equilibria.*
- (ii) *If (\star) does not hold, then there exists a sequence of equilibria along which information aggregates.*
- (iii) *There exists a $\widehat{\delta} < 1$ such that whenever $\delta \in (\widehat{\delta}, \bar{\delta}_\lambda)$ and λ is sufficiently large, there is coexistence of sequences of equilibria along which information aggregates with sequences of equilibria along which aggregation fails.*

The proof hinges on arguments similar to those used in the two-period economy. For (i), we show that the earliest date in which information about the state is supposed to aggregate is similar to the first period in a two-period economy. That is, suppose that information aggregates at some date τ but not before. Because (\star) holds, the option value of waiting for the state to be revealed is sufficiently high to make sellers strictly prefer to delay trade at date τ . But if sellers do not trade in date τ , then no information is revealed, which means that τ cannot possibly be the earliest date of aggregation.

To establish (ii) and (iii), we construct a class of equilibria that essentially share the information aggregation properties of the two-period economy. A feature of this class is that once the belief about the seller weakly exceeds $\bar{\pi}$, all future bids are pooling. When (\star) does not hold, we show that such equilibria exist and that there is an equilibrium sequence within this class along which information aggregates. Then, following arguments similar to those for the proof of Theorem 2, we show that under the conditions stated in (iii), there also exists another sequence of equilibria (still within the class) in which aggregation fails.

5.2 Continuous types

Thus far, we have considered a setting with two types of sellers, which we adopted primarily for tractability as it considerably simplified the manner by which beliefs are updated from histories of trade. The discreteness of types also generated a discontinuity in the second-period price as a function of the posterior belief (see Figure 1). However, such a discontinuity is not essential for our main results. Instead, the important feature is that the low-type's payoff is locally convex in the posterior.

To illustrate this point, consider the following modification of our setting. Assume that the seller values an asset of type θ at $c_\theta + \varepsilon$, where ε is distributed uniformly, and independently among sellers, on the interval $[\underline{\varepsilon}, \bar{\varepsilon}]$ with $\underline{\varepsilon} < 0 < \bar{\varepsilon}$. As before, buyers value the asset at v_θ . Under some conditions, all symmetric equilibria of this modified model are as follows.¹⁵

In the first period, the bid is v_L , which is rejected by all high types, but it is accepted by the low type whenever $\varepsilon < \varepsilon_L^*$. In the second period, the bid is v_L if the posterior π_i is below a threshold, which is accepted by the L -type but rejected by the H -type with probability one. Otherwise, the bid is $V(\hat{\pi}_i)$, which is accepted by all remaining low types but only accepted by the high type if $\varepsilon \leq \varepsilon_H^*$. The belief $\hat{\pi}_i$ is weakly below π_i since not all high types trade in the second period. We note that the second-period price is continuous in π_i if $c_H + \underline{\varepsilon} = v_L$.¹⁶

Observe that in this modified model, ε_L^* plays the role of the trading probability σ and, thus, its behavior is essential for understanding the information aggregation properties of equilibrium. To this end, it is again useful to consider a fictitious economy where the state S is perfectly revealed at the beginning of the second period. We denote by $\pi_i(s; \varepsilon_L^*)$ the buyers' posterior belief, conditional on the L -type trading when $\varepsilon \leq \varepsilon_L^*$ and conditional on $S = s$. The continuation value of the (L, ε) -type is then given by:

$$Q_{L,\varepsilon}^{fict}(\varepsilon_L^*) = (1 - \delta)(c_L + \varepsilon) + \delta \sum_{s=l,h} \mathbb{P}(S = s|L) F_L(\pi_i(s; \varepsilon_L^*)), \quad (19)$$

where F_L maps the posterior π_i to the second-period price. The analog of Condition (\star) is

$$Q_{L,\underline{\varepsilon}}^{fict}(\underline{\varepsilon}) \geq v_L. \quad (\star'')$$

Condition (\star'') states that option value from receiving a pooling price in state h is suffi-

¹⁵The conditions are (i) $c_L + \bar{\varepsilon} \leq v_L$, i.e., there are gains from trade with all L -types, (ii) $c_H + \underline{\varepsilon} \geq v_L$, i.e., high types are not willing to trade at a price of v_L ; (iii) π_0 is below the threshold at which pooling of some high types is possible, which is the analog of Assumption 1.

¹⁶Otherwise, as with discrete types, buyer mixing may be required to ensure the cutoff type is indifferent in equilibrium.

ciently high all low types want to delay trade in the fictitious economy. Analogous to Lemma 1, this is equivalent to requiring that λ and δ are sufficiently close to one.

Using arguments akin to those in Theorems 1 and 2, we can establish that:

- If (\star) holds with strict inequality, then information aggregation fails along any sequence of equilibria.
- If (\star) does not hold, then there can coexist sequences of equilibria, such that information aggregates along some sequences and fails to aggregate along others.

In particular, along non-aggregating sequences of equilibria, the threshold ε_L^* declines to $\underline{\varepsilon}$ at rate N^{-1} , which is slow enough so that trade does not become uninformative but fast enough so that the aggregate state is not revealed as $N \rightarrow \infty$.

6 Concluding Remarks

We study the information aggregation properties of decentralized dynamic markets in which traders have private information about the value of their asset, which is correlated with some underlying ‘aggregate’ state of nature. We provide necessary and sufficient conditions under which information aggregation necessarily fails. Further, we show that when these conditions are violated, there can be a coexistence of non-trivial equilibria in which information about the state aggregates with equilibria in which aggregation fails. Our findings suggest there are important differences in the aggregation properties of multi-asset decentralized markets (as studied here) and single-asset centralized markets as typically explored in the literature.

We then consider the normative implications of our theory. We solve for the optimal information policy of a social planner who observes the trading behavior and chooses what information to communicate to the traders. We show that there is a relationship between information aggregation properties of equilibria and their efficiency. When information fails to aggregate, there is room for the planner to increase trading surplus by concealing information about trading volume when it is sufficiently high. By doing so, the planner reduces the sellers’ incentive to (inefficiently) delay trade. Thus, full transparency is not necessarily optimal from a social perspective.

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A Proofs for Section 2

Proof of Property 1. For (i) and (ii), see the proof of Lemma 1 in Daley and Green (2016). Conditional on reaching the second trading period and the buyers' belief, π_i , the strategic setting for trading a given asset i is identical to theirs. Moreover, by their Lemma A.3, the bid price must earn zero expected profit. To demonstrate (iii), we will show that the bid price must be either v_L or c_H when $\pi_i = \bar{\pi}$ by ruling out all other bids.

Clearly, at $t = 2$, the reservation price of the low-type seller is c_L and the reservation price of the high-type seller is c_H . Hence, if the bid is strictly above c_H , both types will accept w.p.1 and the winning buyer earns negative expected profit. Next, suppose there is positive probability that the bid is strictly less than v_L . Then, for $\epsilon > 0$ small enough, a buyer could earn strictly positive expected profit by deviating and offering $v_L - \epsilon$. Finally, if the bid is strictly between v_L and c_H , the high type will reject, the low type will accept and the winning buyer makes negative profit. Thus, we have shown that the equilibrium bid price at $t = 2$ when $\pi_i = \bar{\pi}$ must be either v_L or c_H . ■

Proof of Property 2. Since $c_H > c_L$ and $F_H \geq F_L$, the continuation value of the low type seller from rejecting the bid v_L in the first period satisfies:

$$\begin{aligned} Q_L^i &= (1 - \delta) \cdot c_L + \delta \cdot \mathbb{E}_L\{F_L(\pi_i, \phi_i)\} \\ &< (1 - \delta) \cdot c_H + \delta \cdot \mathbb{E}_L\{F_L(\pi_i, \phi_i)\} \\ &\leq (1 - \delta) \cdot c_H + \delta \cdot \mathbb{E}_L\{F_H(\pi_i, \phi_i)\}. \end{aligned}$$

Therefore, in order to prove that $Q_H^i > Q_L^i$, it is sufficient to show that $\mathbb{E}_H\{F_H(\pi_i, \phi_i)\} \geq \mathbb{E}_L\{F_H(\pi_i, \phi_i)\}$. Recall that F_H is increasing in π_i and independent of ϕ_i . Hence, the desired inequality is implied by proving that conditional on $\theta_i = H$, the random variable π_i (weakly) first-order stochastically dominates π_i conditional on $\theta_i = L$.

Note that the distribution of π_i in the second period is a function of the trading probabilities of the seller i and of the realization of news from sellers $j \neq i$, $z_i^j \in \{0, 1\}$. Fix the interim belief π_{σ_i} , and consider news \mathbf{z}'_{-i} and \mathbf{z}''_{-i} (which occur with positive probability) such that the posterior π_i satisfies $\pi_i(\mathbf{z}'_{-i}) \geq \pi_i(\mathbf{z}''_{-i})$, i.e., \mathbf{z}'_{-i} is “better news” for seller i than \mathbf{z}''_{-i} . But note that:

$$\frac{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}'_{-i})}{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}'_{-i}) + (1 - \pi_i^{Int}) \cdot \rho_L^i(\mathbf{z}'_{-i})} = \pi_i(\mathbf{z}'_{-i}) \geq \pi_i(\mathbf{z}''_{-i}) = \frac{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}''_{-i})}{\pi_i^{Int} \cdot \rho_H^i(\mathbf{z}''_{-i}) + (1 - \pi_i^{Int}) \cdot \rho_L^i(\mathbf{z}''_{-i})},$$

which implies that $\frac{\rho_H(\mathbf{z}'_{-i})}{\rho_L(\mathbf{z}'_{-i})} \geq \frac{\rho_H(\mathbf{z}''_{-i})}{\rho_L(\mathbf{z}''_{-i})}$, i.e. the ratio of distributions $\frac{\rho_H(\cdot)}{\rho_L(\cdot)}$ satisfies the monotone likelihood ratio property. This in turn implies that $\rho_H(\cdot)$ first-order stochastically dominates $\rho_L(\cdot)$, which establishes the result. ■

Proof of Property 3. We first show that the bid in the first period is v_L w.p.1. From Property 2, the strict ranking of seller continuation values implies that, in any equilibrium, if the high type is willing to accept an offer with positive probability then the low type must accept w.p.1. Thus, given Assumption 1, any bid at or above c_H would lead to negative expected profit. Any bid in (v_L, c_H) also leads to losses since it is only accepted by the low type. If the bid was strictly less than v_L , a buyer can make strictly positive profits by offering

$v_L - \epsilon$, for $\epsilon > 0$ small enough. Thus, any deterministic offer strictly below v_L can be ruled out. The only deterministic bid possible is v_L , at this point there is no profitable deviation for the other buyer than offering v_L as well. The same arguments rule out any mixed strategy equilibrium that has a mass point anywhere other than v_L . Finally, mixing continuously over some interval of offers cannot be an equilibrium. We show this by contradiction. If one of the buyers mixes over some interval $[\underline{b}, \bar{b}]$ with $\bar{b} = v_L$ then the other buyer must be offering v_L with probability 1 because otherwise he would never want to offer v_L , which leads to zero profits w.p.1. If instead $\bar{b} < v_L$, the other buyer's best response can never have \underline{b} (or anything below) as part of its support. This bid will lose with probability 1 and thus earn zero profits, while bidding $\frac{\bar{b} + v_L}{2}$ would lead to strictly positive profits.

Next, it is clear that the high type would reject bid v_L , since $v_L < c_H$. To see that the low type must accept with probability less than one, note that if in equilibrium the low type accepted w.p.1, then the posterior belief would assign probability 1 to the type being high in the next period. The offer in the second period (as argued in Property 1) would then be v_H but, given Assumption 2, the low type seller would then want to deviate and trade in period 2 at v_H rather than at v_L in period 1. ■

Proof of Property 4. The proof that all equilibria involve strictly positive probability of trade in the first period is in the text. We show here that all equilibria must be symmetric. In search of a contradiction, assume there exists an equilibrium in which $\sigma_A > \sigma_B \geq 0$ for some $A, B \in \{1, \dots, N\}$. We establish the result by first showing that the beliefs for seller A are more favorable than for seller B , following all news realizations; then we show that good news about seller A are more likely to arrive than good news about seller B .

Let $\pi_{\sigma_i}^{Int}$ denote the interim belief when (low type) seller i trades w.p. σ_i (by Property 3 the high type does not trade in first period). Consider the posterior belief about seller $i \in \{A, B\}$ following some news $\mathbf{z}_{-i} = (z_i^j)_{j \neq i}$:

$$\pi_i(\mathbf{z}_{-i}) = \frac{\pi_{\sigma_i}^{Int} \cdot \rho_H^i(\mathbf{z}_{-i})}{\pi_{\sigma_i}^{Int} \cdot \rho_H^i(\mathbf{z}_{-i}) + (1 - \pi_{\sigma_i}^{Int}) \cdot \rho_L^i(\mathbf{z}_{-i})}$$

where we can express $\rho_\theta^i(\mathbf{z}_{-i})$ as:

$$\rho_\theta^i(\mathbf{z}_{-i}) = \sum_{s \in \{l, h\}} \mathbb{P}(S = s | \theta_i = \theta) \cdot \mathbb{P}((z_i^j)_{j \neq i, i'} | S = s) \cdot \mathbb{P}(z_i^{i'} | S = s)$$

for $i, i' \in \{A, B\}$ and $i' \neq i$. Note that $\rho_\theta^i(\mathbf{z}_{-i})$ depends on $\sigma_{i'}$ only through the term $\mathbb{P}(z_i^{i'} | S)$. We now show that $\sigma_A > \sigma_B$ implies that:

$$\frac{1 - \pi_{\sigma_A}^{Int}}{\pi_{\sigma_A}^{Int}} \cdot \frac{\rho_L^A(\mathbf{z}_{-i})}{\rho_H^A(\mathbf{z}_{-i})} < \frac{1 - \pi_{\sigma_B}^{Int}}{\pi_{\sigma_B}^{Int}} \cdot \frac{\rho_L^B(\mathbf{z}_{-i})}{\rho_H^B(\mathbf{z}_{-i})}, \quad (20)$$

which will establish that $\pi_A(\mathbf{z}_{-i}) > \pi_B(\mathbf{z}_{-i})$ for all news \mathbf{z}_{-i} . There are two cases to consider, depending on whether $z_i^{i'} = 0$ or $z_i^{i'} = 1$.

If $z_i^{i'} = 1$, then $\mathbb{P}(z_i^{i'} = 1 | S = s) = \sigma_{i'} \cdot \mathbb{P}(\theta_{i'} = L | S = s)$ and the likelihood ratio $\frac{1 - \pi_{\sigma_i}^{Int}}{\pi_{\sigma_i}^{Int}}$.

$\frac{\rho_L^i(\mathbf{z}_{-i})}{\rho_H^i(\mathbf{z}_{-i})}$ decreases in σ_i but is independent of $\sigma_{i'}$. Intuitively, if seller i' traded, her type is revealed to be low, and the intensity with which she trades is irrelevant for updating. But then inequality (20) follows because $\pi_{\sigma_i}^{Int}$ is increasing in σ_i .

If $z_i^{i'} = 0$, then $\mathbb{P}(z_i^{i'} = 0 | S = s) = 1 - \sigma_{i'} \cdot \mathbb{P}(\theta_{i'} = L | S = s)$, and now the likelihood ratio $\frac{1 - \pi_{\sigma_i}^{Int}}{\pi_{\sigma_i}^{Int}} \cdot \frac{\rho_L^i(\mathbf{z}_{-i})}{\rho_H^i(\mathbf{z}_{-i})}$ decreases in both σ_i and $\sigma_{i'}$. However, given that both i and i' did not trade (both are good news for i), inequality (20) follows because the assets i and i' are imperfectly correlated and $\frac{1 - \pi_{\sigma_i}^{Int}}{\pi_{\sigma_i}^{Int}} \cdot \frac{\rho_L^i(\mathbf{z}_{-i})}{\rho_H^i(\mathbf{z}_{-i})}$ is more sensitive to trading probability σ_i than to $\sigma_{i'}$.

Now, note that $\sigma_A > \sigma_B$ also implies that the probability that seller B trades and releases bad news about seller A is lower than the probability that seller A trades and releases bad news about seller B . Since the posteriors following good news are higher than following bad news, this establishes the result.

Finally, the symmetry in ϕ_i follows from monotonicity of Q_L^i in ϕ_i whenever buyer mixing is part of an equilibrium. ■

Proof of Proposition 1. To prove existence of an equilibrium, it suffices to show there exists a $(\sigma, \phi) \in [0, 1]^2$ such that equation (8) holds, i.e., $Q_L(\sigma, \phi) = v_L$ where the second argument states that all other sellers also trade with intensity σ . Note that by varying σ from 0 to 1, Q_L ranges from $[(1 - \delta)c_L + \delta v_L, (1 - \delta)c_L + \delta v_H]$. By continuity of Q_L and Assumption 2, the intermediate value theorem gives the result.

Let us denote by $\pi_i(\mathbf{z}_{-i}; \sigma)$ the posterior belief following news realization \mathbf{z}_{-i} in an equilibrium with trading probability σ . Consider the following two candidate equilibria. The first candidate is an equilibrium in which the posterior belief about the seller satisfies $\pi_i(\mathbf{z}(0); \sigma) = \bar{\pi}$, and the second candidate equilibrium is when the posterior belief about the seller satisfies $\pi_i(\mathbf{z}(N); \sigma) = \bar{\pi}$. Although there can be other equilibria as well, we do not focus on them. We will now show that these two equilibria coexist when λ and δ are large enough.

1. $\pi_i(\mathbf{z}(0); \sigma) = \bar{\pi}$. Note that there is at most one such equilibrium since the trading intensity σ in this category is fully pinned down by the requirement that $\pi_i(\mathbf{z}(0); \sigma) = \bar{\pi}$. Let x be the value of σ such that $\pi_i(\mathbf{z}(0); x) = \bar{\pi}$. As ϕ varies from 0 and 1, $Q_L(x, \phi)$ varies continuously from $(1 - \delta)c_L + \delta v_L$ to $(1 - \delta)c_L + \delta(\rho_L^i(\mathbf{z}(0))c_H + (1 - \rho_L^i(\mathbf{z}(0)))v_L)$ where $\rho_L^i(\mathbf{z}(0)) > 0$. Hence, there exists a $\delta_\lambda^* < 1$, such that $Q_L(x, 1) = v_L$. Clearly, this equilibrium exists if $\delta > \delta_\lambda^*$. Moreover, it is straightforward to show that $\inf_\lambda \rho_L^i(\mathbf{z}(0)) > 0$. Hence, this equilibrium exists if δ is larger than $\delta^* \equiv \sup_{\lambda \in (1 - \pi_0, 1)} \delta_\lambda^* < 1$.

2. $\pi_i(\mathbf{z}(N); \sigma) = \bar{\pi}$. Note that there is at most one such equilibrium since the trading intensity σ is fully pinned down by the requirement that $\pi_i(\mathbf{z}(N); \sigma) = \bar{\pi}$. Let y be the value of σ such that $\pi_i(\mathbf{z}(N); y) = \bar{\pi}$. As ϕ varies from 0 to 1, Q_L varies continuously from

$$(1 - \delta)c_L + \delta \left(\rho_L^i(\mathbf{z}(N))v_L + \sum_{\mathbf{z}_{-i} \neq \mathbf{z}(N)} \rho_L^i(\mathbf{z}_{-i})V(\pi_i(\mathbf{z}_{-i}; y)) \right)$$

to

$$(1 - \delta)c_L + \delta \left(\rho_L^i(\mathbf{z}(N))c_H + \sum_{\mathbf{z}_{-i} \neq \mathbf{z}(N)} \rho_L^i(\mathbf{z}_{-i})V(\pi_i(\mathbf{z}_{-i}; y)) \right).$$

Moreover, it is straightforward to show that $\lim_{\lambda \rightarrow 1} \rho_L^i(\mathbf{z}(N)) = 1$. Therefore, it follows that the range of Q_L converges to the interval $((1 - \delta)c_L + \delta v_L, (1 - \delta)c_L + \delta c_H]$ as λ goes to 1. By Assumption 2, v_L is in this interval. This establishes the existence of the threshold λ_δ^* such that this equilibrium exists whenever $\delta > \delta^*$ and $\lambda > \lambda_\delta^*$.

Thus, we conclude that multiple equilibria exist when $\delta > \delta^*$ and $\lambda > \lambda_\delta^*$. ■

B Proofs for Section 3

Proof of Lemma 1. Uniqueness of equilibrium follows from the fact that $Q_L^{fict} = (1 - \delta)c_L + \delta v_H > v_L$ when $\sigma = 1$, and because Q_L^{fict} is monotonically increasing in σ , and in ϕ when buyer mixing is part of an equilibrium. Hence, the unique equilibrium must feature no trade if $Q_L^{fict}(0, 0) \geq v_L$. Finally, it is straightforward to check that $Q_L^{fict}(0, 0) \geq v_L$ holds if and only if $\lambda \geq \bar{\lambda}$ and $\delta \geq \bar{\delta}$. ■

For the proof of Theorem 1, it will be useful to reference the following lemma, which is straightforward to verify so the proof is omitted. Let $\pi_i(s; \sigma)$ denote the buyers' posterior belief about seller i following a rejection, conditional on observing that the state is s . Then, for $s \in \{l, h\}$, we have:

$$\pi_i(s; \sigma) = \frac{\pi_\sigma^{Int} \cdot \mathbb{P}(S = s | \theta_i = H)}{\pi_\sigma^{Int} \cdot \mathbb{P}(S = s | \theta_i = H) + (1 - \pi_\sigma^{Int}) \cdot \mathbb{P}(S = s | \theta_i = L)},$$

where as before π_σ^{Int} is the interim belief.

Lemma B.1 *Given a sequence $\{\sigma_N\}_{N=1}^\infty$ of trading probabilities corresponding to a sequence of equilibria along which information aggregates, we also have convergence of posteriors: $\pi_i(\mathbf{z}_{-i}; \sigma_N) \rightarrow^p \pi_i(S; \sigma_N)$ as $N \rightarrow \infty$.*

Proof of Theorem 1. Part (i). Suppose to the contrary that (\star) holds with strict inequality, but that information aggregation obtains. Recall that in equilibrium, for any N , we must have:

$$v_L = Q_L^i(\sigma_N, \phi_i) = (1 - \delta)c_L + \delta \sum_{\mathbf{z}_{-i}} \rho_L^i(\mathbf{z}_{-i}) \cdot F_L(\pi_i(\mathbf{z}_{-i}; \sigma_N), \phi_i),$$

where

$$\begin{aligned} \sum_{\mathbf{z}_{-i}} \rho_L^i(\mathbf{z}_{-i}) \cdot F_L(\pi_i(\mathbf{z}_{-i}; \sigma_N), \phi_i) &= \sum_{s=l,h} \mathbb{P}(S = s | \theta_i = L) \sum_{\mathbf{z}_{-i}} \mathbb{P}(\mathbf{z}_{-i} | S = s) \cdot F_L(\pi_i(\mathbf{z}_{-i}; \sigma_N), \phi_i) \\ &> \lambda \cdot v_L + (1 - \lambda) \cdot \sum_{\mathbf{z}_{-i}} \mathbb{P}(\mathbf{z}_{-i} | S = h) \cdot F_L(\pi_i(\mathbf{z}_{-i}; \sigma_N), \phi_i), \end{aligned}$$

because in equilibrium we must have $F_L(\pi_i(\mathbf{z}(0); \sigma_N), \phi_i) > v_L$.

Since by Lemma B.1, $\pi_i(\mathbf{z}_{-i}; \sigma_N) \rightarrow^p \pi_i(h; \sigma_N)$ when the state is h , and because (\star) holding strictly implies that $\pi_i(h; \sigma_N) > \pi_i(h; 0) > \bar{\pi}$, we have that for a given $\epsilon > 0$, if N

is large enough, then:

$$\sum_{\mathbf{z}_{-i}} \mathbb{P}(\mathbf{z}_{-i} | S = h) \cdot F_L(\pi_i(\mathbf{z}_{-i}; \sigma_N), \phi_i) > V(\pi_i(h; \sigma_N)) - \epsilon.$$

Therefore, we conclude that for sufficiently large N :

$$\begin{aligned} v_L = Q_L^i(\sigma_N, \phi_i) &> (1 - \delta) c_L + \delta \cdot (\lambda \cdot v_L + (1 - \lambda) \cdot V(\pi_i(h; \sigma_N))) - \delta \cdot (1 - \lambda) \cdot \epsilon \\ &> (1 - \delta) c_L + \delta \cdot (\lambda \cdot v_L + (1 - \lambda) \cdot V(\pi_i(h; 0))) - \delta \cdot (1 - \lambda) \cdot \epsilon. \end{aligned}$$

Since ϵ was arbitrary, it must be that:

$$v_L \geq (1 - \delta) c_L + \delta \cdot (\lambda \cdot v_L + (1 - \lambda) \cdot V(\pi_i(h; 0))),$$

which violates (\star) holding with strict inequality, a contradiction.

Part (ii). If (\star) does not hold, then in the fictitious economy, the unique equilibrium trading probability in the first period must satisfy $\sigma^* > 0$. We next construct an equilibrium sequence $\{\sigma_N\}$ of the actual economy such that the sequence is uniformly bounded away from zero, which then implies that information aggregates along this sequence. First, consider a sequence $\{\hat{\sigma}_N\}$, not necessarily an equilibrium one, such that $\hat{\sigma}_N = \hat{\sigma} \in (0, \sigma^*)$, i.e., this is a sequence of constant trading probabilities that are positive but strictly below σ^* . Along such a sequence, information clearly aggregates and, by Lemma B.1, $\pi_i(\mathbf{z}_{-i}; \hat{\sigma}_N) \rightarrow^p \pi_i(S; \hat{\sigma}_N)$. Therefore, combined with the fact that $\pi_i(\mathbf{z}_{-i}; \hat{\sigma}_N) = \pi_i(\mathbf{z}_{-i}; \hat{\sigma}) < \pi_i(\mathbf{z}_{-i}, \sigma^*)$, there exists an N^* such that for $N > N^*$, we have:

$$\mathbb{E}_L \{F_L(\pi_i(\mathbf{z}_{-i}; \hat{\sigma}_N), \phi_i)\} < \mathbb{E}_L^{fict} \{F_L(\pi_i(S; \sigma^*))\} = \frac{v_L - (1 - \delta) \cdot c_L}{\delta},$$

where the last equality holds since $\sigma^* > 0$ implies that, in the fictitious economy, the low type must be indifferent to trading at $t = 1$ and delaying trade to $t = 2$. The correspondence $\mathbb{E}_L \{F_L(\pi_i(\mathbf{z}_{-i}; \sigma), \cdot)\}$ is upper hemicontinuous in σ for each N , and has a maximal value of v_H that is strictly greater than $\mathbb{E}_L^{fict} \{F_L(\pi_i(S; \sigma^*))\}$. Hence, for each $N > N^*$, we can find a σ_N such that $\sigma_N \geq \hat{\sigma}_N > 0$ and $\mathbb{E}_L \{F_L(\pi_i(\mathbf{z}_{-i}; \sigma_N), \phi_i)\} = \frac{v_L - (1 - \delta) \cdot c_L}{\delta}$. This delivers the desired equilibrium sequence $\{\sigma_N\}$ along which information aggregates. ■

Proof of Theorem 2. We establish the conditions for the coexistence of aggregating and non-aggregating equilibria. To do so, we first show that if $\lambda > \bar{\lambda}$, there exists a $\delta_2(\lambda) < 1$ such that non-aggregating equilibria exist if $\delta > \delta_2(\lambda)$. Second, we show that for λ large enough $\delta_2(\lambda) < \bar{\delta}_\lambda$. Therefore, both non-aggregating and aggregating equilibria exist if $\delta \in (\delta_2(\lambda), \bar{\delta}_\lambda)$, since (\star) is violated (see Theorem 1).

Consider a candidate sequence of equilibria with trading probabilities $\{\sigma_N\}$, such that $\sigma_N = \kappa_N \cdot N^{-1}$ and:

$$\pi_i(\mathbf{z}(0); \kappa_N \cdot N^{-1}) = \bar{\pi} \tag{21}$$

Solving (21) for κ_N and taking the limit as $N \rightarrow \infty$ gives $\kappa_N \rightarrow \kappa$ where

$$\kappa \equiv \frac{1}{\lambda - \frac{(1-\lambda)(1-\pi_0)}{\pi_0}} \cdot \log \left(\frac{\lambda - \left(\frac{1-\bar{\pi}}{\bar{\pi}} \cdot \frac{\pi_0}{1-\pi_0} \right) \cdot \frac{(1-\lambda)(1-\pi_0)}{\pi_0}}{\left(\frac{1-\bar{\pi}}{\bar{\pi}} \cdot \frac{\pi_0}{1-\pi_0} \right) \cdot \left(1 - \frac{(1-\lambda)(1-\pi_0)}{\pi_0} \right) - (1-\lambda)} \right) \in (0, \infty). \quad (22)$$

Seller i expects to receive an offer of v_L in all events other than $\mathbf{z}(0)$ and an expected offer $\phi_i c_H + (1 - \phi_i)v_L$ for some $\phi_i \in [0, 1]$ in the event $\mathbf{z}(0)$. Therefore, the sequence of trading probabilities defined above constitutes an equilibrium if δ is sufficiently high and the probability of the event $\mathbf{z}(0)$ conditional on the seller's type being low is bounded away from zero. To establish the latter, note that:

$$\begin{aligned} \mathbb{P}(\mathbf{z}(0)|\theta_i = L) &= \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = L) \cdot (1 - \sigma_N \cdot \mathbb{P}(\theta_i = L|S = s))^N \\ &= \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = L) \cdot (1 - \kappa_N \cdot N^{-1} \cdot \mathbb{P}(\theta_i = L|S = s))^N \\ &\rightarrow \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = L) \cdot e^{-\kappa \cdot \mathbb{P}(\theta_i = L|S = s)} > 0, \end{aligned}$$

where the last limit as $N \rightarrow \infty$ follows from Lemma B.2. In these equilibria, information fails to aggregate because as a result $\mathbb{P}(\mathbf{z}(0)|S = s)$ is bounded away from zero in both states of nature (see Lemma B.4). Thus, for each $\lambda > \bar{\lambda}$, we have established the existence of a $\delta_2(\lambda) < 1$ such that non-aggregating equilibria exist whenever $\delta > \delta_2(\lambda)$. Finally, from (22) we have that:

$$\lim_{\lambda \rightarrow 1} \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = L) \cdot e^{-\kappa \cdot \mathbb{P}(\theta_i = L|S = s)} = \frac{1 - \bar{\pi}}{\bar{\pi}} \cdot \frac{\pi_0}{1 - \pi_0} \in (0, 1),$$

and hence $\lim_{\lambda \rightarrow 1} \delta_2(\lambda) < 1$. Letting $\underline{\delta} = \lim_{\lambda \rightarrow 1} \delta_2(\lambda)$ and noting that $\lim_{\lambda \rightarrow 1} \bar{\delta} = 1$ establishes the result.

Next, we establish that when $\lambda < \bar{\lambda}$ or δ is sufficiently small, then only aggregating equilibria exist. First, suppose that $\lambda < \bar{\lambda}$ and assume to the contrary that information aggregation fails along some sequence of equilibria, and pick a subsequence of equilibria with $\sigma_N \rightarrow 0$ as N goes to ∞ (See Lemma B.3 for the existence of such a subsequence). But note that for each N , we have $\pi_i(\mathbf{z}_{-i}; \sigma_N) \leq \pi_i(h; \sigma_N)$, i.e., the posterior beliefs must be weakly lower than if the state were revealed to be high. Since $\pi_i(h; \sigma_N)$ is continuous in σ_N , and since $\lambda < \bar{\lambda}$ implies that $\pi_i(h; 0) < \bar{\pi}$, it follows that for N large enough all posterior beliefs are strictly below $\bar{\pi}$. But then for N large, $Q_L^i < v_L$ and therefore $\sigma_N = 1$, contradicting Property 3.

Second, consider $\dot{\delta}$ defined by $v_L = (1 - \dot{\delta})c_L + \dot{\delta}V(\pi_0)$, and assume that $\delta < \dot{\delta}$ (Note that Assumption 2 can still be satisfied since $V(\pi_0) < c_H$). Suppose to the contrary that information aggregation fails along a sequence of equilibria, and again pick a subsequence of equilibria with $\sigma_N \rightarrow 0$ as N goes to ∞ . By continuity, we must also have that $\pi_{\sigma_N}^{Int} \rightarrow \pi_0$

along this subsequence. But, note that for each N along this subsequence, it must be that:

$$\begin{aligned} v_L &= Q_L^i(\sigma_N, \phi_i) = (1 - \delta)c_L + \delta \mathbb{E}_L\{F_L(\pi_i(\mathbf{z}_{-i}; \sigma_N), \phi_i)\} \\ &\leq (1 - \delta)c_L + \delta \mathbb{E}_L\{V(\pi_i(\mathbf{z}_{-i}; \sigma_N))\} \\ &\leq (1 - \delta)c_L + \delta V(\pi_{\sigma_N}^{Int}). \end{aligned}$$

where the first inequality follows immediately from (6) and the second from the fact that V is a linear function and $\pi_i(\mathbf{z}_{-i}; \sigma_N)$ is a supermartingale conditional on $\theta_i = L$. Since $\delta < \bar{\delta}$ and $V(\pi_{\sigma_N}^{Int}) \rightarrow V(\pi_0)$, the last expression is lower than v_L for N large enough, a contradiction. ■

Proof of Proposition 2. Consider a sequence of equilibria along which information aggregates and let $\{\sigma_N\}$ denote the corresponding sequence of trading probabilities. By Theorem 1, it must be that either (\star) holds with equality or it is violated, i.e. $Q_L^{fict} = v_L$ and $\sigma^{fict} \geq 0$. If $\{\sigma_N\}$ (or any subsequence of it) were strictly above σ^{fict} and bounded away from it, then it is straightforward to show that, for N large enough, due to the convergence of posteriors (see Lemma B.1) we have $Q_{L,N} > Q_L^{fict} = v_L$ and therefore $\sigma_N = 0$, a contradiction. On the other hand, if σ_N were strictly below σ^{fict} and bounded away from it, then again, for N large enough, we would have $Q_{L,N} < Q_L^{fict} = v_L$ and therefore $\sigma_N = 1$, a contradiction. Hence, $\sigma_N \rightarrow \sigma^{fict}$.

Conditional on state s , the fraction of assets traded in the first period converges to $\sigma^{fict} \mathbb{P}(\theta^i = L | S = s)$ and the price is v_L for all N ; in the second period, the buyers' posteriors about the seller converge to $\pi(s; \sigma^{fict})$ (see Lemma B.1), which implies that the prices and the fraction of assets traded converge as well. Thus, conditional on state S , the aggregate volatility of prices and of trading volume (in both periods) goes to zero.

Finally, in equilibrium, we have $Q_{L,N} = v_L = Q_L^{fict}$ and the convergence of posteriors implies that $Q_{H,N} \rightarrow Q_H^{fict}$, which establishes the welfare result. ■

Proof of Proposition 3. The bounds on the equilibrium trading probabilities follow from Lemma B.3.

Along any convergent subsequence of $\{N\sigma_N\}$, by the Poisson Limit Theorem, the distribution of the number of assets traded in the first period in state s goes to Poisson with some parameter $\kappa \cdot \mathbb{P}(\theta^i = L | S = s)$, where $\kappa \in (0, \infty)$. Therefore, even as $N \rightarrow \infty$, conditional on state s , the aggregate volatility of trading volume in the first period remains strictly positive. Since this also implies that the buyers' posteriors in the second period remain uncertain, conditional on state s , the aggregate volatility of the prices and of trading volume in the second period remains strictly positive as well.

We now prove the welfare result. In order to do so, we will use results from Section 4, where we study the optimal information policy. We will use the fact that the agents' welfare is always weakly greater under optimal information policy than in equilibrium, since the laissez-faire equilibrium is in the planner's choice set.

Consider a sequence of equilibria such that aggregation fails along any of its subsequences. By Theorems 1 and 2, there are two cases to consider: either (1) $\lambda > \bar{\lambda}$ and $\delta > \bar{\delta}$ (i.e. (\star) holds with strict inequality) and thus aggregation fails along any sequence of equilibria, or (2) $\lambda > \bar{\lambda}$ and $\delta < \bar{\delta}$ (i.e. (\star) is violated) and thus there is coexistence of aggregating and non-aggregating equilibria. We ignore the non-generic cases where $\lambda = \bar{\lambda}$ or $\delta = \bar{\delta}$.

Case 1. In equilibrium, $Q_{L,N}(\sigma_N, \phi_N) = v_L$. By arguments similar to those in the proof of Lemma C.2, there is an information policy G with support $\{0, \bar{p}(\sigma_N), 1\}$ such that $Q_L^G(\sigma_N, \phi_N) \leq v_L$ and $Q_H^G(\sigma_N, \cdot) \geq Q_{H,N}(\sigma_N, \cdot)$ where:

$$Q_H^G(\sigma, \cdot) = c_H + \delta \cdot \mathbb{P}^G(1|H) \cdot (V(\pi_i(1; \sigma)) - c_H), \quad (23)$$

where $\mathbb{P}^G(1|H) = \mathbb{P}(S = h|\theta_i = H) \cdot \frac{\mu_1}{\pi_0}$, and $\mu_1 = \mathbb{P}^G(1)$. Next, by setting $\phi_N = 0$ and increasing μ_1 such that $Q_L^G(\sigma_N, 0) = v_L$ (possible since $\delta > \bar{\delta}$), we ensure that the low type seller is as well off as in equilibrium and the high type seller is strictly better off. Finally, by increasing μ_1 to π_0 (thus, fully revealing the state), we further increase $Q_H^G(\sigma_N, 0)$, but σ_N drops to zero. Since for N large, equilibrium σ_N is already close to zero, the former effect dominates, implying that $\lim_N W_N < W^{fict}$.

Case 2. In equilibrium, again, $Q_{L,N}(\sigma_N, \phi_N) = v_L$ and, for N large enough, σ_N must be close to zero and thus strictly below $\sigma^{fict} > 0$. Again, there is an information policy G with support $\{0, \bar{p}(\sigma_N), 1\}$ such that $Q_L^G(\sigma_N, \phi_N) \leq v_L$ and $Q_H^G(\sigma_N, \cdot) \geq Q_{H,N}(\sigma_N, \cdot)$ where Q_H^G is as in equation (23). Since $\sigma^{fict} > \sigma_N$ and $\mu_1 \leq \pi_0$, it follows that $Q_H^{fict}(\sigma^{fict}, \cdot) > Q_H^G(\sigma_N, \cdot)$. Thus, for N large enough, the high type seller's payoff is greater in the fictitious economy than in equilibrium, whereas the low type's payoff in both is equal to v_L . Hence, again we have that $\lim_N W_N < W^{fict}$. ■

Lemma B.2 *Let $\{\alpha_x\}$ be any non-negative sequence of real numbers such that $\alpha_x \rightarrow \alpha$ as $x \rightarrow \infty$ where $\alpha \in (0, 1)$. Then $\left(\frac{x-\alpha_x}{x}\right)^x \rightarrow e^{-\alpha}$ as $x \rightarrow \infty$.*

Proof. Assume that for any $\gamma \in (0, 1)$, $\left(\frac{x-\gamma}{x}\right)^x \rightarrow e^{-\gamma}$ as $x \rightarrow \infty$. Then, given $\epsilon > 0$ so that $\epsilon < \alpha < 1 - \epsilon$, if x is large enough then $|\alpha_x - \alpha| < \epsilon$, $\left(\frac{x-\alpha-\epsilon}{x}\right)^x \geq e^{-\alpha-\epsilon} - \epsilon$, and $\left(\frac{x-\alpha+\epsilon}{x}\right)^x \leq e^{-\alpha+\epsilon} + \epsilon$. This in turn implies that:

$$e^{-\alpha-\epsilon} - \epsilon \leq \left(\frac{x-\alpha-\epsilon}{x}\right)^x \leq \left(\frac{x-\alpha_x}{x}\right)^x \leq \left(\frac{x-\alpha+\epsilon}{x}\right)^x \leq e^{-\alpha+\epsilon} + \epsilon.$$

Since ϵ is arbitrary, we conclude that $\left(\frac{x-\alpha_x}{x}\right)^x \rightarrow e^{-\alpha}$ as $x \rightarrow \infty$. Next, we prove the supposition that for any $\gamma \in (0, 1)$, $\left(\frac{x-\gamma}{x}\right)^x \rightarrow e^{-\gamma}$ as $x \rightarrow \infty$. Note that $\left(\frac{x-\gamma}{x}\right)^x = e^{x \cdot \log\left(\frac{x-\gamma}{x}\right)}$ and by L'Hospital's rule:

$$\lim_{x \rightarrow \infty} x \cdot \log\left(\frac{x-\gamma}{x}\right) = \lim_{x \rightarrow \infty} \frac{\log\left(\frac{x-\gamma}{x}\right)}{x^{-1}} = -\lim_{x \rightarrow \infty} \frac{\gamma \cdot x}{x-\gamma} = -\gamma.$$

By continuity, $\lim_{x \rightarrow \infty} e^{x \cdot \log\left(\frac{x-\gamma}{x}\right)} = e^{-\gamma}$. ■

Lemma B.3 *Suppose that there is a sequence of equilibria $\{\sigma_N\}$ along which information aggregation fails. Then there exist a subsequence of equilibria with trading probabilities $\{\sigma_{N_m}\}$ such that for some $0 < \underline{\kappa} < \bar{\kappa} < \infty$, we have $\underline{\kappa} < \sigma_{N_m} N_m < \bar{\kappa}$ for all m .*

Proof. Suppose for contradiction that for all subsequences with trading probabilities $\{\sigma_{N_m}\}$ we have $\lim_{m \rightarrow \infty} \sigma_{N_m} N_m = \infty$. Let X_i denote the indicator that takes value of 1 if seller i

has traded in the first period. Define $Y_{N_m} = N_m^{-1} \cdot \sum_{i=1}^{N_m} X_i$ be the fraction of sellers who have traded in the first period, and note that conditional on the state being s , Y_{N_m} has a mean p_{s,N_m} and variance $N_m^{-1} \cdot p_{s,N_m} \cdot (1 - p_{s,N_m})$, where recall that $p_{s,N_m} = \sigma_{N_m} \cdot \mathbb{P}(\theta_i = L|S = s)$. Since $p_{l,N_m} > p_{h,N_m}$,

$$\begin{aligned} \mathbb{P}\left(Y_{N_m} \geq \frac{p_{h,N_m} + p_{l,N_m}}{2} | S = h\right) &= \mathbb{P}\left(Y_{N_m} - p_{h,N_m} \geq \frac{p_{l,N_m} - p_{h,N_m}}{2} | S = h\right) \\ &\leq \mathbb{P}\left((Y_{N_m} - p_{h,N_m})^2 \geq \left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2 | S = h\right) \end{aligned}$$

And by Markov's inequality:

$$\begin{aligned} \mathbb{P}\left((Y_{N_m} - p_{h,N_m})^2 \geq \left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2 | S = h\right) &\leq \frac{\mathbb{E}\{(Y_{N_m} - p_{h,N_m})^2 | S = h\}}{\left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2} \\ &= \frac{N_m^{-1} \cdot p_{h,N_m} \cdot (1 - p_{h,N_m})}{\left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2} \\ &= 4 \cdot \frac{\sigma_{N_m} \cdot \mathbb{P}(\theta_i = L|S = h) - \sigma_{N_m}^2 \cdot \mathbb{P}(\theta_i = L|S = h)^2}{N_m \cdot \sigma_{N_m}^2 \cdot (\mathbb{P}(\theta_i = L|S = l) - \mathbb{P}(\theta_i = L|S = h))^2} \end{aligned}$$

which by our assumption tends to 0 as $m \rightarrow \infty$. By a similar reasoning, we have that:

$$\begin{aligned} \mathbb{P}\left(Y_{N_m} < \frac{p_{h,N_m} + p_{l,N_m}}{2} | S = l\right) &= \mathbb{P}\left(p_{l,N_m} - Y_{N_m} > \frac{p_{l,N_m} - p_{h,N_m}}{2} | S = l\right) \\ &\leq \mathbb{P}\left((p_{l,N_m} - Y_{N_m})^2 > \left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2 | S = l\right) \\ &\leq \frac{\mathbb{E}\{(Y_{N_m} - p_{l,N_m})^2 | S = l\}}{\left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2} \\ &= \frac{N_m^{-1} \cdot p_{l,N_m} \cdot (1 - p_{l,N_m})}{\left(\frac{p_{l,N_m} - p_{h,N_m}}{2}\right)^2} \\ &= 4 \cdot \frac{\sigma_{N_m} \cdot \mathbb{P}(\theta_i = L|S = l) - \sigma_{N_m}^2 \cdot \mathbb{P}(\theta_i = L|S = l)^2}{N_m \cdot \sigma_{N_m}^2 \cdot (\mathbb{P}(\theta_i = L|S = l) - \mathbb{P}(\theta_i = L|S = h))^2} \end{aligned}$$

which again tends to 0 as $m \rightarrow \infty$. Combining these two observations, we conclude that information about the state must aggregate along all subsequences, a contradiction.

Next, suppose for contradiction that for all subsequences with trading probabilities $\{\sigma_{N_m}\}$ we have that $\lim_{m \rightarrow \infty} \sigma_{N_m} N_m = 0$. Then, given any $\epsilon > 0$ and m large enough, we have:

$$(1 - \sigma_{N_m} \cdot \mathbb{P}(\theta_i = L|S = s))^{N_m} = \left(\frac{N_m - \sigma_{N_m} \cdot N_m \cdot \mathbb{P}(\theta_i = L|S = s)}{N_m}\right)^{N_m} \geq \left(\frac{N_m - \epsilon}{N_m}\right)^{N_m}$$

for $s \in \{l, h\}$, where the last expression converges to $e^{-\epsilon}$ by Lemma B.2. Since ϵ is arbitrary, $(1 - \sigma_{N_m} \cdot \mathbb{P}(\theta_i = L|S = s))^{N_m}$ goes to 1 as $m \rightarrow \infty$. Hence, we have that for $\theta \in \{L, H\}$:

$$\mathbb{P}(Y_{N_m} = 0|\theta_i = \theta) = \sum_{s=l,h} \mathbb{P}(S = s|\theta_i = \theta) \cdot (1 - \sigma_{N_m} \cdot \mathbb{P}(\theta_i = L|S = s))^{N_m} \rightarrow 1.$$

Now, consider the posterior belief about the seller conditional on event that no seller has traded. For any m , since the low type must expect offers above v_L with positive probability and since $\mathbf{z}(0)$ is the best possible news, it must be that:

$$\begin{aligned} \pi_i(\mathbf{z}(0); \sigma_{N_m}) &\geq \bar{\pi} \\ \iff &\frac{\pi_{\sigma_{N_m}}^{Int} \cdot \mathbb{P}(Y_{N_m} = 0|\theta_i = H)}{\pi_{\sigma_{N_m}}^{Int} \cdot \mathbb{P}(Y_{N_m} = 0|\theta_i = H) + (1 - \pi_{\sigma_{N_m}}^{Int}) \cdot \mathbb{P}(Y_{N_m} = 0|\theta_i = L)} \geq \bar{\pi}. \end{aligned}$$

But note that, since $\sigma_{N_m} \rightarrow 0$ and $\pi_{\sigma_{N_m}}$ is continuous, the left-hand side converges to $\pi < \bar{\pi}$, a contradiction. ■

Lemma B.4 *Consider a sequence of equilibria with trading probabilities $\{\sigma_N\}$ such that $\sigma_N N < \bar{\kappa}$ for some $\bar{\kappa} < \infty$. Then $\mathbb{P}(Y_N = 0|S = s)$ is bounded away from zero, uniformly over N , for $s \in \{l, h\}$.*

Proof. We have that $\mathbb{P}(Y_N = 0|S = s) = (1 - p_{s,N})^N$ for $s \in \{l, h\}$. By assumption, $p_{s,N} \leq N^{-1} \cdot \bar{\kappa} \cdot \mathbb{P}(\theta_i = L|S = s)$. Therefore,

$$\mathbb{P}(Y_N = 0|S = s) \geq (1 - N^{-1} \cdot \bar{\kappa} \cdot \mathbb{P}(\theta_i = L|S = s))^N$$

and by Lemma B.2, $\lim_{N \rightarrow \infty} (1 - N^{-1} \cdot \bar{\kappa} \cdot \mathbb{P}(\theta_i = L|S = s))^N = e^{-\bar{\kappa} \cdot \mathbb{P}(\theta_i = L|S = s)} > 0$. ■

C Proofs for Section 4

Notation and Preliminaries. Let $g(p)$ denote the probability of posterior p under the information policy G .¹⁷ Let $g_s(p)$ denote the probability of posterior p conditional on the state s , so that

$$g(p) = \pi_0 \cdot g_h(p) + (1 - \pi_0) \cdot g_l(p), \quad (24)$$

Observe that for any p in the support of G , Bayesian consistency requires that

$$p = \frac{\pi_0 \cdot g_h(p)}{\pi_0 \cdot g_h(p|h) + (1 - \pi_0) \cdot g_l(p)}, \quad (25)$$

Given g , (24) and (25) pin down the conditional probabilities g_h and g_l . Let $\mathbb{P}^G(p|\theta)$ denote the probability that the posterior about the state is p conditional on the seller's type θ :

$$\mathbb{P}^G(p|\theta) = \mathbb{P}(S = h|\theta)g_h(p) + \mathbb{P}(S = l|\theta)g_l(p) \quad (26)$$

¹⁷To streamline notation, we restrict attention to information policies with finite support. Our proofs can easily be extended to rule out policies that assign positive mass to a continuum of beliefs not in the support stated in Lemma C.2.

Given any σ , and p in the support of G , the buyers' posterior belief that the seller is type H can be computed as follows:

$$\pi_i(p; \sigma) \equiv \frac{\pi_\sigma^{Int} \cdot \mathbb{P}^G(p|H)}{\pi_\sigma^{Int} \cdot \mathbb{P}^G(p|H) + (1 - \pi_\sigma^{Int}) \cdot \mathbb{P}^G(p|L)}, \quad (27)$$

where recall that the interim belief is given by $\pi_\sigma^{Int} = \frac{\pi_0}{\pi_0 + (1 - \pi_0)(1 - \sigma)}$. The continuation value of the θ -type seller, under information policy G , is

$$Q_\theta^G(\sigma, \phi) = (1 - \delta)c_\theta + \delta \sum_p F_\theta(\pi_i(p; \sigma), \phi) \mathbb{P}^G(p|\theta), \quad (28)$$

where F_θ is the payoff function defined by the equations (5) and (6). Recall that (G, σ, ϕ) is feasible only if $Q_L^G(\sigma, \phi)$ satisfies (13) and (14).

Lemma C.1 *Define $\bar{\sigma}$ implicitly by $\pi_i(0; \sigma) = \bar{\pi}$ and define $\underline{\sigma} \equiv \inf\{\sigma \in [0, 1], \pi_i(1, \sigma) \geq \bar{\pi}\}$, then σ is feasible only if $\sigma \in [\underline{\sigma}, \bar{\sigma}]$.*

Proof of Lemma C.1. If $\sigma < \underline{\sigma}$, then $\pi_i(1; \sigma) < \bar{\pi}$ and thus the buyers' bid is v_L in the second period w.p.1. But then, $Q_L^G < v_L$, violating (13). If $\sigma > \bar{\sigma}$, then $\pi_i(0; \sigma) > \bar{\pi}$ and thus the buyers' bid is greater than c_H in the second period w.p.1. But then, $Q_L^G > v_L$ (by Assumption 2) and (14) requires $\sigma = 0$, a contradiction. ■

Lemma C.2 *The solution to the informed planner's problem can be achieved with an information policy that has support $\Sigma(\sigma) \subseteq \{0, \bar{p}(\sigma), 1\}$, for some feasible σ and $\bar{p}(\sigma)$ s.t. $\pi_i(\bar{p}(\sigma); \sigma) = \bar{\pi}$.*

Proof of Lemma C.2. First, it is without loss of generality to assume that the optimal policy does not involve posteriors in the interval $(0, \bar{p}(\sigma))$. To see this, note that if the planner were to reassign the weight from any posteriors in $(0, \bar{p}(\sigma))$ to the posteriors in $\{0, \bar{p}(\sigma)\}$ (respecting Bayesian plausibility), she could reduce ϕ to ensure that the low type's continuation value is unaffected. Since this change does not affect the distribution of posteriors above $\bar{p}(\sigma)$, the high type's continuation value (and thus welfare) is also unaffected.

Second, suppose that the planner's policy has posteriors in the interval $(\bar{p}(\sigma), 1)$ in its support. We will demonstrate that such a policy can be (weakly) improved upon. There are two cases to consider, depending on whether (13) is slack.

Case 1. Suppose that (13) holds with equality, and thus $Q_L^G = v_L$ and $\sigma \geq 0$. First, we will argue that if the planner were to reassign weight from the posteriors in $(\bar{p}(\sigma), 1)$ to posteriors in $\{\bar{p}(\sigma), 1\}$, then she would reduce (increase) the low (high) type's continuation value. To see this, consider a candidate information policy G that assigns a positive weight to some posterior $p' \in (\bar{p}(\sigma), 1)$. Next, consider an alternative policy \hat{G} that is the same as G except that it reassigns the weight from p' to the posteriors $\bar{p}(\sigma)$ and 1 to preserve Bayesian plausibility, i.e.,

$$\begin{aligned} g(p') &= \hat{g}(\bar{p}(\sigma)) - g(\bar{p}(\sigma)) + \hat{g}(1) - g(1), \\ g(p') \cdot p' &= \mu(\bar{p}(\sigma)) \cdot \bar{p}(\sigma) + \mu(1) \cdot 1. \end{aligned} \quad (29)$$

Define $\mu^{\widehat{G}}(p) \equiv \widehat{g}(p) - g(p)$. By (26), we have:

$$\underbrace{\mathbb{P}^G(p'|\theta)}_{\equiv \mu_{\theta}^G(p')} = \underbrace{\mathbb{P}^{\widehat{G}}(\bar{p}(\sigma)|\theta) - \mathbb{P}^G(\bar{p}(\sigma)|\theta)}_{\equiv \mu_{\theta}^{\widehat{G}}(\bar{p}(\sigma))} + \underbrace{\mathbb{P}^{\widehat{G}}(1|\theta) - \mathbb{P}^G(1|\theta)}_{\equiv \mu_{\theta}^{\widehat{G}}(1)}. \quad (30)$$

Next, observe that:

$$\begin{aligned} & \mu_L^{\widehat{G}}(\bar{p}(\sigma))V(\pi_i(\bar{p}(\sigma); \sigma)) + \mu_L^{\widehat{G}}(1)V(\pi_i(1; \sigma)) < \mu_L^G(p')V(\pi_i(p'; \sigma)) \\ \iff & \mu_L^{\widehat{G}}(\bar{p}(\sigma))\pi_i(\bar{p}(\sigma); \sigma) + \mu_L^{\widehat{G}}(1)\pi_i(1; \sigma) < \mu_L^G(p')\pi_i(p'; \sigma) \\ \iff & \frac{\mu_L^{\widehat{G}}(\bar{p}(\sigma))}{\mu_L^{\widehat{G}}(\bar{p}(\sigma)) + \mu_L^{\widehat{G}}(1)}\pi_i(\bar{p}(\sigma); \sigma) + \left(1 - \frac{\mu_L^{\widehat{G}}(\bar{p}(\sigma))}{\mu_L^{\widehat{G}}(\bar{p}(\sigma)) + \mu_L^{\widehat{G}}(1)}\right)\pi_i(1; \sigma) < \pi_i(p'; \sigma), \end{aligned} \quad (31)$$

where the last inequality follows from $\frac{\mu_L^{\widehat{G}}(\bar{p}(\sigma))}{\mu_L^{\widehat{G}}(\bar{p}(\sigma)) + \mu_L^{\widehat{G}}(1)} > \frac{\mu_L^G(p')}{\mu_L^G(p') + \mu_L^G(1)}$, $\pi_i(\bar{p}(\sigma); \sigma) < \pi_i(p'; \sigma) < \pi_i(1; \sigma)$, and the fact that

$$\frac{\mu_L^{\widehat{G}}(\bar{p}(\sigma))}{\mu_L^{\widehat{G}}(\bar{p}(\sigma)) + \mu_L^{\widehat{G}}(1)}\pi_i(\bar{p}(\sigma); \sigma) + \left(1 - \frac{\mu_L^{\widehat{G}}(\bar{p}(\sigma))}{\mu_L^{\widehat{G}}(\bar{p}(\sigma)) + \mu_L^{\widehat{G}}(1)}\right)\pi_i(1; \sigma) = \pi_i(p'; \sigma). \quad (32)$$

Since the bid following any posterior $p > \bar{p}(\sigma)$ must be pooling (Property 1), it follows that $Q_L^{\widehat{G}}(\sigma, \phi) < Q_L^G(\sigma, \phi)$. By an analogous argument, we can show that:

$$\mu_H^{\widehat{G}}(\bar{p}(\sigma))V(\pi_i(\bar{p}(\sigma); \sigma)) + \mu_H^{\widehat{G}}(1)V(\pi_i(1; \sigma)) > \mu_H^G(p')V(\pi_i(p'; \sigma)) \quad (33)$$

and, therefore, $Q_H^{\widehat{G}}(\sigma, \phi) > Q_H^G(\sigma, \phi)$. Finally, the planner can increase σ and/or ϕ to some $\widehat{\sigma}$ and $\widehat{\phi}$ to ensure that $Q_L^{\widehat{G}}(\widehat{\sigma}, \widehat{\phi}) = v_L$, and note that both those adjustments increase the high type's continuation value from $Q_H^G(\sigma, \phi)$ to $Q_H^{\widehat{G}}(\widehat{\sigma}, \widehat{\phi})$. Thus, $(\widehat{G}, \widehat{\sigma}, \widehat{\phi})$ generates the same payoff to the low type, but strictly higher payoff for the high type than (G, σ, ϕ) .

Case 2. Suppose that (13) is slack, and thus $Q_L^G(\sigma, \phi) > v_L$ and $\sigma = 0$. By inspecting the planner's objective, we see that it must be that $\phi = 1$; otherwise the planner could implement a higher ϕ and improve welfare without violating (13). Thus, the planner's objective is proportional to $\mathbb{P}(\pi_i \geq \bar{\pi} | \theta_i = H) = \mathbb{P}(\tilde{p} \geq \bar{p}(0) | \theta_i = H)$ (see (11)).

Given the properties just established, we will now construct an alternative policy that is feasible and increases the planner's objective, which completes the proof for Case 2. Recall the supposition we are trying to refute is that the optimal policy assigns weight to posteriors in $(\bar{p}(0), 1)$. The alternative policy G' reassigns the weight from all the posteriors in $[\bar{p}(0), 1]$ to the posteriors $\{0, \hat{p}\}$, where $\hat{p} \equiv \mathbb{E}_G\{\tilde{p} | \tilde{p} \geq \bar{p}(0)\} - \varepsilon$ for some ε small (respecting Bayesian plausibility). If $\varepsilon = 0$, then $\hat{p} > \bar{p}(0)$, the trading surplus is unchanged and the low type's continuation value increases (see argument in Case 1), implying that (13) remains slack. For ε sufficiently small but positive, again $\hat{p} > \bar{p}(0)$ and (13) remains slack by continuity, but the trading surplus strictly increases as Bayesian plausibility implies $\mathbb{P}^{G'}(\bar{p} = \hat{p} | \theta_i = H) > \mathbb{P}^G(\tilde{p} \geq \bar{p}(0) | \theta_i = H)$. ■

Proof of Lemma 2. If (13) is required to hold with equality, then the low type's

payoff is v_L and thus maximization of expected discounted gains from trade is equivalent to maximization of the high type's payoff $Q_H^G(\sigma, \phi)$. By Lemma C.2, we have that:

$$Q_H^G(\sigma, \phi) = (1 - \delta)c_H + \delta (c_H + \mathbb{P}^G(1|H) \cdot (V(\pi_i(h; \sigma)) - c_H)), \quad (34)$$

where $\mathbb{P}^G(1|H) = \mathbb{P}(S = h|\theta^i = H) \cdot \frac{\mu_1}{\pi_0}$. Thus, the planner's objective is increasing in $\mu_1 \cdot (V(\pi_i(h; \sigma)) - c_H)$, which in turn is increasing in μ_1 and in σ . There are two possibilities.

If $Q_L^{fict}(\bar{\sigma}, 0) \leq v_L$, then it is feasible for the planner to set $\mu_1 = \pi_0$ and $\sigma = \bar{\sigma}$, and then choose ϕ so that $Q_L^{fict}(\bar{\sigma}, \phi) = v_L$. This is clearly optimal since both μ_1 and σ are maximized subject to feasibility.

If $Q_L^{fict}(\bar{\sigma}, 0) > v_L$, then $\mu_1 = \pi_0$ and $\sigma = \bar{\sigma}$ is not feasible, since then (13) would hold with strict inequality; thus, either $\mu_1 < \pi_0$, $\sigma < \bar{\sigma}$, or both. Note that the low type's continuation value is given by:

$$Q_L^G(\sigma, \phi) = (1 - \delta)c_L + \delta (v_L + \mathbb{P}^G(\bar{p}(\sigma)|L)\phi(c_H - v_L) + \mathbb{P}^G(1|L) (V(\pi_i(h; \sigma)) - v_L)), \quad (35)$$

where $\mathbb{P}^G(1|L) = \mathbb{P}(S = h|\theta^i = L) \cdot \frac{\mu_1}{\pi_0}$.

We now argue that $\phi = 0$. To see this, note from (35) that $Q_L^G(\sigma, 0)$ is continuous in μ_1 and σ for $\sigma \in [\underline{\sigma}, \bar{\sigma}]$, and $Q_L^G(\bar{\sigma}, 0) = Q_L^{fict}(\bar{\sigma}, 0) > v_L$ when $\mu_1 = \pi_0$. Thus, if ϕ is strictly positive, the planner could reduce ϕ to zero, increase μ_1 and/or σ , and still maintain $Q_L^G = v_L$. But this would increase the trading surplus as Q_H^G is independent of ϕ , but is increasing in μ_1 and σ .

Therefore, at the informed planner's solution, we have $Q_L^G(\sigma, 0) = v_L$, which is equivalent to saying that $\mu_1(V(\pi_i(h; \sigma)) - v_L)$ is equal to some constant $K > 0$, which implies that $\mu_1 = \frac{K}{V(\pi_i(h; \sigma)) - v_L}$. Substituting for μ_1 in $Q_H^G(\sigma, 0)$, we have:

$$Q_H^G(\sigma, 0) = (1 - \delta)c_H + \delta \left(c_H + \mathbb{P}(S = h|\theta^i = H) \cdot \frac{K}{\pi_0} \cdot \frac{V(\pi_i(h; \sigma)) - c_H}{V(\pi_i(h; \sigma)) - v_L} \right), \quad (36)$$

which is increasing in σ . Thus, it is optimal for the planner to increase σ as much as possible (by reducing μ_1). Since $\mu_1 + (1 - \mu_0 - \mu_1)\bar{p}(\sigma) = \pi_0$, it follows that the lowest μ_1 is equal to $\frac{\pi_0 - \bar{p}(\sigma)}{1 - \bar{p}(\sigma)}$, i.e., the planner sets $\mu_0 = 0$. Therefore, the solution to the planner's problem features $\mu_1^* = \frac{\pi_0 - \bar{p}(\sigma^*)}{1 - \bar{p}(\sigma^*)}$ and σ^* is such that $Q_L^G(\sigma^*, 0) = v_L$. ■

Proof of Proposition 4. Let G^* denote the solution to the variant of the informed planner's problem characterized in Lemma 2 in which the inequality (13) is required to hold with equality. It is immediate that either (1) the solution to the original informed planner's problem involves (13) holding with strict inequality, or (2) G^* is the solution to that problem. Therefore, it is sufficient to compare the value of the planner's objective in these two scenarios.

For (1), in Lemma C.3 we show that the informed planner's solution must have $\sigma = 0$, $\phi = 1$, and an information policy, denoted by G_0 , which consists of only two posteriors $\{0, \bar{p}(0)\}$, where by Bayes' rule:

$$\mathbb{P}^{G_0}(\bar{p}(0) | \theta) = \mathbb{P}(S = h|\theta_i = \theta) + \mathbb{P}(S = l|\theta_i = \theta) \left(\frac{1 - \bar{p}(0)}{\bar{p}(0)} \right) \left(\frac{\pi_0}{1 - \pi_0} \right). \quad (37)$$

The trading surplus under $(G, \sigma, \phi) = (G_0, 0, 1)$ is given by:

$$W^{G_0} = \delta \left((1 - \pi_0) (v_L - c_L) + \pi_0 \mathbb{P}^{G_0}(\bar{p}(0) | H) (v_H - c_H) \right). \quad (38)$$

For (2), the trading surplus under $(G, \sigma, \phi) = (G^*, \sigma^*, \phi^*)$ is given by:

$$\begin{aligned} W^{G^*} &= (1 - \pi_0) \cdot (Q_L^{G^*} - c_L) + \pi_0 \cdot (Q_H^{G^*} - c_H) \\ &= (1 - \pi_0) \cdot (v_L - c_L) + \pi_0 \cdot \delta \cdot \mathbb{P}^{G^*}(1|H) \cdot (V(\pi_i(1; \sigma^*)) - c_H), \end{aligned} \quad (39)$$

where $\mathbb{P}^{G^*}(1|H) = \mathbb{P}(S = h|H) \cdot \frac{\mu_1^*}{\pi_0}$ and (μ_1^*, σ^*) are given in Lemma 2.

We will establish the result by showing that (i) $W^{G_0} < W^{G^*}$ when δ is sufficiently small, (ii) $W^{G_0} > W^{G^*}$ when δ is sufficiently large, and (iii) W^{G_0} increases faster in δ than W^{G^*} .

For (i), a necessary condition for G_0 to be preferable to G^* is that (13) holds with strict inequality when $(G, \sigma, \phi) = (G_0, 0, 1)$:

$$v_L < Q_L^{G_0}(0, 1) = (1 - \delta) c_L + \delta \cdot (v_L + \mathbb{P}^{G_0}(\bar{p}(0) | L) \cdot (c_H - v_L)), \quad (40)$$

which is equivalent to requiring that $\delta > \delta_0$, where:

$$\delta_0 \equiv \frac{v_L - c_L}{v_L - c_L + \mathbb{P}^{G_0}(\bar{p}(0) | L) \cdot (c_H - v_L)}. \quad (41)$$

Otherwise, either (13) is violated so that $(G_0, 0, 1)$ is inconsistent with equilibrium, or (13) holds with equality implying $(G_0, 0, 1)$ is sub-optimal (see Lemma 2). Since W^{G_0} and W^{G^*} are continuous in δ , it follows that $W^{G^*} > W^{G_0}$ for δ close to δ_0 .

For (ii), note that $(G, \sigma, \phi) = (G_0, 0, 1)$ would be optimal if δ were equal to 1, since then only the gains from trade in the second period matter for the trading surplus (see Lemma C.3). Since W^{G_0} and W^{G^*} are continuous in δ , it follows that $W^{G^*} < W^{G_0}$ for δ close to 1.

For (iii), for all $\delta \in (\delta_0, 1)$, we have:

$$\frac{dW^{G_0}}{d\delta} = (1 - \pi_0) \cdot (v_L - c_L) + \pi_0 \cdot \mathbb{P}^{G_0}(\bar{p}(0) | H) \cdot (v_H - c_H) \quad (42)$$

$$> \pi_0 \cdot \mathbb{P}(S = h|H) \cdot (v_H - c_H), \quad (43)$$

since $\mathbb{P}^{G_0}(\bar{p}(0)|H) > \mathbb{P}^{G_0}(S = h|H)$. On the other hand, using the same arguments as in the proof of Lemma 2, we have that $\mu_1^* \cdot (V(\pi_i(1; \sigma^*)) - c_H)$ is decreasing in δ and thus

$$\frac{dW^{G^*}}{d\delta} < \pi_0 \cdot \mathbb{P}(S = h|H) \cdot (v_H - c_H) \quad (44)$$

since $\mu_1^* \leq \pi_0$ and $V(\pi_i(1; \sigma^*)) < v_H$. Thus, $\frac{dW^{G_0}}{d\delta} > \frac{dW^{G^*}}{d\delta}$ for $\delta \in (\delta_0, 1)$. ■

Proof of Lemma 3. If $\sigma_N = 0$ for some N , the planner's information policy would be uninformative. Hence, in the second period, the buyers' posterior would equal their prior, which is strictly below $\bar{\pi}$ (Assumption 1) and, thus, their bid would equal to v_L w.p.1. But this would imply that $Q_L^{M_N}(0, \cdot) < v_L$, a contradiction. ■

Proof of Proposition 5. Consider the following information policy for the planner.

(i) When the planner observes that the number of assets that have traded in the first period is weakly greater than threshold τ_N , then she says that “the trading volume is high.” For convenience, we denote this report by $\omega_N = \emptyset$.

(ii) When instead the planner observes that the number of assets that have traded in the first period is lower than τ_N , then she discloses the trading volume fully, i.e., her report is $\omega_N = \sum_j z^j$ where $z^j \in \{0, 1\}$ is the indicator for whether seller j has traded.

There are two cases to consider, depending on whether the informed planner’s optimal policy characterized in Lemma 2 is fully revealing.

Consider first the case where the informed planner’s policy is fully revealing, i.e., part (i) of Lemma 2. Suppose that the planner sets $\tau_N = N$, i.e., she always fully discloses the trading volume. Next, note that condition (\star) does not hold. Since all trading behavior is observed by the agents, by Theorem 1, there exists a sequence of equilibria along which information aggregates and, using the same arguments as in the proof of Proposition 2, along any such sequence $\sigma_N \rightarrow \sigma^{fict} = \bar{\sigma}$ and $Q_H^{\mathcal{M}^N}(\sigma_N, \phi_N) \rightarrow Q_H^{fict}(\bar{\sigma}, \phi^*) = Q_H^{G^*}(\bar{\sigma}, \phi^*)$, where the latter is the payoff of the high-type seller under the optimal information policy of the informed planner.

Consider next the case where the informed planner’s policy is not fully revealing, i.e., part (ii) of Lemma 2. Conjecture that there is an equilibrium sequence $\{\sigma_N\}$ such that σ_N is bounded away from zero. Along such a sequence, conditional on the aggregate state S , the distribution of the random variable $X_N \equiv N^{-1} \sum_j z^j$ converges to a normal with mean $p_{s, \sigma_N} = \sigma_N \mathbb{P}(\theta^i = L | S = s)$ and variance $N^{-1} p_{s, \sigma_N} (1 - p_{s, \sigma_N})$. Suppose that the planner sets τ_N such that $\Phi\left(\frac{N^{-1}\tau_N - p_{h, \sigma_N}}{\sqrt{N^{-1}p_{h, \sigma_N}(1-p_{h, \sigma_N})}}\right) = \mu_1^*$, where $\Phi(\cdot)$ is the standard normal cdf and where μ_1^* is given in part (ii) of Lemma 2. By construction, $N^{-1}\tau_N$ converges to $\sigma_N \mathbb{P}(\theta^i = L | S = h)$, i.e., the mean of X_N in state h . By Lemma 3, in equilibrium, we must have:

$$v_L = Q_L^{\mathcal{M}^N}(\sigma_N, \phi_N) = (1 - \delta) c_L + \delta \cdot \sum_{\omega_N} \mathbb{P}^{\mathcal{M}^N}(\omega_N | \theta^i = L) \cdot F_L(\pi_i(\omega_N; \sigma_N), \phi_N), \quad (45)$$

where $\mathbb{P}^{\mathcal{M}^N}(\omega_N | \theta^i = \theta)$ denotes the probability distribution over the planner’s report ω_N , given the seller’s type; and $\pi_i(\omega_N; \sigma_N)$ is the posterior about the seller i given the planner’s report ω_N . Clearly, along the sequence $\{\sigma_N\}$, it must be that $\mathbb{P}^{\mathcal{M}^N}(\omega_N \neq \emptyset | S = h) = \mathbb{P}^{\mathcal{M}^N}(X_N < N^{-1}\tau_N | S = h) \rightarrow \mu_1^*$ and $\mathbb{P}^{\mathcal{M}^N}(\omega_N \neq \emptyset | S = l) = \mathbb{P}^{\mathcal{M}^N}(X_N < N^{-1}\tau_N | S = l) \rightarrow 0$. Therefore, $\pi_i(\omega_N; \sigma_N) \rightarrow^p \pi_i(1; \sigma_N)$ conditional on $\omega_N \neq \emptyset$, where $\pi_i(1; \sigma_N)$ is the posterior about the seller i given that the state is revealed to be h .

Next, if any subsequence of $\{\sigma_N\}$ were below (above) and bounded away from σ^* , as defined in part (ii) of Lemma 2, then it is straightforward to show that along it the continuation value of the low type $Q_L^{\mathcal{M}^N}(\sigma_N, 0)$ would eventually be strictly below (above) $Q_L^{G^*}(\sigma^*, 0)$, which cannot be part of equilibrium since $v_L = Q_L^{G^*}(\sigma^*, 0)$. Using continuity arguments, $\sigma_N \rightarrow \sigma^*$ is part of equilibrium, and our conjecture is verified since $\sigma_N > 0$ for all N and $\sigma^* > 0$ by Property 4 and Lemma 2 respectively. Given the above results, the equilibrium

continuation value of the high type satisfies:

$$Q_H^{\mathcal{M}_N}(\sigma_N, \phi_N) \geq c_H + \delta \cdot \sum_{\omega_N \neq \emptyset} \mathbb{P}^{\mathcal{M}_N}(\omega_N | \theta^i = H) \cdot F_H(\pi_i(\omega_N; \sigma_N), \phi_N) \rightarrow Q_H^{G^*}(\sigma^*, 0). \quad (46)$$

Since also $Q_H^{\mathcal{M}_N}(\sigma_N, \phi_N) \leq Q_H^{G^*}(\sigma^*, 0)$, i.e., the informed planner always does weakly better than the uninformed, it follows that $Q_H^{\mathcal{M}_N}(\sigma_N, \phi_N) \rightarrow Q_H^{G^*}(\sigma^*, 0)$.

This establishes the result since, under both the informed planner's and the uninformed planner's policies, the buyers break even and the low type's payoff is v_L . ■

Proof of Corollary 1. Follows directly from our characterization of the optimal information policy. ■

Proof of Corollary 2. Follows directly from our characterization of the optimal information policy, together with the fact that:

$$Q_L^{fict}(\bar{\sigma}, 0) = (1 - \delta)c_L + \delta(v_L + (1 - \lambda) \cdot \pi_i(1, \bar{\sigma}) \cdot (v_H - v_L)) \quad (47)$$

with $\bar{\sigma}$ as defined in Lemma C.1. ■

Lemma C.3 *Suppose that at the solution to the informed planner's problem (13) holds with strict inequality. Then, the solution must have $\sigma = 0$, $\phi = 1$ and an information policy that only consists posteriors $\tilde{p} \in \{0, \bar{p}(0)\}$.*

Proof. First, if (13) holds with strict inequality, then $\sigma = 0$ by (14). Second, if (13) holds with strict inequality for some $\phi < 1$, it must also hold with strict inequality for $\phi = 1$ as $Q_L^G(0, \phi)$ is increasing in ϕ . By inspection of the planner's objective, we see that the trading surplus is increasing in ϕ . Thus, it must be that $\phi = 1$.

Given the above observations, the planner's objective reduces to choosing the information policy G in order to maximize $\mathbb{P}^G(\tilde{p} \geq \bar{p}(0) | H)$. We prove the last claim by way of contradiction. Suppose that the optimal information policy, G , consists of posteriors outside $\{0, \bar{p}(0)\}$.

- If there are posteriors in $(0, \bar{p}(0))$, then consider the alternative policy G' that reassigns the weight from all such posteriors to the posteriors 0 and $\bar{p}(0)$ (respecting Bayesian plausibility). Note that $Q_L^{G'}(0, 1) > Q_L^G(0, 1)$ since the bid is v_L for any $p < \bar{p}(0)$, which ensures that (13) is satisfied under G' , and it implies that $\mathbb{P}^{G'}(\tilde{p} \geq \bar{p}(0) | H) > \mathbb{P}^G(\tilde{p} \geq \bar{p}(0) | H)$ by Bayesian plausibility, which contradicts the optimality of G .
- If there are posteriors above $\bar{p}(0)$, then consider the alternative policy G' , which reassigns the weight from all the posteriors in $[\bar{p}(0), 1]$ to the posteriors 0 and $\hat{p} \equiv \mathbb{E}_G\{\tilde{p} | \tilde{p} \geq \bar{p}(0)\} - \varepsilon$ (respecting Bayesian plausibility). Using the same arguments as in Case 2 in the proof of Lemma C.2, if ε is small enough, then (13) continues to be satisfied while trading surplus strictly increases, which again contradicts the optimality of G .

■

D Proof of Theorem 3

We establish parts (i)-(iii) of Theorem 3 separately.

Proof of Theorem 3, part (i). We proceed by contradiction and suppose to the contrary that there is some finite date t at which information aggregates. In particular, suppose that information has not aggregated before t , but it aggregates at t . Consider seller i who trades with probability in $(0, 1)$ at t . We know that the number of such sellers must grow to ∞ with N , since otherwise there would be insufficient information learned at t . Without loss of generality assume that all sellers trade with probability in $(0, 1)$ at t . By the skimming property, the bid for this seller's asset must be v_L , which the high type rejects whereas the low type accepts with some probability $\sigma_{i,N} \in (0, 1)$.

Let $Q_{L,t}^{i,N}$ denote the low type seller i 's continuation value from rejecting a bid v_L at time t . Define

$$\bar{Q}_t^N \equiv (1 - \delta) \cdot c_L + \delta \cdot (\lambda_{L,t} \cdot v_L + (1 - \lambda_{L,t}) \cdot V(\pi_i(h; 0))), \quad (48)$$

where (i) $\lambda_{L,t} = \mathbb{P}_t(S = l | \theta_i = L)$ is the posterior belief that the state is l conditional on trading history up to period t and the seller's type being L , and (ii) $\pi_i(h; 0)$ is the posterior belief about the seller i conditional on the state being h . In Lemma D.1, we show that:

$$\lim_{N \rightarrow \infty} \mathbb{P}_t \left(Q_{L,t}^{i,N} \geq \bar{Q}_t^N \right) = 1, \quad (49)$$

i.e., \bar{Q}_t^N provides a lower bound on the low type's continuation value. Next, we use this result to show that with probability bounded away from zero in both states of nature, if N is large enough, then $Q_{L,t}^{i,N} > v_L$. This immediately implies that the low types strictly prefer to delay trade at t , contradicting aggregation and thus establishing our result.

Since (\star) holds, $\bar{Q}_1^N > v_L$. Thus, information aggregation must fail in the first period. In Lemma D.2, we show that failure of information aggregation at t implies that the probability of the event that no seller trades in that period must be bounded away from zero, uniformly over N , in both states of nature. Because this event is 'good' news about the state, then following it in the first period, we have $\lambda_{L,2} < \lambda_{L,1}$ and, thus, $\bar{Q}_2^N > v_L$. But then again information aggregation must fail in the second period and, therefore, the probability that no seller trades in the second period must remain bounded away from zero in both states of nature. Repeating this argument until period t , we can construct a history that occurs with probability bounded away from zero in both states of nature, in which $\bar{Q}_t^N > v_L$, as was stated above. ■

Proof of Theorem 3, part (ii). The proof is by construction. Consider a candidate equilibrium in which for any period t , the following properties hold:

- (i) If $\pi_{i,t} < \bar{\pi}$, then the bid is v_L , which the low type accepts w.p. $\sigma_t \in [0, 1)$ whereas the high type rejects w.p.1.
- (ii) If $\pi_{i,t} > \bar{\pi}$, then the bid is $V(\pi_{i,t})$ and both types accept it w.p.1.
- (iii) If $\pi_{i,t} = \bar{\pi}$, then the bid is $V(\pi_{i,t})$ w.p. ϕ_t (and both types accept it w.p.1) and is v_L w.p. $1 - \phi_t$ (and both types reject it).

The only off-equilibrium path event in a candidate satisfying (i)-(iii) is a rejection when $\pi_{i,t} > \bar{\pi}$, in which case the interim belief as given by Bayes rule is not well defined. For such cases, we specify $\pi_{\sigma_{i,t}} = \pi_{i,t}$ (i.e., unexpected rejections are attributed to random trembles).¹⁸

We will now verify that an equilibrium satisfying (i)-(iii) exists (with off-path beliefs as specified immediately above). To do so, consider any history and let N_t (\mathcal{N}_t) denote the number (set) of sellers who have not yet traded at the beginning of period t . Notice that the seller's value function under the proposed equilibrium is the same as in (5) and (6), where (π_i, ϕ_i) is replaced by $(\pi_{i,t}, \phi_t)$. By symmetry of the candidate, $\pi_{i,t} = \pi_{j,t} = \pi_t$ for all $j \neq i \in \mathcal{N}_t$. We now show that there exists (σ_t, ϕ_{t+1}) such that profitable deviations do not exist:

- (i) Suppose that $\pi_t < \bar{\pi}$. Since continuation values in period $t + 1$ are the same as in the two-period model, we know by Proposition 1, that for any N_t , there exists at least one (σ_t, ϕ_{t+1}) pair such that the low types' continuation value is exactly v_L . Hence, a low-type seller is willing to mix. Clearly, a high-type seller strictly prefers to reject.
- (ii) Suppose that $\pi_t > \bar{\pi}$ and seller i rejects, since all other sellers accept w.p.1. there is no information revealed by other sellers and therefore (given the off-path specification above) $\pi_{i,t+1} = \pi_t$. Therefore, rejecting the offer leads to a payoff of $(1 - \delta)c_\theta + \delta F_\theta(\pi_t) < V(\pi_t)$.
- (iii) Suppose that $\pi_t = \bar{\pi}$. If trade does not occur, buyers in period $t + 1$ attribute all rejections to a low offer made by buyers in period t . Hence, if the seller rejects, $\pi_{i,t+1} = \pi_{i,t}$ and by the same argument as in (ii), such a deviation is not profitable for the seller.

That buyers do not have a profitable deviation from the candidate follows by a similar reasoning to the argument for Property 1 in the two-period model. Belief consistency is by construction. Thus, there exists an equilibrium of the candidate form.

Notice that, by construction, the second-period payoff to a type- θ seller is the same as in the two-period model. Therefore, if (\star) does not hold then following the same argument as in the proof of Theorem 1, we can construct a sequence of first-period trading probabilities $\{\sigma_{N,1}\}$ that are uniformly bounded away from zero, which ensures information aggregates in the first period. ■

Proof of Theorem 3, part (iii). From Theorem 3, part (ii), when (\star) does not hold, there is a sequence of equilibria along which information aggregates. In the class of equilibria constructed in the proof of Theorem 3 (ii), equilibrium play in the first play coincides with the equilibrium play of all equilibria in the two-period economy of Section 3. As a result, under the same conditions as in Theorem 2 (namely, that $\delta \in (\underline{\delta}, \bar{\delta})$), there exists a sequence of equilibria along which information aggregation fails in the first period. Furthermore, by construction of this sequence of equilibria (see proof of Theorem 2), (i) the probability of the event that no seller trades in the first period remains bounded away from zero in both states of nature and (ii) the posterior belief about the seller in the second period following this event

¹⁸The precise specification of off-path beliefs is not crucial for the construction, any $\pi_{\sigma_{i,t}} \leq \min\{g(\pi_{i,t}), 1\}$ will suffice, where $g(\pi_{i,t}) > \pi_{i,t}$ is such that $(1 - \delta)c_H + \delta V(g(\pi_{i,t})) = V(\pi_{i,t})$ and ensures that a high type cannot profitably deviate from rejecting.

is equal to $\bar{\pi}$. But then, following this event, by construction no additional information about the state is revealed through trade. ■

In what follows, we prove the two lemmas used in the proof of Theorem 3, part (i).

Lemma D.1 *Suppose that (\star) holds, and information aggregates in period t but not before. Then $\lim_{N \rightarrow \infty} \mathbb{P}_t \left(Q_{L,t}^{i,N} \geq \bar{Q}_t^N \right) = 1$.*

Proof. The low type's continuation value from rejecting bid v_L at date t is:

$$\begin{aligned} Q_{L,t}^{i,N} &= (1 - \delta) \cdot c_L + \delta \cdot \left(\lambda_{L,t} \cdot \mathbb{E}_t \left\{ F_{L,t+1}^{i,N} | S = l \right\} + (1 - \lambda_{L,t}) \cdot \mathbb{E}_t \left\{ F_{L,t+1}^{i,N} | S = h \right\} \right) \\ &> (1 - \delta) \cdot c_L + \delta \cdot \left(\lambda_{L,t} \cdot v_L + (1 - \lambda_{L,t}) \cdot \mathbb{E}_t \left\{ F_{L,t+1}^{i,N} | S = h \right\} \right), \end{aligned}$$

where $\mathbb{E}_t \left\{ F_{L,t+1}^{i,N} | S = s \right\}$ denotes the low type's expected payoff conditional on history up to t and the state being s . For the inequality, we used the fact that the payoffs at $t + 1$ must be strictly above v_L with positive probability, since otherwise no seller would be willing to delay trade to $t + 1$. We next show that, for any $\varepsilon > 0$,

$$\lim_{N \rightarrow \infty} \mathbb{P}_t \left(F_{L,t+1}^{i,N} \geq V(\pi_i(h; 0)) - \varepsilon | S = h \right) = 1, \quad (50)$$

which, since ε is arbitrary, will establish the result.

Suppose that the state is h and let T be the smallest number of periods such that:

$$(1 - \delta^T) \cdot c_H + \delta^T \cdot v_H < V(\pi_i(h; 0)),$$

which is finite since (\star) implies $V(\pi_i(h; 0)) > c_H$. Since information aggregates at t (by hypothesis), we can choose N large enough so that (w.p. close to 1) the agents' belief that the state is h is close to 1 in the periods $t + 1$ through $t + 1 + T$. Let us consider histories in which this is the case. If we show that (w.p. going to 1 as N goes to ∞) the bid at $t + 1$ is pooling and both seller types accept the bid, then we are done.

Suppose to the contrary that for any N , there is strictly positive probability (bounded away from zero) that the bid is not pooling at $t + 1$. There are two cases to consider at t . First, it could be that, with probability bounded away from zero, the buyers make a bid that is rejected by both types. Second, it could be that, with probability bounded away from zero, the bid is v_L and the low types accept it with positive probability.

The first case is straightforward to rule out, since otherwise the buyers could profitably deviate and attract both seller types to trade at date t . For the second case, note that also at $t + 2$, with probability bounded away from zero, the bid v_L must be made and accepted by the low type with some probability. Otherwise, if the pooling bid were made instead (w.p. close to 1), the low type would not be willing to trade at $t + 1$ (Assumption 2). We can repeat this argument until and including period T and construct sub-histories that occur with probability bounded away from zero, in which the buyers make a bid v_L which is accepted with positive probability by the low types in periods $t + 1$ through $t + 1 + T$.

Let Ω_τ denote the set of sub-histories at $\tau \in \{t + 1, \dots, t + 1 + T\}$ in which the bid is v_L in periods $t + 1$ through τ , and let ω_τ denote an element of Ω_τ . For $\tau' > \tau$, let $\Omega_{\tau'} | \omega_\tau$ denote

the sub-histories in $\Omega_{\tau'}$ that have ω_{τ} as a predecessor. Now, for any $\omega_{\tau} \in \Omega_{\tau}$, in order for buyers not to be able to attract the high type at τ , it must be that:

$$V(\pi_{i,\tau}(\omega_{\tau})) \leq Q_{H,\tau}^{i,N}(\omega_{\tau}), \quad (51)$$

i.e., the high type would weakly prefer to reject a pooling offer and get his continuation value. The high type's continuation value in turn satisfies:

$$Q_{H,\tau}^{i,N}(\omega_{\tau}) \leq (1 - \delta) \cdot c_H + \delta \cdot \max \left\{ \mathbb{E}_H \{ Q_{H,\tau+1}^{i,N} | \{ \Omega_{\tau+1} | \omega_{\tau} \} \}, \mathbb{E}_H \{ V(\pi_{i,\tau+1}) | \{ \Omega_{\tau+1} | \omega_{\tau} \}^c \} \right\}.$$

Since the beliefs that the state is h are arbitrarily close to 1 in all periods $\tau \in \{t+1, t+1+T\}$, the posterior beliefs about the seller are arbitrarily close to each other in any such period τ . Hence, combining with (51), for any $\varepsilon > 0$, we can choose N large enough so that:

$$Q_{H,\tau}^{i,N}(\omega_{\tau}) \leq (1 - \delta) \cdot c_H + \delta \cdot \mathbb{E}_H \{ Q_{H,\tau+1}^{i,N} | \{ \Omega_{\tau+1} | \omega_{\tau} \} \} + \varepsilon$$

for all $\tau \in \{t+1, t+1+T\}$, which implies that:

$$Q_{H,t+1}^{i,N}(\omega_{t+1}) \leq (1 - \delta^T) \cdot c_H + \delta^T \cdot \mathbb{E}_H \{ Q_{H,t+1+T}^{i,N} | \{ \Omega_{t+1+T} | \omega_{t+1} \} \} + \hat{\varepsilon},$$

where $\hat{\varepsilon}$ can be made small by choosing ε small. Since the value to the seller in any period cannot exceed v_H , then with probability approaching 1 as N goes to ∞ ,

$$V(\pi_i(h; 0)) - \hat{\varepsilon} \leq V(\pi_{i,t+1}(\omega_{t+1})) \leq (1 - \delta^T) \cdot c_H + \delta^T \cdot v_H + \hat{\varepsilon},$$

which, since $\hat{\varepsilon}$ is arbitrary, contradicts our choice of T . ■

Lemma D.2 *Suppose that information aggregation fails at t along a sequence of equilibria with time- t trading probabilities $\{\sigma_{i,N}\}$. Then there is a subsequence of equilibria along which the probability that no seller trades at t remains bounded away from zero, in both states of nature.*

Proof. Assume that there is a subsequence of equilibria with trading probabilities $\{\sigma_{N_m}\}$ with $\sum_{i=1}^{N_m} \sigma_{i,N_m} < \bar{\kappa} < \infty$ for some $\bar{\kappa} > 0$ and all m . Note that $1 - \sigma_{i,N_m} \cdot P(\theta_i = L | S = s) \geq e^{-\sigma_{i,N_m} \cdot K}$ for any K satisfying $1 - P(\theta_i = L | S = l) \geq e^{-K}$. But for any such K , we have:

$$\begin{aligned} P(\text{no seller trades at } t | S = s) &= \prod_{i=1}^{N_m} (1 - \sigma_{i,N_m} \cdot P(\theta_i = L | S = s)) \\ &\geq \prod_{i=1}^{N_m} e^{-\sigma_{i,N_m} \cdot K} \\ &= e^{-K \cdot \sum_{i=1}^{N_m} \sigma_{i,N_m}} \\ &\geq e^{-K \cdot \bar{\kappa}} > 0, \end{aligned}$$

which establishes the result.

We are left to prove the assertion that there is a subsequence $\{\sigma_{N_m}\}$ with $\sum_{i=1}^{N_m} \sigma_{i,N_m} < \bar{\kappa} < \infty$ for some $\bar{\kappa} > 0$ and all m . Suppose to the contrary that for all subsequences $\lim_{m \rightarrow \infty} \sum_{i=1}^{N_m} \sigma_{i,N_m} = \infty$. Let $X_i \in \{0, 1\}$ denote the indicator that seller i has traded

and $Y_{N_m} = N_m^{-1} \sum_{i=1}^{N_m} X_i$ denote the fraction of sellers who have traded. Let $p_{i,N_m}(s) = \sigma_{i,N_m} \cdot \mathbb{P}(\theta_i = L | S = s)$ and note that:

$$\mu_{N_m}(s) \equiv \mathbb{E}\{Y_{N_m} | S = s\} = N_m^{-1} \cdot \sum_{i=1}^{N_m} p_{i,N_m}(s),$$

and

$$\nu_{N_m}(s) \equiv \mathbb{E}\{(Y_{N_m} - \mu_{N_m}(s))^2 | S = s\} = N_m^{-2} \cdot \sum_{i=1}^{N_m} p_{i,N_m}(s) \cdot (1 - p_{i,N_m}(s)).$$

Since $\mu_{N_m}(l) > \mu_{N_m}(h)$,

$$\begin{aligned} P\left(Y_{N_m} \geq \frac{\mu_{N_m}(h) + \mu_{N_m}(l)}{2} | S = h\right) &= P\left(Y_{N_m} - \mu_{N_m}(h) \geq \frac{\mu_{N_m}(l) - \mu_{N_m}(h)}{2} | S = h\right) \\ &\leq P\left((Y_{N_m} - \mu_{N_m}(h))^2 \geq \left(\frac{\mu_{N_m}(l) - \mu_{N_m}(h)}{2}\right)^2 | S = h\right). \end{aligned}$$

And, by Markov's inequality:

$$\begin{aligned} P\left((Y_{N_m} - \mu_{N_m}(h))^2 \geq \left(\frac{\mu_{N_m}(l) - \mu_{N_m}(h)}{2}\right)^2 | S = h\right) &\leq \frac{\nu_{N_m}(h)}{\left(\frac{\mu_{N_m}(l) - \mu_{N_m}(h)}{2}\right)^2} \\ &= \frac{N_m^{-2} \cdot \sum_{i=1}^{N_m} p_{i,N_m}(h) \cdot (1 - p_{i,N_m}(h))}{\left(N_m^{-1} \cdot \sum_{i=1}^{N_m} \frac{p_{i,N_m}(l) - p_{i,N_m}(h)}{2}\right)^2} \\ &= 4 \cdot \frac{\sum_{i=1}^{N_m} \sigma_{i,N_m} \cdot P(\theta_i = L | S = h) - \sum_{i=1}^{N_m} \sigma_{i,N_m}^2 \cdot P(\theta_i = L | S = h)^2}{\left(\sum_{i=1}^{N_m} \sigma_{i,N_m}\right)^2 \cdot (P(\theta_i = L | S = l) - P(\theta_i = L | S = h))^2}, \end{aligned}$$

which by our assumption tends to 0 as m goes to ∞ . By a similar reasoning, we have that:

$$\mathbb{P}\left(Y_{N_m} < N_m^{-1} \cdot \sum_{i=1}^{N_m} \frac{p_{i,N_m}(l) + p_{i,N_m}(h)}{2} | S = l\right) \rightarrow 0,$$

as m goes to ∞ . Combining these two observations, we conclude that information about the state must aggregate along all subsequences, a contradiction. ■