



## Detrending and business cycle facts

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### Abstract

This paper examines the business cycle properties of a small set of a real US macroeconomic time series using a variety of detrending methods. It is shown that both quantitatively and qualitatively ‘stylized facts’ of US business cycles vary widely across detrending methods and that alternative detrending filters extract different types of information from the data. Implications and suggestions for current macroeconomic practice are provided. © 1998 Elsevier Science B.V. All rights reserved.

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For the drama lies in this – in the conscience that I have, that each one of us has. We believe this conscience to be a single thing, but it is many sided ... . We have this illusion of being one person for all ... but it is not true.

Luigi Pirandello

### 1. Introduction

Since the influential work of Hodrick and Prescott (1980) it has become increasingly popular to characterize the behavior of macroeconomic variables over the business cycle using a set of uncontroversial summary statistics (examples include Baxter and Stockman (1989), Kydland and Prescott (1990),

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Stock and Watson (1990) and Backus and Kehoe (1992)). The compilation of stylized facts of the business cycle is important for two reasons. First, it gives a coarse summary of the complex comovements existing among aggregates in the economy, allows a rough calculation of the magnitude of the fluctuations in economic variables and may guide researchers in choosing leading indicators for economic activity. Second, it provides a set of ‘regularities’ which macroeconomists use as a benchmark to examine the validity of numerical versions of theoretical models.

Any empirical examination of the business cycle, however, involves the delicate and controversial issue of detrending. There are two problems connected with detrending. The first concerns the lack of a professional consensus on what constitutes business fluctuations. The second concerns the use of a statistically-based approach vs. an economic-based approach to detrending.

Consider first the issue of what business cycles are. Business cycle fluctuations are typically identified with deviations from the trend of the process. However, within the empirical literature, there is fundamental disagreement on the properties of the trend and on its relationship with the cyclical component of a series. In the past the representation and extraction of the secular component was handled in a very simple way. The trend was represented with deterministic polynomial functions of time, assumed to be independent of the cyclical component and extracted using simple regression methods. More recently, following Nelson and Plosser’s (1982) findings, Beveridge and Nelson (1981), Watson (1986), Hamilton (1989) and Quah (1992) have proposed alternative definitions of the trend, different assumptions about the relationship between the trend and the cycle and novel methods for estimating the two components. Since the issue of what is an ‘appropriate’ statistical representation of the trend cannot be solved in small samples and since the choice of the relationship between the cyclical and secular components is arbitrary, statistical based approaches to detrending raise questions about the robustness of certain ‘facts’. As Singleton (1988, p. 372) observes, ‘The stylized facts motivating recent specifications of the business cycle models may have been distorted by prefiltering procedures’. Moreover, it is now clear that different statistical representations for the trend embed different economic concepts of business cycle fluctuations and choosing one detrending method over another implies selecting one particular economic object over another. Documenting the properties of different types of business cycles may therefore help us, on one hand, to provide a more exhaustive description of the data, and, on the other, to highlight the sense in which they are economically different.

The second problem connected with detrending – the question of a statistical vs. an economic based decomposition – arises from a standard ‘measurement without theory’ concern. It is often argued that before variables can be selected and facts reported, a theory explaining the mechanism generating economic fluctuations is needed. This point of view has been advocated by those who use

economic theory to choose an economic-based decomposition of the actual time series in deriving business cycle regularities (see, e.g. Singleton (1988), King et al. (1989) or King et al. (1991)) and also by those who employ economic theory as an organizing principle for time series analysis but use arbitrary filtering procedures, which reflect the preferences of the researcher and the question to be investigated, to establish business cycle facts (see, e.g. Kydland and Prescott (1990) or Stock and Watson (1990)).

Dynamic economic theory, however, does not indicate the type of economic trend that series may display nor the exact relationship between secular and cyclical components. Models have been proposed where the long-run component may be either deterministic or stochastic and may or may not be related to the cyclical component (see Dellas (1993) for an example where trend and cycle interact in a non-trivial way). In other words, without a set of statistical facts pinning down the properties of the secular component of a time series, the theoretical relationship between trend and cycle is unknown and the choice among various economic-based decompositions arbitrary. This issue is particularly relevant because there has been surprisingly little discussion in the literature on whether particular economic representations provide an appropriate characterization of the actual business cycles or whether they, instead, leave out important sources of fluctuations (an exception is Watson (1993)). Because of this circularity, all economic-based decompositions are, at best, attempts to approximate unknown features of a series and therefore subject to specification errors.

Compared to the vastness of the problems raised in this introduction, the focus of the paper is modest. I report the cyclical properties of a small set of real series using a number of different detrending methods. The approach of the paper is agnostic. Modern dynamic theory of real economic fluctuations is used only to select the variables of interest for this study. None of the detrending filters employed is believed to be the correct one. Instead, I assume that all procedures are approximations which isolate aspects of the secular and cyclical components of the series. In this sense, different detrending methods are alternative windows which look at series from different perspectives. The crucial question is not which method is more appropriate but whether different concepts of cycle are likely to produce alternative information which can be used to get a better perspective into economic phenomena and to validate theories. The idea of the paper is to organize this information in a systematic manner in an attempt (i) to identify whether there exists a set of relationships which is invariant to the definition of cycle employed, (ii) point out some situations where choosing a standard concept of cycle provides misleading impressions of the comovements of the data and (iii) provide evidence on certain ‘data anomalies’ which have motivated recent efforts in the theoretical literature and pose new ‘puzzles’ which may guide future developments.

I choose to concentrate on a small set of real variables to maintain comparability with the existing real business cycle (RBC) literature but it should be clear that the problems outlined in this introduction are not unique to this literature and concern all approaches which use ‘stylized facts’ as qualitative or quantitative benchmarks to compare the properties of theoretical models. The lack of monetary and financial series from the list of variables examined does not make the substance of the arguments weaker in any sense.

I compare the properties of the cyclical components of seven real series (GNP, Consumption, Investment, Hours, Real Wage, Productivity and Capital Stock) obtained using seven univariate (Hodrick-Prescott, Beveridge-Nelson, Linear, Segmented, First Order Differencing, Unobservable Components, Frequency Domain Masking) and three multivariate (Cointegration, Common Linear and Multivariate Frequency Domain) detrending techniques. For each method I report sample moments, a few terms of the cross correlation function and the impulse response function of each of the seven variables when GNP is shocked.

Antecedents of the type of research carried out here are Baxter and Stockman (1989), Baxter (1991), King and Rebelo (1993), Harvey and Jaeger (1993) and Cogley and Nason (1995). They demonstrated how the mechanical application of the Hodrick and Prescott filter to series which are either integrated or driven by deterministic trends may induce spurious results and how particular quantitative features of the business cycles are not robust to the choice of detrending.

The paper documents that the second-order properties of the estimated cyclical components of the seven series vary widely across detrending procedures but that only minor differential effects emerge in higher moments. I show that different detrending methods extract different ‘types’ of business cycle information from the original series and that, even among the class of filters which produce cycles with similar duration features, significant qualitative differences may emerge. I argue that the qualitative responses of consumption, investment, hours and real wage to a typical shock in GNP exhibit two typical patterns: one broadly consistent with technology driven and one broadly consistent with a demand driven idea of business cycles. Quantitatively, a variety of relative responses emerge. Finally, I show how the information produced can be used to shed light on some contradictory empirical evidence. I note that in some situations, e.g. in determining whether labor hoarding occurs or not, economic theory broadly suggests which class of detrending methods should be employed to examine the relevance of the phenomena. However, I also show that in certain cases concentrating on a standard definition of cycle may waste information, e.g. in examining the cyclicity of productivity, and this has implications for how we believe the economy functions.

The analysis of the paper completely ignores the possibility that measurement errors are present in the raw data. This is potentially a serious problem since the

data collected by statistical agencies is massaged in so many ways that spurious results may obtain (see, e.g. Wilcox (1992)). The crucial issue here is whether these filtering procedures (which include sectoral and temporal aggregations, various adjustments and the use of proxies) induce differing amounts of measurement errors at different frequencies. Given the lack of information on the construction of various aggregates, I reluctantly assume that measurement errors are negligible and constant across frequencies.

The paper is organized as follows: the next section describes the detrending procedures. The emphasis here is on the different assumptions characterizing the trend and the relationship with its cyclical component. Section 3 describes the properties of the cyclical components obtained with different detrending methods. Section 4 analyzes certain ‘stylized facts’ in light of the results of Section 3. Section 5 concludes and discusses the implications of our findings for macroeconomic practice.

## 2. Alternative detrending methods

This section reviews the procedures I use to extract trends from the observable time series. I divide the methods into two broad categories: ‘statistical’ methods, which assume that the trend and the cycle are unobservable but use different statistical assumptions to identify the two components, and ‘economic’ methods, where the choice of trend is dictated by an economic model, by the preferences of the researcher or by the question being asked. Since only trend and cycle are assumed to exist, all the procedures implicitly assume that either data has previously been seasonally adjusted or that the seasonal and the cyclical component of the series are lumped together and that irregular (high frequency) fluctuations play little role. Although these assumptions are not without consequences, the implications of these restrictions will be neglected as a first approximation. Throughout the paper I denote the natural logarithm of the time series by  $y_t$ , its trend by  $x_t$  and its cyclical component by  $c_t$ .

### 2.1. Statistical procedures

#### 2.1.1. Polynomial functions of time

This procedure is the simplest and the oldest one. It assumes that trend and cycle of the (log) of the series are uncorrelated and that  $x_t$  is a deterministic process which can be approximated with polynomial functions of time.

These assumptions imply a model for  $y_t$  of the form

$$y_t = x_t + c_t, \tag{1}$$

$$x_t = a + \sum_j^q b_{1j} f_j(t - t_0) \quad \text{if } t \leq \bar{t},$$

$$x_t = a + \sum_j^q b_{2j} f_j(t - t_1) \quad \text{if } \bar{t} + 1 \leq t \leq T, \tag{2}$$

where  $q$  is typically chosen to be small,  $t_0$  and  $t_1$  are given points in time scaling the origin of the trend. In Eq. (2), I allow for the possibility of a structural break in the secular component at a known time  $\bar{t}$ . I present results for  $f_j(t - t_0) = t - t_0$  and  $\bar{t} = T$  (LT in the tables), and for  $f_j(t - t_0) = t - t_0$ ,  $f_j(t - t_1) = t - t_1$  and  $t_1 = \bar{t} = 1973,3$  (SEGM in the tables). The trend is estimated by fitting  $y_t$  to a constant and to scaled polynomial functions of time using least squares and by taking the predicted value of the regression. The cyclical component is the residual from Eq. (1). The results I present are broadly insensitive to the choice of  $\bar{t}$  in the range [1973,1–1975,4].

2.1.2. First order differences

The basic assumptions of a first-order differencing procedure (FOD in the tables) are that the secular component of the series is a random walk with no drift, the cyclical component is stationary and that the two components are uncorrelated. In addition, it is assumed that  $y_t$  has a unit root which is entirely due to the secular component of the series. Therefore  $y_t$  can be represented as:

$$y_t = y_{t-1} + \varepsilon_t \tag{3}$$

the trend is defined as  $x_t = y_{t-1}$  and an estimate of  $c_t$  is obtained as  $\hat{c}_t = y_t - y_{t-1}$ .

2.1.3. Beveridge and Nelson’s procedure

The key identifying assumption of Beveridge and Nelson’s (1981) procedure is that the cyclical component of the series is stationary while the secular component accounts for its non-stationary behavior. Let  $w_t = (1 - \ell)y_t$  be a stationary ARMA process with moving average representation  $w_t = \mu + \gamma(\ell)\epsilon_t$ , where  $\epsilon_t \sim \text{i.i.d.}(0, \sigma^2)$  and  $\gamma(\ell) = \phi(\ell)^{-1}\theta(\ell)$  is a polynomial in the lag operator with the roots of  $\phi(z) = 0$  outside the unit circle.

Beveridge and Nelson show that the secular component of a series can be defined as the long-run forecast of  $y_t$  adjusted for its mean rate of change  $k\mu$ ; i.e.

$$x_t \equiv y_t + \hat{w}_t(1) + \dots + \hat{w}_t(k) - k\mu \tag{4}$$

with  $\hat{w}_t(i) = E_t(w_{t+i} | y_t, y_{t-1}, \dots) = \sum_{j=0}^{i-1} (\sum_{l=j+1}^k \gamma_l) \epsilon_{t-j}$ . For  $k$  sufficiently large, the trend is the value the series would take if it were on the long-run path.

Therefore, for  $k \rightarrow \infty$  Eq. (4) collapses to:  $x_t = x_{t-1} + \mu + (\sum_{i=1}^{\infty} \gamma_i) \epsilon_t$ . The cyclical component of the series is then

$$c_t = \hat{w}_t(1) + \dots + \hat{w}_t(k) - k\mu = \chi(\ell)\epsilon_t. \quad (5)$$

Two characteristics of this decomposition should be noted. First, since trend and cycle are driven by the same shock, this decomposition has the remarkable property that the secular and the cyclical components are perfectly correlated. Second, since estimates of the  $\gamma$ 's and forecasts  $\hat{w}_t(i)$  are obtained from an ARIMA model, the problems inherent to ARIMA specifications are carried over to this method. For example, as Christiano and Eichenbaum (1990) have pointed out, there are several ARIMA models which fit the sample autocorrelations of a data set fairly well. However, because ARIMA models having the same short-run properties may have very different long-run features, alternative specifications may lead to very different decompositions into trend and cycle. Also, as Maravall (1993) has argued, because ARIMA models are designed to fit the short-run properties of the data they are very ill-suited to capture their long-run features.

Since the results vary considerably with the choice of  $\theta(\ell)$  and  $\phi(\ell)$ , both in terms of the magnitude of the fluctuations and of the path properties of the data, I examined various ARIMA specifications. Here I present results obtained using  $\theta(\ell) = 1 \forall \ell$ , five lags for  $\phi(\ell)$ , the actual value of GNP at 1955,2 as the initial condition and the quick computational approach of Coddington and Winters (1987) (BN in the tables).

#### 2.1.4. Unobserved components model

The key identifying assumptions of this procedure are that the secular component follows a random walk with drift and that the cyclical component is a stationary finite order AR process. Also, contrary to a FOD procedure, a UC approach allows for correlation between the trend and the cycle. The most recent Unobservable Components (UC) literature assumes that the drift term in the random walk may drift over time as well (see, e.g. Harvey and Jaeger, 1993). However, since the task here is to compare methodologies, not to find the best model specification, I do not consider this possibility. UC models are usually cast in a state space framework (see Harvey (1985) and Watson (1986) among others). The measurement equation is given by

$$y_t = x_t + c_t + \varepsilon_t, \quad t = 1, \dots, T, \quad (6)$$

where  $\varepsilon_t \sim N(0, \sigma^2)$  for all  $t$  and  $E(\varepsilon_t \varepsilon_{t-i}) = 0$  for  $i \neq 0$ . The transition equations are

$$\begin{aligned} x_t &= x_{t-1} + \delta + u_t, \\ c_t &= \phi(\ell)c_{t-1} + v_t \end{aligned} \quad (7)$$

where  $\delta$  is a parameter and the  $q$  roots of  $\phi(z) = 0$  lie outside the unit circle. The properties of  $x_t$  and  $c_t$  are fully characterized by the assumption that the distribution of  $u_t$  and  $v_t$  are jointly normal with covariance matrix  $\Sigma$  and by the fact that  $\varepsilon_t$  is uncorrelated with  $u_t$  and  $v_t$ . The parameters  $\beta = (\sigma^2, \sigma_u^2, \sigma_v^2, \sigma_{uv}, \delta, \phi_j, j = 1, \dots, q)$  are typically estimated using the prediction error decomposition of the likelihood and a smoothing algorithm which revises recursive estimates (see, e.g. Harvey (1985)). To simplify, estimates of  $\beta$ 's are obtained using the autocovariances of  $w_t = (1 - \ell)y_t$  (see Carvalho et al., 1979). Given the estimates of  $\beta$  and a zero mean and a diagonal covariance matrix with large but finite elements as initial conditions, recursive estimates of the state vector  $\alpha_t = [x_t, c_t, \dots, c_{t-q}, 1]'$  are obtained with the Kalman filter.

Here I report results obtained using 2 lags for  $\phi(\ell)$  when no smoothing of recursive estimates is undertaken (UC in the tables). The results are not very sensitive to the choice of lag length for  $\phi(\ell)$  in the range [2,4].

### 2.1.5. Frequency domain methods

The frequency domain procedure employed here draws from Sims (1974). The procedure assumes that the cyclical and secular components of the series are independent, that the secular component has most of its power in a low-frequency band of the spectrum and that away from zero the power of the secular component decays very fast. The identification assumptions do not restrict the trend to be either deterministic or stochastic and allows for changes in the trend over time as long as the changes are not too frequent. The secular component can be recovered from  $y_t$  using

$$a(\omega)F_y(\omega) = F_x(\omega) \quad (8)$$

where  $a(\omega)$  is a 'low' pass filter and  $F_y(\omega)$  and  $F_x(\omega)$  are the Fourier transforms of  $y_t$  and  $x_t$ . In the time domain the polynomial  $a(\ell)$ , the inverse Fourier transform of  $a(\omega)$ , has the form

$$a(\ell) = \frac{\sin(\omega_2\ell) - \sin(\omega_1\ell)}{\pi\ell} \quad (9)$$

(see, e.g. Priestley (1981) (p. 275)) where  $\omega_1$  and  $\omega_2$  are the upper and lower limits of the frequency band where the secular component has all its power. An estimate of the cyclical component is then  $(1 - a(\ell))y_t$ . The key to this procedure is the appropriate selection of the upper and lower limits of the frequency band. Following the NBER taxonomy, which describes as business cycle those fluctuations with 2–6 yr periodicity, and the conventional wisdom that no complete cycle has exceeded 8 yr in length, I chose  $\omega_1 = 0$  and  $\omega_2 = \pi/15$ . Since the spectrum is symmetric around the origin, this filter wipes out all the power of the series in the band  $(-\pi/15, \pi/15)$  and cycles with length less than 30 quarters are

all assumed to belong to the cyclical component of  $y_t$  (FREQ1 in the tables). The results I present are not too sensitive to choices of  $\omega_2$  leaving in  $c_t$  cycles with maximum length between 20 and 30 quarters.

The above filter leaves a considerable amount of undesirable high-frequency variability, which need not necessarily be identified with business cycle fluctuations. For this reason, I also consider a decomposition of  $y_t$  as in Eq. (6) where  $\varepsilon_t$  is identified by the assumption that it has most of its power located in a high-frequency band of the spectrum (as, e.g. in Englund et al., 1991). In this case the cyclical component of the series is obtained with a filter which, in addition to eliminating all cycles with period greater than thirty quarters, eliminates all cycles with period less than six quarters. This is achieved by choosing  $a(\omega)$  to be:

$$a(\omega) = \begin{cases} 1 & \text{if } \omega \in [0, \pi/15] \cup [\pi/3, \pi], \\ 0 & \text{otherwise.} \end{cases}$$

The results are presented as FREQ2 in the tables. It is worthwhile noting that this filter has approximately the same properties as the ‘Batterworth’ filter used by Stock and Watson (1990) and the band pass filter of Baxter and King (1994).

#### 2.1.6. One-dimensional index model

The final procedure in the statistical group is multivariate and assumes that while each series is trending, either deterministically or stochastically or both, some linear combination of them does not have trends (see, e.g. Stock and Watson, 1989). The key assumption is that in the low frequencies of the spectrum there exists a one-dimensional process (a secular component) which is common to all series (see Quah and Sargent (1993) for a two-index model). This process is characterized by the property that it has all its power at low frequencies and that away from zero it decays very fast. The model for  $y_t$  is given by Eq. (1) where now  $y_t$  is an  $n \times 1$  vector,  $x_t = Az_t$  and  $z_t$  is a scalar process with  $0 < S_z(\omega) < M$ ,  $\forall \omega \in [\bar{\omega}, \pi]$  where  $S_z(\omega)$  is the spectral density of  $z_t$ ,  $M$  is a small number,  $A$  is an  $n \times 1$  vector of loadings and  $x_t$  is an  $n \times 1$  vector independent of  $c_t$ . An estimate of  $x_t$  is obtained using a multivariate version of the procedure used for the UC model and  $\hat{c}_t$  is obtained residually from Eq. (1) (MINDEX in the tables).

## 2.2. Economic procedures

### 2.2.1. A model of common deterministic trends

King et al. (1988) present a neoclassical model of capital accumulation with labor supply choices where there is deterministic labor augmenting technical

progress. Their model implies that all endogenous variables have a common deterministic trend (the growth rate of labor augmenting technical progress) and that fluctuations around the common linear trend are all of a transitory nature. Each time series is therefore generated by a model like Eq. (1) where the secular and cyclical components are independent, where  $x_t$  is common to all series and given by

$$x_t = x_0 + \delta t \tag{10}$$

where  $\delta$  is the growth rate of technological progress. To construct a deterministic trend which is common to all series I use data on GNP, Consumption, Investment, Real Wage and Capital and select  $x_0$  to be an estimate of the unconditional mean of each series. Since the hours series is measured in absolute terms, I detrend it using the growth rate of population (about 0.3% per quarter over the sample 1955,3–1986,3). The resulting estimate of  $\delta$  is 0.7%, which differs from the one of King et al. (0.4%) because they do not use the capital stock in the calculations and employ a different sample (MLT in the tables).

2.2.2. *A model of common stochastic trends*

King et al. (1991) propose a version of model of King et al. (1988) where the long-run properties of the endogenous variables are driven by the same non-stationary technological shock. The corresponding statistical common trend representation, developed in Stock and Watson (1988), implies that all the endogenous variables have a common trend. This approach produces, as a by-product, a decomposition into secular (non-stationary) and cyclical (stationary) components which is the multivariate counterpart of the method of Beveridge and Nelson. Let  $w_t$  be an  $n \times 1$  vector of time series,  $w_t = (1 - \ell)y_t$  with moving average representation  $w_t = \delta + C(\ell)\varepsilon_t + B(\ell)z_t$  where  $\alpha' C(1) = 0$ ,  $\varepsilon_t = G^{1/2}v_t$  with  $v_t \sim$  i.i.d.(0, $I$ ) and  $z_t$  is a set of cointegrating vectors. Stock and Watson show that the model implies that

$$x_t = y_0 + A\tau_t = y_0 + \delta t + C(1)\zeta_t, \tag{11}$$

$$c_t = D(\ell)\varepsilon_t, \tag{12}$$

where  $A$  is an  $n \times k$  vector,  $\tau_t = \mu + \tau_{t-1} + \eta_t$ ,  $\eta_t$  is a serially uncorrelated random noise,  $\dim(\tau_t) = k \leq n$ ,  $D_j = -\sum_{i=1}^{\infty} {}_{i+j}C_i$  and  $\zeta_t = \sum_{s=1}^t \varepsilon_s$ . Rather than testing whether there is a cointegrating vector  $z_t$ , I estimate a vector error correction model (VECM) and use one lag of two cointegrating vectors (GNP/consumption, GNP/investment) to obtain estimates of  $\delta, C(\ell)$  and  $\varepsilon_t$ . An estimate of the transitory component is obtained by taking  $\hat{c}_t = y_t - y_0 - \hat{\delta}t - \hat{C}(1)\hat{\zeta}_t$ .

As in the Beveridge–Nelson decomposition, estimates of  $x_t$  and  $c_t$  differ for different specifications of the VECM model (both in terms of the number of

variables and lag length). Here I present results obtained using data on GNP, Consumption, Investment, Hours, Real Wage and Capital and five lags for each variable (COIN in the tables).

### 2.2.3. The Hodrick and Prescott's filter

The Hodrick and Prescott (HP) (1980) filter has two justifications: one intuitive and one statistical.

In the Real Business Cycle (RBC) literature the trend of a time series is not intrinsic to the data but it is a representation of the preferences of the researcher and depends on the economic question being investigated. The popularity of the HP filter among applied macroeconomists results from its flexibility to accommodate these needs since the implied trend line resembles what an analyst would draw by hand through the plot of the data (see, e.g. Kydland and Prescott, 1990).

The selection mechanism that economic theory imposes on the data via the HP filter can be justified using the statistical literature on curve fitting (see, e.g. Wabha, 1980).<sup>1</sup> In this framework the HP filter optimally extracts a trend which is stochastic but moves smoothly over time and is uncorrelated with the cyclical component. The assumption that the trend is smooth is imposed by assuming that the sum of squares of the second differences of  $x_t$  is small. An estimate of the secular component is obtained by minimizing:

$$\min_{\{x_t\}_{t=1}^T} [\sum_{t=1}^T c_t^2 + \lambda \sum_{t=2}^T ((x_{t+1} - x_t) - (x_t - x_{t-1}))^2] \quad \lambda > 0 \quad (13)$$

where  $T$  is the sample size and  $\lambda$  is a parameter that penalizes the variability of trend. As  $\lambda$  increases, the penalty imposed for large fluctuations in the secular component increases and the path for  $\hat{x}_t$  becomes smoother. In this context, the 'optimal' value of  $\lambda$  is  $\lambda = \sigma_x^2 / \sigma_c^2$ , where  $\sigma_x$  and  $\sigma_c$  are the standard deviation of the innovations in the trend and in the cycle.

Users of the HP filter select  $\lambda$  a priori to isolate those cyclical fluctuations which belong to the specific frequency band the researcher wants to investigate. With quarterly data,  $\lambda = 1600$  is typically chosen and the filter leaves in the data cycles of average duration of 4–6 yr. While this approach is meaningful from the point of view of a business cycle researcher, the assumed magnitude of  $\lambda$  is debatable. Nelson and Plosser (1982) estimated  $\lambda$  to be in the range  $[\frac{1}{6}, 1]$  for most of the series they examine. This implies that much of the variability that the Hodrick and Prescott filter attributes to the cyclical component is, in fact, part of the trend. To investigate this possibility I experimented with two values of  $\lambda$ :

<sup>1</sup> Harvey and Jaeger (1993) offer also an unobservable component interpretation of this filter.

a standard one (HP1600 in the tables) and one obtained by assuming that the relative standard deviation of the components is 2 (HP4 in the tables).<sup>2</sup>

In practical terms the procedure involves the solution of a system of  $T$  linear simultaneous equations in  $T$  unknowns, of the form  $\hat{x} = Ay$  where  $x = [x_1, x_2, \dots, x_T]'$  and  $y = [y_1, y_2, \dots, y_T]'$ . An estimate of the cyclical component is obtained from Eq. (1).

Some of the properties of the HP filter when  $T \rightarrow \infty$  and the penalty function is two-sided have been highlighted by King and Rebelo (1993) and Cogley and Nason (1995).

Before proceeding with the analysis it is useful to note that the information used to compute the trend of the series differs with detrending method. While most procedures employ information up to  $T$ , FOD and UC only use the information available at  $t - s$  to compute the trend for  $t - s + 1$ . This should be kept in mind when comparing the outcomes across detrending methods. In addition, because the UC model assumes the presence of both an irregular and a cyclical component, care should be exercised in comparing the properties of  $c_t$  obtained with UC and other methods.

### 3. The properties of the cyclical components

In this section I describe some of the properties of the cyclical components of seven real variables and present plots of the cyclical components of GNP. The analysis of this section is descriptive. The next section discusses more substantive issues.

#### 3.1. The raw data

In this paper I use the logarithms of seasonally adjusted quarterly US time series for the period 1955.3–1986.3. GNP, Consumption, Investment, Hours and Real Wage Compensation are obtained from the Citibase data base. GNP measures Real Gross National Product in 1982 dollars (Citibase name: GNP82), consumption measures consumption expenditure by domestic residents on nondurables and services in 1982 dollars (Citibase names: GSC82 and GCN82), investment measures total fixed investment in plants and equipment plus consumer durables in 1982 dollars (Citibase names: GINPD82 and GCD82), hours measures the total number of hours of labor input as reported by establishment survey data (Citibase name: LPMHU) and the real wage is constructed using

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<sup>2</sup> A previous version of the paper also reported a decomposition where  $\lambda$  was separately estimated for each series by maximum likelihood. Results obtained were intermediate between the two considered here and are not reported.

nominal total compensation of nonagricultural employees (Citibase name: GCOMP) and a measure of price (Citibase name: PUNEW). A quarterly series for the capital stock is reconstructed using the net capital stock (residential and nonresidential) for 1954, the quarterly series for investment and a depreciation rate of 2.5% per quarter. Finally, I also consider a productivity series constructed taking the difference between  $\log(\text{GNP})$  and  $\log(\text{Hours})$ .

While this set of variables is standard in aggregate analyses of the business cycle, different authors have used alternative measures of hours, real wage, productivity and capital. For example, Kydland and Prescott (1990) do not include residential capital in their capital stock series. To assess the sensitivity of the results to choice of series, I examined, in addition to the variables studied here, total consumption and consumption of nondurables only, hours measured by household survey data, real wage measured as output per man-hour in manufacturing and productivity (Citibase name: LBOU). The results for these series are contained in an appendix available on request.

Time plots for the log of the data, their estimated pseudo log spectrum and the estimated pseudo coherence of each series with GNP appear in Fig. 1.<sup>3</sup> Shaded areas in the time series plots indicate recessions according to NBER chronology. Shaded areas in the plots of the spectra and the coherence comprise cycles with periodicity of 2–6 yr.

### 3.2. *The plots*

The plots of the estimates of the cyclical component of GNP, appearing in Fig. 2, provide important visual information on the cyclical characteristics induced by different detrending methods. For example, detrending methods that impose a random walk on the secular component of the series (e.g. FOD, BN and UC) generate low cyclical variability in GNP. At the opposite end LT, MLT and COIN leave the largest variability in the cyclical component of GNP.

Visual similarities also emerge in the time path of several estimated cyclical components of GNP. For example, those obtained with linear and segmented filters look quite similar but have a slightly different mean value; those obtained with BN, FOD and HP4 filters resemble each other and those obtained with  $\text{FREQ1}$  and  $\text{HP1600}$  are almost indistinguishable. Finally, the three multivariate methods produce cyclical components of GNP which are similar to each other and significantly different from those obtained using univariate methods (except, perhaps, LT).

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<sup>3</sup> Pseudo spectra and pseudo coherences are computed knocking out frequency zero and smoothing the periodogram for each series. The elimination of frequency zero is necessary because spectra and coherences do not exist for variables which may contain a unit root.

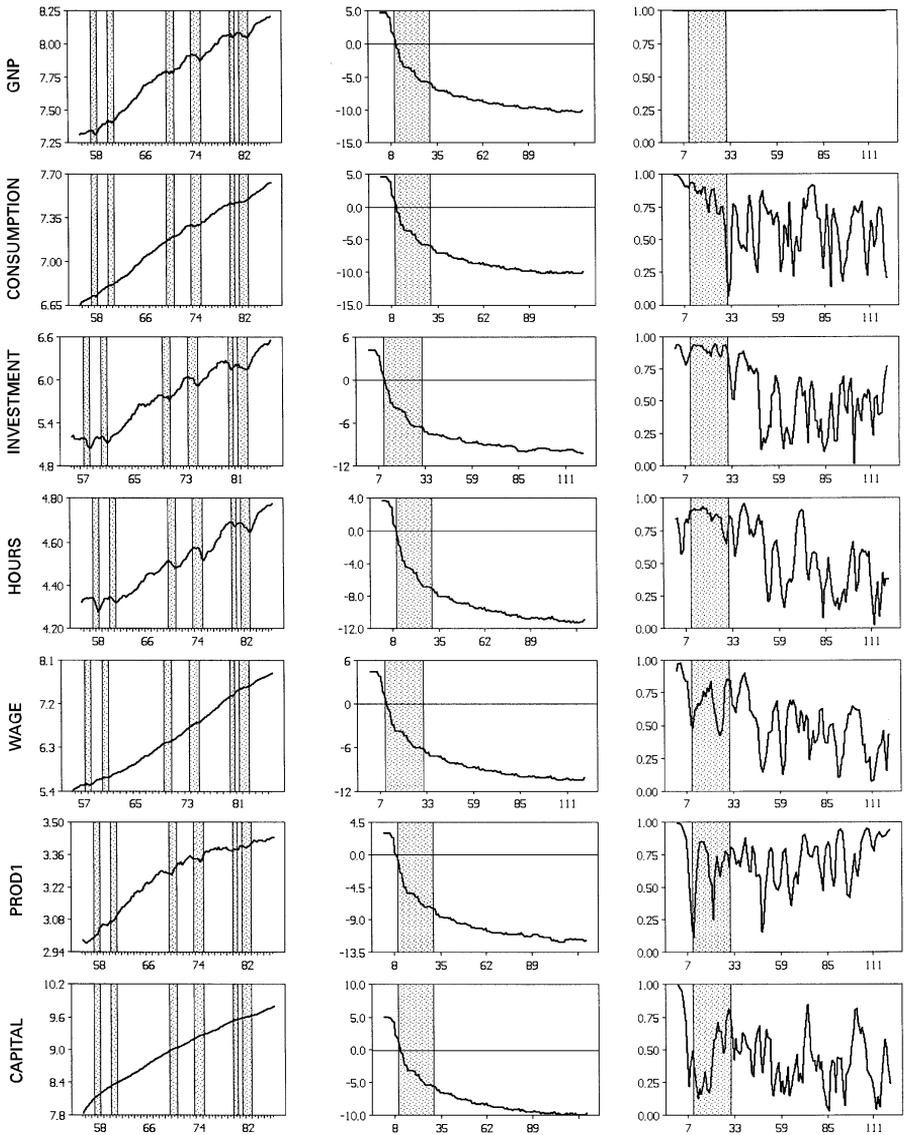


Fig. 1. Time series, spectra and coherence.

In general, three general types of cyclical patterns are present in Fig. 2. With HP1600, SEGM, the frequency domain filters and, to a lesser extent, UC the cyclical component of GNP displays cycles with average duration of 4–6 yr and turning points for expansions and contractions which approximately reproduce

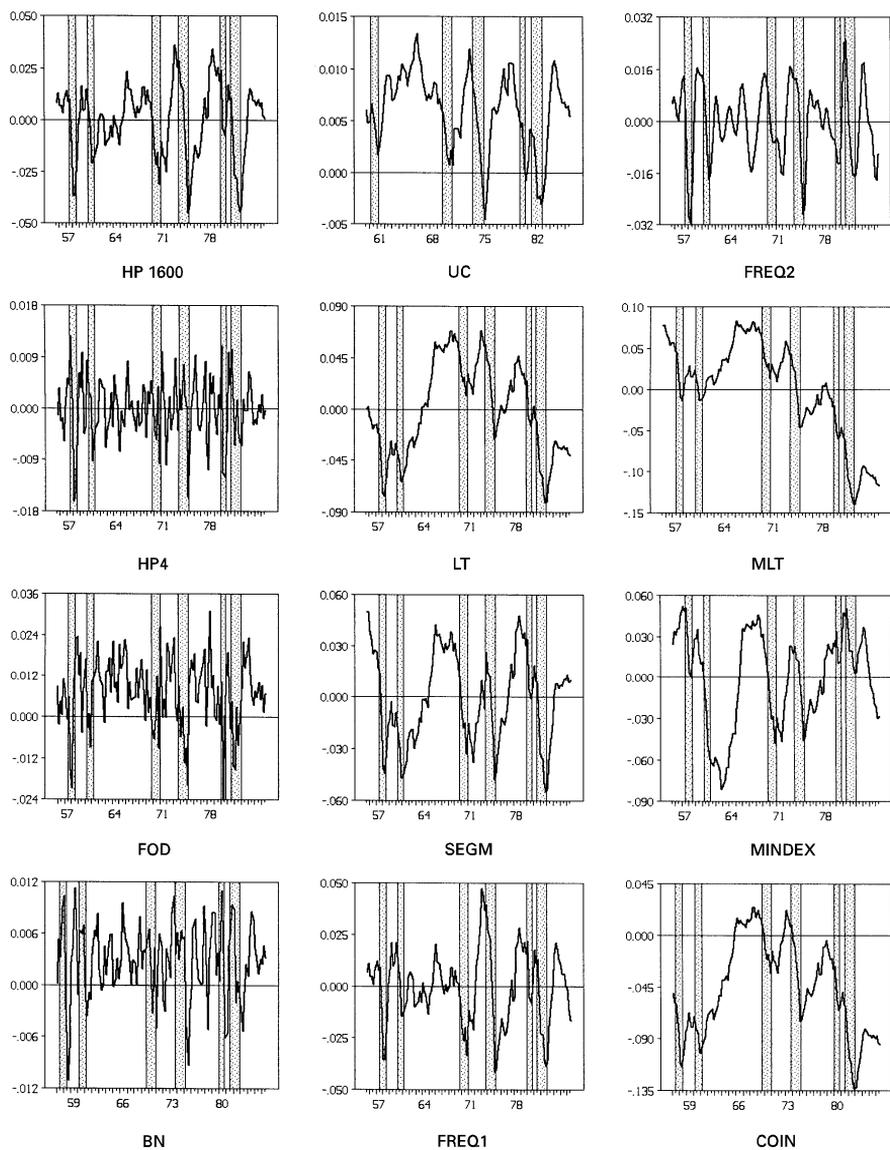


Fig. 2. Cyclical components.

NBER dating. With linear detrending and the three multivariate procedure we see cycles which are generally long (average duration 8–10 yr) and turning points do not correspond to NBER chronology. Finally, methods which impose a unit root on the trend generate cyclical components which are very erratic, display

cycles of short length (average duration 2–3 yr) whose turning points have little agreement with NBER dating.

To obtain additional information on the type of cycles that each method extracts, it is instructive to examine the characterization of the 1979 and 1981–1982 contractions given by each procedure. With most detrending methods the 1979 contraction was mild, i.e. the decline in GNP below its trend was small. In three cases (UC, SEGM and MINDEX) the 1979 contraction appears simply as a slowdown, i.e. the cyclical component of GNP did not cross the trend line in this episode. Finally, with FOD, MLT and COIN, the 1979 contraction was sufficiently severe. However, with MLT and COIN, GNP is below its long-run trend from 1974 up to 1986 and the 1979 contraction appears as a relatively minor incident in that long cycle. For the 1981–1982 contraction all methods but BN and MINDEX locate the trough of the cycle sometime between 1981 and 1982 but there is substantial disagreement regarding its magnitude relative to the trend. With MINDEX the 1981–1982 contraction appears as a minor slowdown, while with BN it shows up as an expansion and the trough of the cycle occurs only in late 1983, when NBER dating indicated that an expansion was well under way.

The plots of the cyclical components of the other six variables have essentially the same features and are not reported for reason of space but are available on request. There are two conclusions that can be drawn from these observations. First, different detrending methods leave cycles of different average duration in the data, some of which are too long and some too short relative to the standard business cycle classification. Second, as a consequence of the above, different detrending methods have different implications for the timing of turning points and the severity of standardly classified contractions (see Canova (1994)).

### 3.3. Summary statistics

To summarize the properties of the cyclical components of the data, I report a few moments of the distribution, various short-term cross correlations and the responses of the variables to a 1% standard error innovation in GNP. Table 1 reports the absolute standard deviations of the cyclical component of GNP and the relative standard deviations of the other six variables, in percentage of GNP standard deviations. Table 2 presents cross correlations at lags  $(-1, 0, 1)$ . In both tables a ‘\*’ indicates that the statistic in the cell differs at the 5% significance level from the statistic obtained with the HP1600 filter.<sup>4</sup>

<sup>4</sup> Under standard regularity conditions outlined, e.g. in Newey and West (1987), the statistics  $J_1 = (\text{var}_x(i) - \text{var}_x(\text{HP1600})) V_1^{-1} (\text{var}_x(i) - \text{var}_x(\text{HP1600}))$  and  $J_2 = (\text{cov}_{x,\text{GNP}}(i) - \text{cov}_{x,\text{GNP}}(\text{HP1600})) V_2^{-1} (\text{cov}_{x,\text{GNP}}(i) - \text{cov}_{x,\text{GNP}}(\text{HP1600}))$  are distributed  $\chi^2(1)$  where  $i$  stands for detrending method,  $x$  for the particular series examined and  $V_1$  and  $V_2$  are the asymptotic covariance matrices of the random variables  $[\text{var}_x(i), \text{var}_x(\text{HP1600})]$  and  $[\text{cov}_{x,\text{GNP}}(i), \text{cov}_{x,\text{GNP}}(\text{HP1600})]$  respectively.

Table 1  
Standard deviations

Method	GNP	Consumption as % of GNP	Investment as % of GNP	Hours as % of GNP	Real Wage as % of GNP	Productivity as % of GNP	Capital as % of GNP
HP1600	1.76	0.49	2.82	1.06	0.70	0.49	0.61
HP4	0.55*	0.48*	2.70*	0.89*	0.65*	0.69*	0.14*
FOD	1.03*	0.51*	2.82*	0.91*	0.98*	0.67*	0.63*
BN	0.43*	0.75*	3.80*	1.64*	2.18*	1.14*	2.64*
UC	0.38*	0.34*	6.72*	4.14*	2.24	4.09*	1.22*
LT	4.03*	0.69*	2.16*	0.69*	1.71*	1.00*	1.56*
SEGM	2.65*	0.52*	3.09*	1.01*	1.10*	0.54*	0.97*
FREQ1	1.78	0.46	3.10	1.20	1.07*	0.66*	1.41*
FREQ2	1.14*	0.44*	3.00*	1.16*	1.11	0.69	1.26*
MLT	6.01*	0.67*	2.36*	0.46*	1.21*	1.00*	1.05*
MINDEX	3.47*	0.98*	2.65*	1.14*	1.27*	0.72*	1.85*
COIN	4.15*	0.71*	3.96*	0.75*	1.68*	1.09*	1.30*

\*Indicates a rejection at the 5% confidence level of the null hypothesis that the variance of the cyclical component of the series is identical to the one obtained using the HP1600 filter.

Table 2  
Cross-correlations

Lag	Methods												
	HP1600	HP4	FOD	BN	UC	LT	SEGM	FREQ1	FREQ2	MLT	MINDEX	COIN	
C-GNP	-1	0.75	0.16*	0.35*	0.23*	0.79	0.90*	0.76	0.75	0.68	0.93*	0.79	0.82*
	0	0.75	0.31*	0.46*	0.42*	0.74	0.91*	0.81*	0.73	0.69	0.96*	0.84*	0.83*
	1	0.62	0.01*	0.21*	0.38*	0.61	0.89*	0.76*	0.57	0.52*	0.93*	0.85*	0.82*
I-GNP	-1	0.76	0.07*	0.25*	-0.08*	0.82	0.73	0.78	0.73	0.58*	-0.26*	0.81	0.28*
	0	0.91	0.65*	0.71*	0.45*	0.85	0.77*	0.86	0.86	0.85	-0.26*	0.84	0.30*
	1	0.84	0.28*	0.39*	0.38*	0.73*	0.76*	0.79	0.80	0.85	-0.26*	0.80	0.31*
H-GNP	-1	0.67	0.11*	0.28*	0.27*	0.01*	0.25*	0.71	0.63	0.48*	0.17*	0.71	0.16*
	0	0.88	0.73*	0.75*	0.72*	0.17*	0.34*	0.85	0.83	0.80	0.22*	0.77*	0.24*
	1	0.90	0.44*	0.54*	0.30*	0.28*	0.37*	0.86	0.85	0.87	0.23*	0.78*	0.27*
W/P-GNP	-1	0.81	0.12*	0.34*	0.05*	0.80	0.89*	0.79	0.73	0.59*	0.79	0.64*	0.89*
	0	0.81	0.49*	0.69*	0.52*	0.85	0.92*	0.88	0.84	0.81	0.82	0.67*	0.91*
	1	0.63	-0.30*	0.42*	0.45*	0.79*	0.90*	0.84*	0.75*	0.79*	0.81*	0.65	0.90*
APL-GNP	-1	0.26	0.12*	0.04*	-0.06*	0.20	0.78*	0.33	0.11*	0.36	0.80*	0.17	0.76*
	0	0.10	0.49*	0.45*	-0.16*	0.06	0.76*	0.25*	-0.02	0.08	0.89*	0.14*	0.74*
	1	-0.25	-0.30	-0.30	-0.09*	-0.07*	0.70*	0.05*	-0.27*	-0.28	0.85*	0.06*	0.68*
H-W/P	-1	0.43	-0.08*	0.39	0.48	-0.16*	0.06*	0.79*	0.53*	0.65*	0.06*	0.19*	0.12*
	0	0.67	0.39*	0.68	0.64	-0.05*	0.10*	0.85*	0.67	0.80*	0.10*	0.23*	0.17*
	1	0.80	0.34*	0.53*	0.21*	0.05*	0.13*	0.86	0.69*	0.73	0.11*	0.24*	0.20*
H-APL	-1	-0.55	-0.36*	-0.37*	-0.13*	-0.95*	-0.38*	-0.41*	-0.67*	-0.73*	-0.25*	-0.53*	-0.49
	0	-0.24	-0.22	-0.24	-0.79*	-0.97*	-0.34	-0.29	-0.55*	-0.52*	-0.23	-0.50*	-0.46*
	1	-0.07	0.12*	0.00	-0.11	-0.89*	-0.26*	-0.13	-0.34*	-0.18	-0.19*	-0.43*	-0.39*

\*Indicates a rejection at the 5% confidence level of the null hypothesis that the correlation coefficient in the cell is identical to the correlation coefficient obtained using the HP1600 filter.

Table 3 displays the estimated coefficients of skewness and Table 4 contains the estimated coefficients of excess kurtosis. A ‘\*’ in these two tables indicates that the Kendall and Stuart (1958) test rejects the null hypothesis that the moment is the same as one of a normal random variable at the 5% significance level.

### 3.3.1. *Standard deviations*

The magnitudes of the standard deviations vary greatly across detrending methods. The absolute variability of the cyclical component of GNP is smallest for UC (0.38) and largest for MLT (6.01) while the HP1600 filter generates, approximately, the median value. Note that those methods which leave cycles of long mean duration in the data typically generate high variability while methods which leave cycles of short mean duration typically induce small variability.

The range of relative variabilities is large as well. Consumption variability ranges between 34% and 98% of the variability of GNP, relative investment variability ranges from 216% to 672% and hours from 50% to 414%. The relative variability of real wage to GNP varies between 65% and 224% and the relative variability of productivity is between 49% and 409%, with the HP filters producing the lowest value in both cases. Qualitatively, the capital stock series displays an almost identical pattern to productivity although the range of relative variabilities is smaller (from 14% to 185%). Finally, hours can be either much less or much more volatile than productivity (ranging from 46% to 212%) (see also Baxter, 1991).

While it is relatively simple to group approaches when the absolute variability of GNP is considered, it is much harder to draw general conclusions regarding relative variabilities. For methods which extract cycles of short mean duration, no regularity seems to appear. For those methods which emphasize cycles of medium mean duration three features warrant mention. First, the magnitude of relative variabilities of HP filtered series are among the lowest, regardless of the value of the smoothing parameter employed. Second, the ordering of relative variabilities obtained with UC and FREQ filters differs substantially from those obtained with HP filters, with consumption, hours, real wage and productivity being the most affected. Third, the relative variabilities generated with FOD are close to those obtained with the HP1600 and HP4, confirming some of the properties of the two filters described by King and Rebelo (1993). Finally, the size and ordering of relative variability is more coherent across methods which emphasize longer cycles (say 8–10 yr). For example, hours are always less volatile than GNP and productivity while investment is about twice as volatile as GNP.

### 3.3.2. *Cross correlations*

The cross correlations of the cyclical components are also very sensitive to detrending. For example, the contemporaneous cross correlation of consumption and GNP varies from 0.31 to 0.96 and that of hours and GNP varies from

Table 3  
Skewness

Method	GNP	Consumption	Investment	Hours	Real Wage	Productivity	Capital
HP1600	-0.024	-0.034	-0.367	-0.400*	-0.310	-0.235	-0.247
HP4	0.174	0.196	0.058	0.303	0.065	-0.186	0.082
FOD	-0.045	-0.322	-0.367	-0.328	0.048	0.006	-0.351
BN	-0.243	-0.141	-0.402*	0.326	-0.165	-0.415*	-0.259
UC	-0.028	-0.207	-0.342	-0.179	0.155	0.384	-0.220
LT	-0.114	-0.253	-0.460*	-0.389	-0.059	0.143	-0.320
SEGM	0.086	-0.322	-0.459*	-0.350	0.050	0.085	4.490*
FREQ1	-0.048	0.090	-0.316	-0.310	-0.209	-0.147	-0.187
FREQ2	0.156	0.056	-0.104	-0.252	0.026	0.139	0.584*
MLT	-0.210	-0.283	-0.478*	-0.385	-0.032	0.038	-0.320
MINDEX	0.125	-0.269	-0.309	-0.275	0.022	0.193	0.383
COIN	-0.146	-0.239	-0.423*	-0.376	0.188	0.025	-0.226

\*Indicates a rejection at the 5% level of the null hypothesis that the value of the skewness in each cell is identical to the value appearing under normality.

Table 4  
Excess kurtosis

Method	GNP	Consumption	Investment	Hours	Real Wage	Productivity	Capital
HP1600	0.066	-0.077	1.382*	0.953	0.613	-0.068	0.949
HP4	-0.131	-0.616	0.512	0.455	0.115	0.134	0.395
FOD	-0.222	-0.220	0.788	0.111	0.063	-0.050	0.660
BN	-0.026	-0.576	0.906	0.087	0.126	0.560	0.625
UC	-0.153	-0.568	0.781	-0.050	0.630	0.062	0.758
LT	0.206	0.162	1.089*	0.938	0.575	-0.269	1.051*
SEGM	0.438	0.428	1.002*	0.744	0.769	0.605	38.08*
FREQ1	-0.068	0.490	1.336*	0.649	0.671	-0.191	0.517
FREQ2	0.464	-0.265	0.234	-0.052	-0.126	-0.518	1.570*
MLT	0.120	0.041	0.935	0.938	0.497	-0.259	1.051*
MINDEX	-0.048	-0.189	0.829	0.641	0.553	-0.477	0.599
COIN	-0.065	0.064	0.906	0.561	0.653	-0.264	0.854

\*Indicates a rejection at the 5% level of the null hypothesis that the value of the excess kurtosis in each cell is identical to the value appearing under normality.

0.17 to 0.88. Even more striking is the range of cross correlations between productivity and GNP which varies from  $-0.16$  to about  $0.75$  and of hours and the real wage, from  $-0.05$  to  $0.85$ . Similarly, there is a wide range of cross correlations between productivity and past GNP (range  $-0.06$ – $0.80$ ) or real wage and past GNP (range  $0.05$ – $0.89$ ). In general, the largest range in the lead and lag correlations occurs for hours and GNP while the smallest range occurs for consumption and GNP. In some cases, e.g. the contemporaneous relationship between productivity and GNP, it is hard even to sign the correlation with sufficient accuracy.

Among detrending methods, the HP1600 filter produces the highest contemporaneous correlation between hours and GNP and investment and GNP. In fact, most of the contemporaneous correlations with GNP obtained with the HP1600 filter are significantly larger than those obtained with other methods (the exception are data detrended with frequency domain methods) and the hypothesis that the two sets of correlations are identical is frequently rejected. Hence, even among those methods extracting cycles which approximately cover the standard business cycle periodicity (4–6 yr), the magnitude or even the sign of various correlations differs substantially.

### 3.3.3. *Higher moments*

Current work cataloging properties of business cycles<sup>5</sup> typically reports only second moments. Linger in the background are one of two assumptions: either that the series are zero mean normal stochastic processes so that second moments summarize all that is contained in the data or that higher moments do not carry crucial information about the cyclical properties of the data. Recent work by Neftci (1984), Falk (1986), DeLong and Summers (1986) and Pfann (1991) have considered higher moments in an attempt to detect asymmetries or fat tails in the distribution of the cyclical components of GNP and employment. Here I study the third and fourth moments to (i) examine the sensitivity of estimated higher moments to detrending and (ii) indicate whether any detrending procedures induce significant distortions in the properties of the data.

Perhaps surprisingly, the estimated skewness has similar properties across detrending methods for 5 of 7 series and the estimated excess kurtosis has similar properties for almost all detrending methods for 4 out of 7 series. The major discrepancies occur with the investment series, which is strongly left skewed with 5 methods (BN, LT, SEGM, MLT, COIN) and leptokurtic with 4 methods (HP1600, LT, SEGM, FREQ1) and with the capital series which is leptokurtic

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<sup>5</sup> Examples include Kydland and Prescott (1990) or Stock and Watson (1990) for the US, Englund et al. (1991) for Sweden, Danthine and Girardin (1989) for Switzerland, Brandner and Neusser (1992) for Germany and Austria, Blackburn and Ravn (1991) for European countries, Backus and Kehoe (1992) for G-10 countries and Fiorito and Kollintzas (1994) for the G-7.

in 5 cases. To understand the differences note that all methods but *FREQ2* generate both negative skewness and positive excess kurtosis, although their magnitudes vary. Because *FREQ2* eliminates high-frequency variability, the skewed and leptokurtic behavior of investment is primarily due to irregular fluctuations rather than to business cycle movements. Note also that the leptokurtic behavior of the capital stock appears only with those methods which leave medium-long cycles in the data. Finally, for *LT*, *SEGM*, *HP1600* and *FREQ1* detrended data the assumption that the cyclical component is normal is clearly inappropriate.

The size of the distortions induced by detrending can be evaluated by comparing the skewness and the excess kurtosis obtained before and after detrending.<sup>6</sup> For the original data all series are slightly left skewed but the coefficient of skewness is never significantly different from zero. Investment, real wage and capital, on the other hand, display marginally significant leptokurtosis. Hence, although different detrending methods induce very different second moments in the cyclical component of the data, they appear to leave the higher-order properties of the original series intact.

#### 3.3.4. *Impulse responses*

Another statistic typically examined to study the propagation of cyclical shocks is the impulse response function (IRF) when cyclical GNP is shocked by one standard deviation. Here I perform the exercise using a VAR system which includes the cyclical component of six variables (GNP, Hours, Real Wage, Consumption, Investment, and Capital). Because the IRF is a linear transformation of the data, the results for the average productivity can be read off directly from the responses of GNP and Hours. The lag length of the system is method dependent and is chosen so that the innovations satisfy the white noise assumption. Some detrending methods induce near MA unit roots in the estimates of the cyclical components so for some decompositions very long lags are needed to whiten the residuals.

Because I will concentrate on the responses of the system to a shock in cyclical GNP, I will not attempt a behavioral identification of the system. While this may be problematic when it comes to study the structure of the dynamic interrelationship across variables, it is not so crucial when the task is to compare the properties of the cyclical components obtained with various methods through a particular window, regardless of the fact that it is misspecified or not. In addition, the identification of a disturbance in the cyclical component of GNP only requires one restriction while the identification of a fully behavioral

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<sup>6</sup> Since the tests for skewness and excess kurtosis are invalid in the presence of serial correlation, both the original and the filtered series are prewhitened with 12 lags before the statistics are computed.

Table 5  
Summary statistics for the impulse response function

Method	Cycle length	Size and location of the peak response											
		Con-sumption		Invest-ment		Hours		Real Wage		Produc-tivity		Capital	
HP1600	20*	2	0.28	3	1.93	3	0.76	1	0.46	1	2.00	6	0.30
HP4	8	1	0.17	1	1.50	1	0.58	1	0.37	1	1.70	2	0.05
FOD	6	1	0.25	1	1.50	1	0.57	1	0.53	1	1.82	4	0.11
BN	8	1	0.30	1	2.10	1	1.24	1	1.42	1	0.84	5	2.18
UC	21	1	0.23	1	6.02	5	2.38	4	1.18	1	0.98	6	0.54
LT	48	3	0.26	3	1.80	3	0.79	3	0.56	1	2.03	44	0.54
SEGM	19	1	0.24	4	1.86	3	0.77	1	0.52	1	2.02	6	0.36
FREQ1	17	4	0.30	3	1.98	3	0.82	3	0.60	1	2.10	4	0.31
FREQ2	12	4	1.12	4	10.20	4	3.25	4	2.08	4	2.06	25	1.53
MLT	48	2	0.26	2	1.84	3	0.79	3	0.52	1	1.83	7	0.28
MINDEX	39	2	0.28	4	1.91	2	0.81	3	0.55	1	1.67	44	0.36
COIN	24	3	1.32	4	6.23	4	3.18	6	1.78	1	0.74	10	0.42

\*Note: Cycle length measures the span of time, in quarters, needed to complete a cycle in GNP. If multiple peaks occur, size and location refer to the first peak.

system necessitates many, possibly debatable, restrictions. The assumption I use to identify an innovation in cyclical GNP is that, within a quarter, no shock other than its own affects cyclical GNP. Table 5 reports summary measures of the IRF. Fig. 3 plots the IRF for HP1600 and COIN detrended data.

The properties of the IRF differ across detrending methods in several respects. First, the average duration of a GNP cycle in response to a GNP shock varies with detrending procedure. For example, the average cycle is about 3.5 yr with the HP1600 filter and about 1 yr with the FOD filter. Second, the response of investment has varying degrees of persistence: it is zero after 4 quarters when FOD is used while it is still sizable after 24 quarters with UC detrended data. Third, the size of the peak responses in consumption and investment is method dependent. For example, the peak response in consumption varies from 0.17 (with HP4) to 1.3 (with COIN) of the shock in GNP and peak investment response varies from 1.5 (with HP4 and FOD) to about 10.5 (with MINDEX). Finally, the timing of the peak responses falls into two categories. In the first category, which includes most univariate methods (both HP filters, RW, BN, LT and FREQ1), a shock to GNP produces a peak response in GNP and real wage instantaneously, a 1–2 quarters lagged peak response in investment, a 2–4 quarters lagged peak response of consumption and hours, and 4–6 quarters delayed peak in capital. The exact timing of the peak response in hours

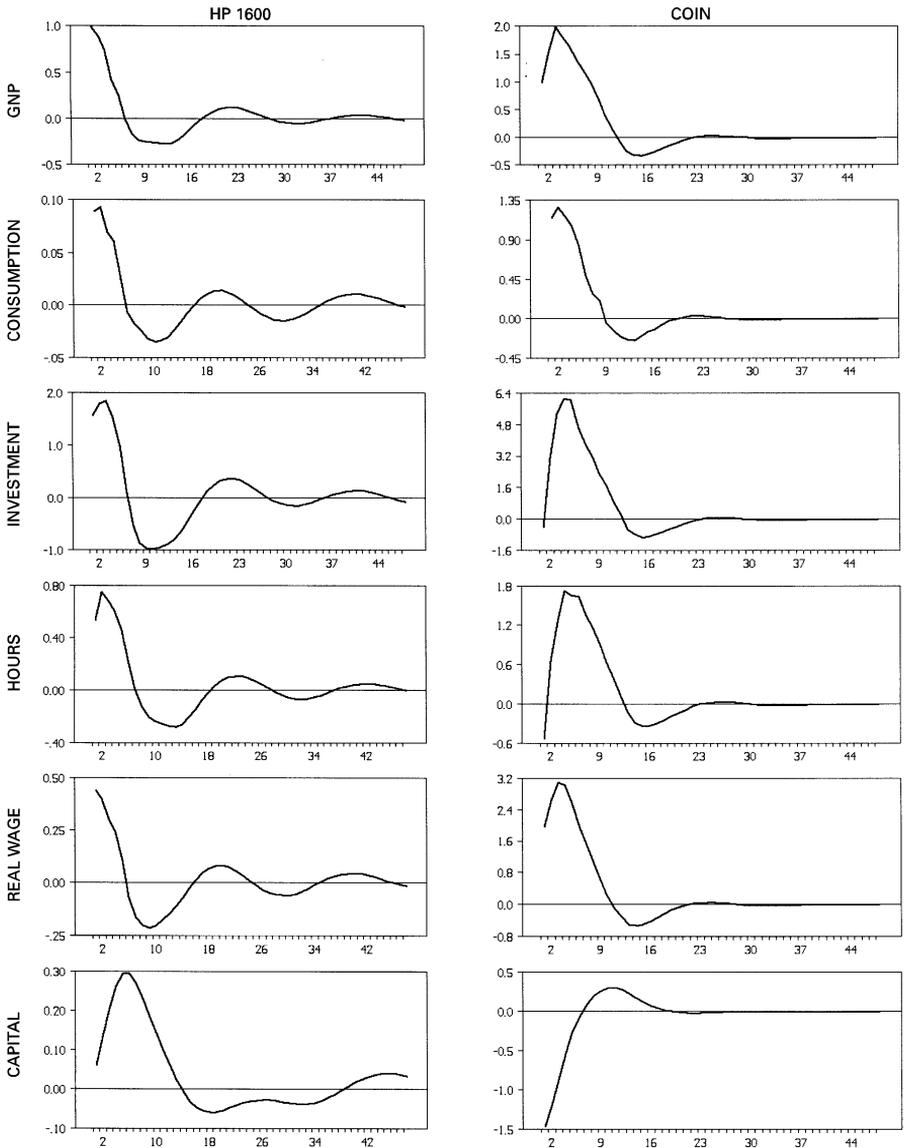


Fig. 3. Impulse responses.

constitutes the major difference among these methods, although the longest delay does not exceed 4 quarters. In all cases but UC, the size of the instantaneous response in productivity is always greater than the size of the instantaneous response in real wages.

In the second category which includes COIN and MINDEX, GNP and the real wage display a peak response which lags the initial shock by 4–6 quarters, the peak in hours lags 2–3 quarters, the peaks in consumption and investment lag 2–4 quarters and the peak in capital about 10 quarters. Here the magnitude of consumption responses exceeds the magnitude of GNP responses over the first 2–3 quarters of the cycle, the immediate response of investment and capital is negative and the response of productivity is negative, at least in the first few quarters of the cycle.<sup>7</sup> Finally, the size of the peak response in all variables but capital exceeds the size of the disturbance in GNP.

To summarize, the results show that qualitatively and quantitatively the second-order properties of the data and the transmission mechanism of a shock in cyclical GNP depend on the detrending procedure used. However, higher moments of the cyclical component of the data are broadly insensitive to the choice of detrending. I conclude that, except in a few cases, a quantitative assessment of the relationship across the seven variables is method dependent and even qualitatively there is not one single set of facts. Different detrending methods imply different sets of economic relationships because they generate different economic concepts of the business cycle. Moreover, even within the class of methods which extract cycles with durations close to the conventional 4–6 yr periodicity, several qualitative differences emerge.

In the next section I discuss the implication of these findings for some stylized facts of the business cycle. In particular I examine the evidence concerning the relative volatilities of consumption, productivity and GNP, the cross correlation of productivity, hours and GNP and of real wages and hours and discuss what the evidence on the transmission of GNP shocks tell us about sources of business cycle fluctuations.

## 4. Some stylized facts of the business cycle revisited

### 4.1. *Relative variabilities*

A number of stylized facts of the business cycle are stated in terms of the magnitude of the relative variability of one variable to GNP. For example, Kydland and Prescott (1990) or Backus and Kehoe (1992) suggest that consumption is less volatile than output. The relative volatility of consumption to GNP is also crucial for tests of the permanent income hypothesis. Deaton (1987), for example, indicates that if GNP has a unit root, consumption is too smooth to be consistent with the permanent income hypothesis and this result

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<sup>7</sup> The negative contemporaneous response of investment to a shock in GNP has also been found by Warne and Vredin (1991) using a COIN filter on Swedish data.

has spurred substantial work in an attempt to rationalize this finding (see, e.g. Quah, 1989).

Qualitatively speaking, Table 1 indicates that consumption is uniformly less volatile than output. However, a quantitative statement on the size of the relative variability is difficult: the range is between 0.34 and 0.98. Among the methods which impose or allow for a unit root in GNP, Deaton's paradox holds, i.e. consumption tends to be less volatile than output. However, in at least three cases the relative variability exceeds 0.7 and in one case is 0.98. Hence, even within this class of procedures whether consumption is excessively smooth or not depends on detrending and with many methods the paradox is less dramatic than previously thought.

The relative variabilities of productivity to GNP and to hours are two commonly used statistics to gauge the state of labor markets over the cycle. Prescott (1986) (Table 1) claims that the variability of productivity is less than the variability of GNP. Mankiw (1989) (p. 86) claims that 'Over the typical business cycle, employment varies substantially while determinants of the labor supply – the real wage and the real interest rate – vary only slightly'.

Because the existing literature has measured productivity in different ways, I experimented with two alternative measures. For 7 of the 12 methods I find that a standard measure of productivity is less volatile than GNP and with four methods it is, approximately, as volatile. When the real wage is used in place of productivity (see Burnside et al. (1993) and next subsection for some arguments which may justify this switch) its relative variability exceeds that of GNP in 9 out of 12 cases.

To try to account for both the differences between productivity and real wage and the variety in the outcomes it is useful to examine the spectra of GNP and of these two variables (see Fig. 1). It turns out that productivity is significantly more volatile than GNP in those frequencies corresponding to cycles of 8–10 yr length and significantly less volatile than GNP for cycles of 4–6 yr. This variability is eliminated from the cyclical component extracted with methods which emphasize cycles of medium or short average duration (like HP and FOD), but it appears intact with methods like LT which emphasize cycles of this length. The case of real wage is somewhat different since, quantitatively speaking, the proportion of the variability of the real wage series in the region corresponding to 4–10 yr cycles is slightly but uniformly larger than the proportion of the GNP series. This implies that differences across detrending methods are less marked although filters like HP and FOD, which carve out only a portion of this region, produce a smaller relative variability relative to other methods.

One consequence of these results is that the relative variability of hours to productivity depends both on the measure of productivity used and on the detrending method. For example, a standard measure of productivity is more

volatile than hours for those methods which leave long cycles in the cyclical component (LT, MLT or COIN methods). When the real wage is used, results are mixed and unrelated with the type of cycles each method extracts.

Two general points need to be emphasized here. By focusing on a precise concept of cycle (for example, 4–6 yr periodicity) and selecting those methods which primarily extract these cycles, it is possible to produce a more uniform view about the size of relative variabilities. However this approach need not be satisfactory because it neglects valuable information included in cycles of slightly different duration. This is particularly evident in the case of productivity where a substantial portion of variability lies outside the commonly defined business cycles frequencies.

#### 4.2. *Procyclical productivity*

A second set of stylized facts of the business cycle comes in the form of comovements across variables. Relationships which have attracted the attention of researchers include the correlations among productivity, real wage, hours and GNP. In this subsection I examine the question of the procyclicality of productivity. The existing literature has found evidence of countercyclicality (Chirinko, 1980), of acyclicity (Geary and Kennan, 1982), and of procyclicality (Barsky and Solon, 1988; Waldman and Delong, 1991) of productivity. Whether productivity is procyclical or not has important implications for the functioning of the labor market over the business cycle. Procyclicality is, in fact, consistent with the idea that labor demand has shifted in response to shifts in the production function. Countercyclicality suggests that shifts in the supply of labor are the primary source of disturbances in the labor market.

In examining this relationship, it is common to interchange the real wage and productivity (see, e.g. Prescott (1986), McCullum (1989) or Bernanke and Parkinson (1991)). In a competitive world the real wage equals, in equilibrium, the marginal product of labor (MPL). Because productivity here measures the average product of labor (APL) the equality need not hold. Christiano and Eichenbaum (1992) argue that using APL in place of real wages is a reasonable approximation as one should expect the equality to hold on average, not on a period by period basis. In addition, since in many models MPL and APL are proportional, the results should be approximately similar.

As expected from the discussion of Section 4.1, this substitution is problematic. When a measure of real wage is used, procyclicality appears with each detrending method and the magnitude of the correlation is consistently above 0.5. When a measure of productivity is used the magnitude of the correlations is, in general, much smaller (the mean value around 0.10), the range of values is very large and in two cases the correlation is negative, albeit small (BN and *FREQ1*).

With those methods which extract cycles of 4–6 yr average periodicity one gets the impression that productivity is acyclical.<sup>8</sup>

To explain the differences it is useful to examine the coherence among pairs of series (see Fig. 1). While the correlation between real wage and GNP is approximately constant over a large band of frequencies up to cycles of about 8 quarters, the magnitude of the correlation coefficient between productivity and GNP is very different by frequency: it is low in the region corresponding to 6–8 yr cycles and to 4 yr cycles and high in the region corresponding to 4–6 yr cycles. Because of this uneven behavior different detrending methods, even those which extract cycles of similar duration, produce different results.

In sum, several conclusions can be drawn. First, the identification of the average productivity with the real wage may lead to serious inconsistencies. The existence of noncompetitive aspects may be one reason for the divergence (see, e.g. Bernanke and Parkinson, 1991). Second, within a wide range of business cycle frequencies, the real wage is procyclical and highly correlated with GNP. Third, the magnitude and even the sign of the correlations of productivity with GNP depend on detrending and this is true even for methods which extract cycles of similar length (see also McCullum (1989)). This result therefore strengthens the idea that productivity and GNP have economic cycles with different features, variability and durations and elicits the need for theoretical work to provide reasons for why this phenomenon occurs.

#### 4.3. *The Dunlop–Tarshis puzzle*

A recurrent anomaly in the business cycle literature is the so-called Dunlop–Tarshis paradox, i.e. the fact that the correlation between the return to working and the numbers of hours worked is very small. Kydland and Prescott (1988), for example, report that the contemporaneous correlation between a measure of hours and the real wage is approximately zero when HP1600 detrended data are used. Many models, both in the neoclassical and Keynesian tradition fail to account for this observation. Because both types of models share the assumption that real wages and hours worked are on a fixed downward sloped marginal product of labour schedule, real wages and hours worked should be strongly negatively correlated. On the other hand, current RBC models driven by technology shocks, generate procyclical movements in hours and real wage via cyclical shifts in the production function. The response to the discrepancy between theory and the data has been of two types. Kydland and

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<sup>8</sup> The results obtained with the alternative productivity series presented in the appendix show less heterogeneity. All correlations are in fact positive even though the range is still large. With the alternative measure of wages significant countercyclical behavior emerges in 3 cases (LT, MLT, COIN).

Prescott, for example, suggest that measurement errors may be important and attempt to reconstruct a real wage series which is free from these errors while Christiano and Eichenbaum (1992) have modified existing RBC models to generate a theoretical correlation between hours and real wage which is approximately zero.

Table 2 shows that when real wage is used the correlation is almost always positive and greater than 0.40 in half of the cases. When a standard measure of productivity is used the correlations are all negative and in 5 cases smaller than  $-0.50$ .<sup>9</sup> Note that the correlation obtained with HP1600 detrended data ( $-0.24$ ) is very similar to the one reported by Christiano and Eichenbaum (1992) ( $-0.16$ ) even though they use a different hours series. Also, only with the real wage and LT, MLT and UC detrended data is the correlation statistically close to zero.

The sign change occurring when APL is used in place of the real wage is easy to explain. In many cases, productivity is countercyclical up to the mid 1960s (the range is  $[-0.32, 0.03]$ ) and procyclical afterwards but the negative sign obtained in the first part of the sample dominates.

The sign of the correlation between real wage and hours is surprisingly robust across detrending methods. The association is strong for cycles with 4–6 yr average duration and it is weaker for cycles of 8–10 yr or less than 4 yr duration but the correlation is positive and significant, a result which is entirely consistent with the idea that shifts in the production function may drive the business cycle in labor markets. The strength of the association between productivity and hours shows no clear pattern. It appears to be unrelated to the duration of the fluctuations each method extracts and, even for fluctuations included in the standard definition of business cycle, differences are significant.

Although the patterns I have described may be the consequence of measurement errors and sampling uncertainty in the hours series (see, e.g. Christiano and Eichenbaum (1990b) (appendix)), the results suggest that the Dunlop–Tarshis paradox seem to be less of a puzzle than previously thought: a small and insignificant association between productivity (or real wage) and hours occurs in only a few cases. The sign of the correlation, however, depends on whether real wage or productivity is used and on the sample period, while the strength of the association depends, to some extent, on the economic concept of cycle employed.

#### 4.4. *Labor hoarding*

The final stylized fact I examine is the relationship between productivity and lagged measures of economic activity. Some authors (e.g., Summers (1986) and

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<sup>9</sup> When the alternative measure of real wage is used all correlations exceed 0.50, while with a more direct measure of productivity the range of correlations is  $[-0.43, 0.82]$ .

McCullum (1989)) have claimed that a negative correlation indicates the presence of labor hoarding, i.e. because of hiring and firing costs, firms adjust their workforce slowly and the cyclical behavior of productivity primarily reflects the cyclical behavior of output (see Rothemberg and Summers, 1990).<sup>10</sup> In examining this relationship, a further complication to the choice between productivity and real wage measures arises because some authors have used hours in place of GNP as an indicator of cyclical activity (see, e.g. Burnside et al., 1993).

At first glance, Table 3 suggests that whether labor hoarding is present or not depends on what measures of productivity and cyclical activity are used and on what detrending method is employed. For example, when the standard measure of productivity is used the sign of the correlation ( $APL_t, GNP_{t-1}$ ) is almost equally split between positive and negative values, while when the real wage is used, it is mainly positive and significant. When hours are used as an indicator for cyclical activity, the correlation with productivity is always negative, while the correlation with real wage and lagged hours is almost always positive.<sup>11</sup> Note also that for each pair of variables different detrending methods produce a wide range of outcomes contributing to the impression that the qualitative relationship between lagged productivity and GNP is frequency dependent.

In order to gain some intuition for why the descriptions of the phenomena contrast, it is useful to study the differences in one set of correlations across detrending methods. This exercise allows us to further highlight some features of various detrending filters and stress that a simple theoretical characterization of labor hoarding phenomena may suggest which class of detrending methods should be used. When hoarding labor firms must compare the costs of keeping idle workers with the benefits of not having to rehire and retrain new workers when demand picks up. These costs increase if the current recession is expected to persist for a long time. Therefore, even if labor hoarding is an important economic phenomena, it is unlikely to be detectable with methods which extract long cycles in the data. If one hopes to find evidence of labor hoarding via the simple correlation measure employed here, one should look for detrending methods which emphasize short cyclical fluctuations (say 1–3 yr), where this phenomenon may be prevalent.

Among the available methods there are two procedures which emphasize this type of cycle: HP4 and FOD. These procedures give, regardless of the pair of variables used, a negative although small lagged correlation with real activity

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<sup>10</sup> Although the intuition is simple, the mechanics of signing this coefficient is somewhat obscure. In particular, it seems necessary to assume that output is mean reverting to obtain a negative sign.

<sup>11</sup> The use of alternative measures of productivity, real wage and hours does not clarify the qualitative features of the relationship. The other measure of productivity is positively correlated with lagged GNP and with hours in half of the cases, while the other measure of real wage is positively correlated with lagged GNP in 9 of the 12 cases, and with hours in all but one case.

(around  $-0.30$ ), a result which is consistent with labor hoarding. For filters which extract cycles of medium length (UC, HP1600, *FREQ1* and *FREQ2*) the correlation is still negative but closer to zero, while for *SEGM* it is positive but only marginally so. Finally, filters which extract long cycles (*LT* and the three multivariate methods) induce positive correlation between productivity and lagged GNP, regardless of the productivity measure used.

Two conclusions can be drawn from the above discussion. First, because the labor hoarding hypothesis restricts the type of cycles to be examined, one should focus attention on those methods which describe the relationship within the acceptable band of fluctuations. Second, although the difference is not large, the sign of the correlation changes as we move from short to medium to long cycles. This suggests the presence of instabilities within business cycle frequencies but this pattern is revealed only when the analysis is conducted with several detrending methods. For the case of labor hoarding, this instability conforms with economic intuition. For other cases, e.g. productivity, switches of this type warrant careful theoretical examination.

#### *4.5. Is the cycle driven by supply or by demand?*

I conclude this section by examining the implications of the patterns of impulse responses for questions concerning the generation of cycles. Impulse response analysis is becoming increasingly popular in non-structural analyses of business cycles (see, e.g. Stock and Watson, 1990), in semistructural ones (e.g. Ahmed et al., 1993) or in completely structural ones (see, e.g. King et al., 1991). The exercise I conduct is only suggestive because I do not attempt a complete identification of the behavioral disturbances of the system. However, it may be useful in two respects. First, to stress the fact that the relationship among variables at different business cycle frequencies may be consistent with contrasting theories of business cycle fluctuations. Second, to warn users of impulse response analysis against informally linking reduced form evidence to theories taking one concept of cycle as if it was the ‘correct’ one.

The first pattern of responses discussed in Section 3.3.4 seems to fit a RBC tale: a temporary shock to output increases labor demand, so that hours and the real wage go up within a year’s time. As the real wage increases, consumption increases and investment follows. Since the average productivity increases more than the real wage, profits increase and payments to holders of capital rise as well (average product of capital =  $\text{GNP}/\text{capital}$  is positive in the first stages of the cycle). Therefore the real return per unit of capital invested increases. This increase is correlated with the increase in hours. Hence hours move together with this measure of the real rate of return, a result which is consistent with the RBC emphasis on intertemporal substitution of labor. In addition, the responses of productivity are approximately coincident with those of GNP, a result which goes against the labor-hoarding explanation of business cycle fluctuations.

The second pattern of responses, on the other hand, fits a neoKeynesian perspective better. A one standard error shock in GNP instantaneously increases consumption by about 1.2 times that amount and, because of wealth effects, decreases the amount of hours worked. To achieve this consumption increase, the economy depletes the capital stock. At least in the first phase of the cycle, the response of the average productivity of labor is negatively related to (and lags) output responses, a pattern which fits the labor-hoarding story discussed in Section 4.4. The demand driven expansion caused by the increase in consumption induces a further increase in output in the short run, possibly through the use of idle capacity or overtime and this drives hours and real wages up. When the consumption boom is exhausted, previous decisions are reverted: agents enjoy increasing amounts of leisure pushing hours below their long-run path in the medium run, investments decrease and the deterioration of the capital stock is reversed. The reconstruction of the capital stock is completed in about 8 quarters and convergence to its steady-state path occurs after about 15 quarters. Finally, because the capital stock is countercyclical, the real interest rate is large and positive in the first few quarters of the cycle. Despite large interest rates and real wage movements, hours move, relatively speaking, only by a small amount, a result which agrees with recent neoKeynesian descriptions of the business cycle (see, e.g. Mankiw, 1989).

## 5. Conclusions and implications for macroeconomic practice

In this paper I examine how different detrending methods affect the cyclical properties of some US real variables. I compare the properties of the cyclical components of seven variables (GNP, Consumption, Investment, Hours, Real Wage, Productivity and Capital) obtained using seven univariate (Hodrick-Prescott (HP), Beveridge-Nelson (BN), Linear (LT), Segmented (SEGM), First-Order Differencing (FOD), Unobservable Components (UC), Frequency Domain Masking (FD)) and three multivariate (Common deterministic trend (MLT), One-dimensional index (MINDEX) and Cointegration (COIN)) detrending techniques for seasonally adjusted data over the sample 1955–1986. For each method I report moments of the data, the short-term cross correlations and the impulse response function of the seven variables when GNP is shocked.

The paper documents a wide range of outcomes with little agreement in both the quantitative and the qualitative properties of the second moments, even among those methods which extract cycles of comparable duration from the data. Higher moments are less sensitive to the issue of detrending but these statistics are seldom considered by business cycle researchers. We also argue that the qualitative response to a GNP shock can result in two broad patterns which provide different characterizations of the transmission mechanism of shocks. The paper also discusses the implications of the results for selected

stylized facts of the business cycle. Here I show that although in certain situations theory suggests the type of cycles the applied analyst should investigate, in many occasions it is silent. In this case focusing the analysis on one type of cycle only throws away information which can be used to establish interesting observations or refute existing theories.

A few conclusions can be drawn from the exercise. First, the practice of solely employing the HP1600 filter in compiling business cycle statistics is problematic. The HP1600 filter produces results which are similar to those obtained with conventional band-pass filters (e.g. frequency domain masking the low frequency components of the data or standard MA filters) and concentrates the attention of the researcher on cycles with an average duration of 4–6 yr. However, there are instances where selecting cycles with this particular duration may inappropriately characterize a phenomenon (e.g. labor hoarding), throw away a large portion of the variability of a series (e.g. productivity) or induce extreme second-order properties in the detrended data and misdirect theoretical research trying to cope with them (see, e.g. Hansen's (1985) effort to remedy Kydland and Prescott's (1982) failure to replicate the variability of hours or Christiano's (1988) attempt to replicate the magnitude of investment volatility). Second, the idea that there is a single set of facts which is more or less robust to the exact definition of business cycle is misleading since different concepts of business cycle generate different economic objects which need not have similar characteristics. Sweeping these differences under the rug may lead to sterile discussion, inconsistencies in the characterization of the relationship among economic variables and misplaced emphasis on particular cyclical components. Our recommendation for empirical practice is to compile statistics using a variety of shrewdly selected detrending methods so as to gain information on the behavior of variables at different business cycle frequencies and pursue a more interactive relationship between theory and practice. Theory may indicate which concept of cycle is the object of research and therefore implicitly dictate a class of detrending procedures and empirical practice should indicate whether this choice leaves out important features of the data or produces distortions of various kinds.

Third, the empirical characterization of the business cycle obtained with multivariate detrending methods which have their base in dynamic economic theory is different from the one obtained with statistically based univariate procedures. However, since there is very weak evidence of common (deterministic or stochastic) trends, at least with the data set used here, caution should be exercised in deriving business cycle regularities or structural conclusions regarding sources and propagations using theoretical restrictions which are far from being satisfied in the data.

Fourth, since both the quantitative and qualitative interrelationships among real variables display substantial differences across a broad range of business cycle frequencies, the practice of building theoretical models whose numerical

versions quantitatively match one set of regularities obtained with a particular concept of cyclical fluctuation warrants a careful reconsideration. At a minimum, the data generated by numerical versions of the theory should be passed through a variety of detrending filters which emphasize different business cycle concepts in order to check the implications of theoretical models over a wide range of cyclical frequencies.

Finally, because the focus of the paper is in documenting and organizing the information at business cycle frequencies evidence, we have refrained from asking questions like: which detrending method produces cyclical components whose features ‘best’ replicate the conventional characteristics of the business cycle as given by, say, NBER researchers (see Canova (1994) for this type of exercise). As already mentioned, there are situations when the adoption of a conventional notion of the business cycle may distort the representation of the dynamic interrelationships existing in the data and a broader empirical point of view may be more useful for theoretical work. On the other hand one should be aware that some methods extract trends which have undesirable features (e.g. BN trends are in some cases more volatile than the series themselves). This recognition may help to reduce the number of detrending procedures which economists consider reasonable for the purpose of documenting features of the business cycle.

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