Optimal monetary and fiscal policy in a currency union ☆

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ABSTRACT

We lay out a tractable model for the analysis of optimal monetary and fiscal policy in a currency union. The monetary authority sets a common interest rate for the union, whereas fiscal policy is implemented at the country level, through the choice of government spending. In the presence of country-specific shocks and nominal rigidities, the policy mix that is optimal from the viewpoint of the union as a whole requires that inflation be stabilized at the union level by the common central bank, whereas fiscal policy has a country-specific stabilization role, one beyond the efficient provision of public goods.

1. Introduction

The creation of the European Monetary Union (EMU) has led to an array of new challenges for policymakers. Those challenges have been reflected most visibly in the controversies surrounding the implementation and proposed reforms of the Stability and Growth Pact, as well as in the frequent criticisms of the interest rate policy implemented by the European Central Bank. From the perspective of macroeconomic theory, the issues raised by EMU have created an urgent need for an analytical framework that would allow us to evaluate alternative monetary and fiscal policy arrangements for EMU, or other monetary unions that may emerge in the future. In the present paper we propose a tractable framework suitable for the analysis of fiscal and monetary policy in a currency union, and study its implications for the optimal design of such policies.

In our opinion that analytical framework has to meet several desiderata. First, it has to incorporate some of the main features characterizing the optimizing models with nominal rigidities that have been developed and used for monetary policy analysis in recent years. Secondly, it should contain a fiscal policy sector, with a purposeful fiscal authority. Thirdly, the framework should comprise many open economies, linked by trade and financial flows.

The framework we propose aims at meeting the three desiderata listed above. First, we introduce nominal rigidities by assuming a staggered price setting structure, analogous to the one embedded in the workhorse model used for monetary policy analysis in closed economies, which we treat as a useful benchmark. Secondly, we incorporate a fiscal policy sector, by allowing for

country-specific levels of public consumption, and by having the latter yield utility to domestic households. Finally, we model the currency union as being made up of a *continuum* of small open economies, subject to imperfectly correlated productivity shocks. That modelling choice stands in contrast with most optimizing sticky price models of the world economy found in the literature, where tractability often requires that they be restricted to two-country world economies.[1] Yet, while such a framework may be useful to discuss issues pertaining to the links between two large economies (say, the U.S. and the euro area), it can hardly be viewed as a realistic description of the incentives and constraints facing policymakers in a monetary union like EMU, currently made up of thirteen countries (each with an independent fiscal authority), but expected to accommodate as many as twelve additional members over the next few years. Clearly, and in contrast with models featuring two large economies, the majority of the countries in EMU are small relative to the union as a whole. As a result, their policy decisions, taken in isolation, are likely to have very little impact on other countries. By looking at the limiting case of a continuum of economies, with each economy of negligible size relative to the rest of the world, we overcome the tractability problems associated with "large *N*".

Our analysis focuses on the optimal fiscal and monetary policies from the view-point of the currency union as a whole. In particular we determine the monetary and fiscal policy rules that maximize a second-order approximation to the integral of utilities of the representative households inhabiting the different countries in the union.

Two main results emerge from that analysis. First, we show that it is optimal for the (common) monetary authority to stabilize inflation in the union as a whole. Attaining that goal generally requires offsetting the threats to price stability that may arise from the joint impact of the fiscal policies implemented at the country level. Our finding would thus seem to provides a rationale for a monetary policy strategy like the one adopted by the European Central Bank, i.e. one that focuses on attaining price stability for the union as a whole. It is important to stress, however, that the optimality of that policy is conditional on the national fiscal authorities simultaneously implementing their part of the optimal policy package. The latter implies a neutral fiscal stance in the aggregate – in a sense to be made precise below – which poses no inflationary pressures on the union. We argue that, in the absence of such coordinated response by the national fiscal authorities, the union's central bank may find it optimal to deviate from a strict inflation targeting policy.

Second, under the optimal policy arrangement, each country's fiscal authority plays a dual role, trading-off between the provision of an efficient level of public goods and the stabilization of domestic inflation and output gap. Interestingly, we find that the existence of such a stabilizing role for fiscal policy is desirable not only from the viewpoint of each individual country, but also from that of the union as a whole. Our simulations under the optimal policy mix of a representative economy's response to an idiosyncratic productivity shock show that the strength of the countercyclical fiscal response increases with the importance of nominal rigidities. Such finding may call into question the desirability of imposing external constraints on a currency union's members ability to conduct countercyclical fiscal policies, when the latter seek to limit the size of the domestic output gap and inflation differentials resulting from idiosyncratic shocks.

Before we turn to a description of our model we make a brief reference to the related literature. Several recent papers have also used a microfounded DSGE frame-work to analyze the nature of optimal policy in a currency union.[2] Benigno (2004) develops a model of a two-country monetary union, where each country is subject to idiosyncratic shocks. In contrast with the present paper, Benigno's analysis does not deal with fiscal policy, focusing instead on the characterization of the optimal monetary policy by the common monetary authority. When countries differ only in size, he shows that the optimal policy requires that the price level for the union as a whole be fully stabilized, a result consistent with the one we obtain under coordinated policies.

Closer to the present paper, though written independently, Beetsma and Jensen (2005; BJ, henceforth) have also analyzed the role of fiscal stabilization policy in the context of a monetary union. Although our optimal policy implications are similar to theirs, our paper differs in several respects, both in terms of modelling choices and the type of exercises conducted. Perhaps most noticeably, our model features a continuum of small open economies, whereas BJ's framework is a more conventional two-country model. The flexibility of this setup allows, among other things, to assess the optimal policy problem for a small open economy considered in isolation (inside or outside the monetary union), since the negligible size of each country implies the absence of feedback effects from the rest of economies. In general, we believe our setting, while clearly an abstraction, may be capturing better the environment facing a majority of current and future members of the euro area.[3]

Finally, Ferrero (2006) also analyzes optimal monetary and fiscal policy setting in a two-country currency area. His contribution is complementary to ours, in that he includes a role for distortionary taxation and government debt. The presence of government debt implies that the government budget balance is an explicit constraint in the policy problem, leading to a modified optimal targeting rule for the union as a whole, in which both current and past inflation (as opposed to current inflation only as in our case) are proportional to the rate of change in the output gap. At the level of each individual country, however, the optimal response to

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[1] See, among others, Obstfeld and Rogoff (1995), Corsetti and Pesenti (2001), Benigno and Benigno (2003), Bacchetta and van Wincoop (2000), Devereux and Engel (2003), Pappa (2004), Kollmann (2001), Chari , Kehoe and McGrattan (2002). Only a subset of these contributions feature a role for a fiscal sector. For a recent analysis of monetary-fiscal policy interaction in a two-country setting and flexible exchange rates see Lombardo and Sutherland (2004). For a two-country analysis more specifically tailored to a monetary union, see Ferrero (2006).

[2] We leave out of our discussion examples of currency union models without explicit microfoundations. See, e.g., Uhlig (2003), which provides a discussion of the losses from non-cooperation in a static model with many fiscal authorities and a central bank.

[3] In addition to the assumption of a continuum of countries, our model features two differences relative to BJ, which in our opinion make it more appealing. First, we introduce home bias - thus allowing for deviations from PPP and CPI inflation differentials - whereas PPP holds continuously in BJ, implying identical CPI inflation rates across union members. Second, our framework generates an approximate welfare loss function featuring only the squares of inflation, output gap and a fiscal gap, whereas the objective function in BJ takes a more complicated form, involving interaction terms between selected endogenous variables.

country-specific shocks still implies sizeable variability in the spending and tax gaps, even if policy in each country is set optimally from a union-wide perspective.[4]

The paper is organized as follows. In Section 2 we develop the basic model. In Section 3 we characterize the equilibrium dynamics in a currency union, from the perspective of both a single member economy and of the union as a whole. In Section 4 we study the efficient allocation under flexible prices. In Section 5 we highlight the policy tradeoffs for both the Union as a whole and for each individual country. In Section 6 we analyze optimal monetary and fiscal policy in a currency union under nominal rigidities. Section 7 concludes and suggests extensions for future work.

2. A currency union model

We model the currency union as a closed system, made up of a *continuum of small open economies* represented by the unit interval. Each economy, indexed by $i \in 2\,[0,1]$ is of measure zero; as a result, its domestic policy decisions do not have any impact on the rest of the union. While different economies are subject to imperfectly correlated shocks, we assume that they share identical preferences, technology, and market structure.[5]

Next we describe in detail the problem facing households and firms in our model economy.

2.1. Households

Consider a typical country belonging to the monetary union (say, country i). We assume it is inhabited by an infinitely-lived representative household seeking to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t^i, N_t^i, G_t^i\right) \tag{1}$$

where C_t^i, N_t^i denote, respectively, private consumption and hours of work, while G_t^i is an index of public consumption, described in a separate section below.

More precisely, C_t^i is a composite consumption index defined by

$$C_t^i \equiv \frac{\left(C_{i,t}^i\right)^{1-\alpha}\left(C_{F,t}^i\right)^{\alpha}}{(1-\alpha)^{(1-\alpha)}\alpha^{\alpha}} \tag{2}$$

where $C_{i,t}^i$ is an index of country i's consumption of domestic goods (i.e., goods produced in country i itself) given by the CES function

$$C_{i,t}^i \equiv \left(\int_0^1 C_{i,t}^i(j)^{\frac{\epsilon-1}{\epsilon}}\mathrm{d}j\right)^{\frac{\epsilon}{\epsilon-1}} \tag{3}$$

where $j \in [0,1]$ denotes the type of good (within the set produced in country i).[6]

Variable $C_{F,t}^i$ is an index of country i's consumption of imported goods, given by:

$$C_{F,t}^i \equiv exp \int_0^1 c_{f,t}^i\,\mathrm{d}f$$

where $c_{f,t}^i \equiv \log C_{f,t}^i$ is, in turn, the log of an index of the quantity of goods consumed by country i's households that are produced in (and, hence, imported from) country f. That index is defined in a way symmetric to (3), that is:

$$C_{f,t}^i \equiv \left(\int_0^1 C_{f,t}^i(j)^{\frac{\epsilon-1}{\epsilon}}\mathrm{d}j\right)^{\frac{\epsilon}{\epsilon-1}} \tag{4}$$

Notice that in the specification of preferences described above $\alpha \in [0,1]$ is the weight of imported goods in the utility of private consumption. Given that the weight of the home economy in the union is infinitesimal, a value for α strictly less than one reflects the presence of home bias in private consumption, implying that households in different countries will have different consumption baskets.[7] Equivalently, we can think of α as an index of openness.

Finally, notice that parameter $\epsilon > 1$ denotes the elasticity of substitution between varieties produced within any given country, independently of the producing country.

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[4] In addition, the presence of distortionary taxes prevents Ferrero from using lump-sum taxes to correct for the steady-state market power distortion. This requires employing the methodology of Benigno and Woodford (2005) to derive an approximated welfare objective for the union.

[5] In Galí and Monacelli (2005) we used a similar modelling approach, though the focus of that paper-the design of monetary policy in a single, small open economy with its own central bank-is very different from the one in the present paper.

[6] As discussed below, each country produces a continuum of differentiated goods, represented by the unit interval. Each good is produced by a separate firm. No good is produced in more than one country.

[7] As a result, CPI inflation differentials across countries may emerge, even if the law of one price holds for each individual good.

Maximization of Eq. (1) is subject to a sequence of budget constraints of the form:

$$\int_0^1 P_t^i(j)C_{i,t}^i(j)dj + \int_0^1 \int_0^1 P_t^f(j)C_{f,t}^i(j)djdf + E_t\left\{Q_{t,t+1}D_{t+1}^i\right\} \leq D_t^i + W_t^iN_t^i - T_t^i \tag{5}$$

for t=0, 1, 2,…, where $P_t^f(j)$ is the price of good j produced in country f (expressed in units of the single currency). D_{t+1}^i is the nominal payoff in period $t+1$ of the portfolio held at the end of period t (and which may include shares in firms, local and foreign), W_t^i is the nominal wage, and T_t^i denotes lump-sum taxes.

We assume that households have access to a complete set of contingent claims, traded across the union. $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs, common across countries. Also, implicit in the notation in Eq. (5) – which features a single country index for each price – is the assumption that the *law of one price* holds across the union.

The optimal allocation of any given expenditure on the goods produced in a given country yields the demand functions:

$$C_{i,t}^i(j) = \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\epsilon}C_{i,t}^i; \qquad C_{f,t}^i(j) = \left(\frac{P_t^f(j)}{P_t^f}\right)^{-\epsilon}C_{f,t}^i \tag{6}$$

for all i, f, $j \in [0, 1]$. $P_t^i = \left(\int_0^1 P_t^i(j)^{1-\epsilon}dj\right)^{\frac{1}{1-\epsilon}}$ represents country i's *domestic* price index (i.e., an index of prices of domestically produced goods), for all $i \in [0, 1]$. Notice that, as a consequence of the law of one price, $P_t^f \equiv \left(\int_0^1 P_t^f(j)^{1-\epsilon}dj\right)^{\frac{1}{1-\epsilon}}$ is the price index for the bundle of goods imported from country f, as well as the latter's domestic price index. It follows from Eq. (6) that $\int_0^1 P_t^i(j)C_{i,t}^i(j)dj = P_t^iC_{i,t}^i$ and $\int_0^1 P_t^f(j)C_{f,t}^i(j)dj = P_t^fC_{f,t}^i$.

Furthermore, the optimal allocation of expenditures on imported goods by country of origin implies:

$$P_t^fC_{f,t}^i = P_t^*C_{F,t}^i \tag{7}$$

for all $f \in [0, 1]$, where $P_t^* \equiv \exp\int_0^1 p_t^f df$ is the union-wide price index. From the viewpoint of any individual country, P_t^* is also a price index for imported goods. Notice that Eq. (7) implies that we can write total expenditures on imported goods as $\int_0^1 P_t^fC_{f,t}^i df = P_t^*C_{F,t}^i$.

Finally, and letting $P_{c,t}^i \equiv (P_t^i)^{1-\alpha}(P_t^*)^\alpha$ denote the *consumer price index* (CPI) for country i, the optimal allocation of expenditures between domestic and imported goods in that country is given by:

$$P_t^iC_{i,t}^i = (1-\alpha)P_{c,t}^i \ C_t^i; \quad P_t^*C_{F,t}^i = \alpha P_{c,t}^iC_t^i \tag{8}$$

Combining all previous results, we can write total consumption expenditures by country i's households as $P_t^iC_{i,t}^i + P_t^*C_{F,t}^i = P_{c,t}^iC_t^i$. Thus, and conditional on an optimal allocation of expenditures, the period budget constraint can be rewritten as:

$$P_{c,t}^iC_t^i + E_t\left\{Q_{t,t+1}D_{t+1}^i\right\} \leq D_t^i + W_t^iN_t^i + T_t^i \tag{9}$$

In what follows we assume that the period utility takes the simple form

$$U(C, N, G) \equiv (1-\chi)\log C + \chi\log G - \frac{N^{1+\varphi}}{1+\varphi} \tag{10}$$

where parameter $\chi \in [0, 1)$ measures the weight attached to public consumption (relative to private consumption).

The remaining optimality conditions for country i's households are thus given by:

$$C_t^i\left(N_t^i\right)^\varphi = (1-\chi)\frac{W_t^i}{P_{c,t}^i} \tag{11}$$

$$\beta\left(\frac{C_t^i}{C_{t+1}^i}\right)\left(\frac{P_{c,t}^i}{P_{c,t+1}^i}\right) = Q_{t,t+1} \tag{12}$$

which are assumed to hold for all periods and states of nature (at t and $t+1$, in the case of Eq. (12)). Taking conditional expectations on both sides of Eq. (12) and rearranging terms we obtain a conventional Euler equation:

$$\beta R_t^*E_t\left\{\left(\frac{C_t^i}{C_{t+1}^i}\right)\left(\frac{P_{c,t}^i}{P_{c,t+1}^i}\right)\right\} = 1 \tag{13}$$

where $R_t^* = \frac{1}{E_t\{Q_{t,t+1}\}}$ is the gross nominal return on a riskless one-period discount bond paying off one unit of the common currency in $t+1$ or, for short, the (gross) nominal interest rate. Below we assume that the union's central bank uses that interest rate as its main instrument of monetary policy.

For future reference it is useful to note that Eqs. (11) and (13) can be respectively written in log-linearized form as:

$$w_t^i - p_{c,t}^i = c_t^i + \varphi n_t^i - \log(1-\chi)$$
$$c_t^i = E_t\{c_{t+1}^i\} - \left(r_t^* - E_t\{\pi_{c,t+1}^i\} - \rho\right) \tag{14}$$

where, as before, lower case letters denote the logs of the respective variables, $\rho \equiv -\log \beta$ is the time discount rate, and $\pi_{c,t}^i \equiv p_{c,t}^i - p_{c,t-1}^i$ is CPI inflation. The above optimality conditions hold for all $i \in [0, 1]$.

2.1.1. Some definitions and identities

Before proceeding with our analysis, we introduce several assumptions and definitions, and derive a number of identities that are extensively used below.

We start by defining the *bilateral terms of trade* between countries i and f as $\mathcal{S}_{f,t}^i \equiv \frac{P_t^f}{P_t^i}$, i.e., the price of country f's domestically produced goods in terms of country i's. The *effective terms of trade* for country i are thus given by

$$\mathcal{S}_t^i = P_t^* P_t^i = exp \int_0^1 \left(p_t^f - p_t^i \right) \mathrm{d}f = exp \int_0^1 s_{f,t}^i \mathrm{d}f$$

where $s_{f,t}^i \equiv \log \mathcal{S}_{f,t}^i$. Equivalently, in logs, we have $s_t^i = \int_0^1 s_{f,t}^i \, df$.

Notice also that the CPI and the domestic price levels are related according to:

$$P_{c,t}^i = P_t^i \left(\mathcal{S}_t^i \right)^\alpha$$

or, in logs:

$$p_{c,t}^i = p_t^i + \alpha s_t^i \tag{15}$$

Hence, it follows that *domestic inflation* – defined as the rate of change in the price index for domestically produced goods, i.e., $\pi_t^i \equiv p_t^i - p_{t-1}^i$ – and *CPI inflation* are linked according to the equation:

$$\pi_{c,t}^i = \pi_t^i + \alpha \Delta s_t^i \tag{16}$$

which makes the gap between our two measures of inflation proportional to the percent change in the terms of trade, with the coefficient of proportionality given by the index of openness α.

Notice that the distinction between CPI inflation and domestic inflation, while meaningful at the level of each country, vanishes for the currency union as a whole. Formally, integrating Eq. (15) over $i \in [0, 1]$ and using the fact that $\int_0^1 s_t^i \, di = 0$, yields the basic equality:

$$p_{c,t}^* = p_t^*$$

and, hence, $\pi_{c,t}^* = \pi_t^*$.

2.1.2. International risk sharing

Under the assumption of complete markets for state-contingent securities across the union, a first order condition analogous to Eq. (12) will hold for the representative house-hold in any other country, say country f:

$$\beta \left(\frac{C_t^f}{C_{t+1}^f} \right) \left(\frac{P_{c,t}^f}{P_{c,t+1}^i} \right) = Q_{t,t+1} \tag{17}$$

Combining Eqs. (12) and (17), we obtain:

$$C_t^i = \vartheta_i C_t^f \left(\mathcal{S}_{f,t}^i \right)^{1-\alpha} \tag{18}$$

for all $i, f \in [0, 1]$ and all t, and where ϑ_i is a constant which will generally depend on initial conditions. Henceforth, and without loss of generality, we assume symmetric initial conditions (i.e., zero net foreign asset holdings for all countries, combined with an ex-ante identical environment), in which case we have $\vartheta_i = \vartheta = 1$ for all $i \in [0, 1]$.

Taking logs on both sides of Eq. (18) and integrating over f we obtain

$$c_t^i = c_t^* + (1 - \alpha) s_t^i \tag{19}$$

where $c_t^* \equiv \int_0^1 c_t^f df$ is the (log) aggregate consumption index for the union as a whole.

2.2. Optimal allocation of government purchases

Country i's public consumption index is given by

$$G_t^i \equiv \left(\int_0^1 G_t^i(j)^{\frac{\epsilon-1}{\epsilon}} \mathrm{d}j \right)^{\frac{\epsilon}{\epsilon-1}}$$

where $G_t^i(j)$ is the quantity of domestic good j purchased by the government. For simplicity, we assume that government purchases are fully allocated to domestically produced goods.[8]

For any given level of public consumption G_t^i (whose determination is a central focus of the analysis below), the government is assumed to allocate expenditures across goods in order to minimize total cost. This yields the following set of government demand schedules, analogous to those associated with private consumption:

$$G_t^i(j) = \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\epsilon} G_t^i$$

In order to focus our attention on the determination of its aggregate level and its effects (rather than the distortions induced by its financing), we assume that government spending is entirely financed by means of lump sum taxes (paid by domestic residents).

2.3. Firms

2.3.1. Technology

Each country has a continuum of firms represented by the interval [0, 1]. Each firm produces a differentiated good with a linear technology:

$$Y_t^i(j) = A_t^i N_t^i(j) \tag{20}$$

for all $i, j \in [0, 1]$, where A_t^i is a country-specific productivity shifter. The latter is assumed to follow an AR(1) process (in logs):

$$a_t^i = \rho_a a_{t-1}^i + \varepsilon_t^i$$

where $a_t^i \equiv \log A_t^i$, $\rho_a \in [0, 1]$, and $\{\varepsilon_t^i\}$ is white noise.

The assumption of a linear technology implies that the real marginal cost (expressed in terms of domestic goods) is common across firms in any given country, and given (in logs) by

$$mc_t^i = \log\left(1 - \tau^i\right) + w_t^i - p_t^i - a_t^i$$

where τ^i is a (constant) employment subsidy whose role is discussed below.

Let $Y_t^i \equiv \left[\int_0^1 Y_t^i(j)^{\frac{\epsilon-1}{\epsilon}} dj\right]^{\frac{\epsilon}{\epsilon-1}}$ denote the aggregate output index for country i. The amount of labor hired is thus given by

$$N_t^i = \int_0^1 N_t^i(j) dj = \frac{Y_t^i Z_t^i}{A_t^i} \tag{21}$$

where $Z_t^i \equiv \int_0^1 \frac{Y_t^i(j)}{Y_t^i} dj$. In the Appendix A we show that equilibrium variations in $z_t^i \equiv \log Z_t^i$ around the perfect foresight steady state are of second order. Thus, and up to a first order approximation, the following relationship between aggregate employment and output holds for all $i \in [0, 1]$:

$$y_t^i = a_t^i + n_t^i \tag{22}$$

2.3.2. Price setting

Firms are assumed to set prices in a staggered fashion, as in Calvo (1983). Hence, a measure $1-\theta$ of (randomly selected) firms sets new prices each period, with an individual firm's probability of re-optimizing in any given period being independent of the time elapsed since it last reset its price. As is well known, the optimal price-setting strategy for the typical firm resetting its price in period t can be approximated by the (log-linear) rule:[9]

$$\bar{p}_t^i = \mu + (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta)^k E_t\left\{mc_{t+k}^i + p_{t+k}^i\right\} \tag{23}$$

where $\bar{p}_t^i$ denotes the (log) of newly set prices in country i (same for all firms reoptimizing), and $\mu \equiv \log \frac{\epsilon}{\epsilon-1}$ is the (log) of the optimal markup in the corresponding flexible price economy (or, equivalently, the markup prevailing in a zero inflation steady state).

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[8] For OECD countries, there is evidence of strong home bias in government procurement, over and above that observed in private consumption. See for instance Trionfetti (2000) and Brulhart and Trionfetti (2004).

[9] The approximation is carried out around a zero inflation steady state. See the appendix in Galí and Monacelli (2005) for a derivation in the context of a model with an identical price-setting block.

3. Equilibrium dynamics

3.1. Aggregate demand and output determination

The clearing of market for good j produced in country i requires

$$
\begin{aligned}
Y_t^i(j) &= C_{i,t}^i(j) + \int_0^1 C_{i,t}^i(j)\,df + G_t^i(j) \\
&= \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon}\left[(1-\alpha)\left(\frac{P_{c,t}^i}{P_t^i}\right)C_t^i + \alpha\int_0^1\left(\frac{P_{c,t}^f}{P_t^i}\right)C_t^f\,df + G_t^i\right] \\
&= \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon}\left[(1-\alpha)(S_t^i)^\alpha C_t^i + \alpha(S_t^i)^\alpha\int_0^1(S_t^i)^{1-\alpha}C_t^f\,df + G_t^i\right] \\
&= \left(\frac{P_t^i(j)}{P_t^i}\right)^{-\varepsilon}\left[C_t^i(S_t^i)^\alpha + G_t^i\right]
\end{aligned}
\tag{24}
$$

and where the last equality makes use of Eq. (18). An analogous condition must hold for all $i, j \in [0, 1]$ and all t.

Plugging the previous condition into the definition of country i's aggregate output $Y_t^i \equiv \left(\int_0^1 Y_t^i(j)^{1-\frac{1}{\varepsilon}}dj\right)^{\frac{\varepsilon}{\varepsilon-1}}$ we obtain the following aggregate goods market clearing condition for country i:

$$
Y_t^i = C_t^i\left(S_t^i\right)^\alpha + G_t^i
\tag{25}
$$

A log-linear approximation to that market clearing condition around a (symmetric) steady state is given by:[10]

$$
\widehat{y}_t^i = (1-\gamma)\left(\widehat{c}_t^i + \alpha s_t^i\right) + \gamma\,\widehat{g}_t^i
\tag{26}
$$

where a "^" symbol is used to denote log deviations of a variable from its steady state value, e.g., $\widehat{x}_t \equiv x_t - x$, and where $\gamma \equiv \frac{G}{Y}$ denotes the steady state government spending share.

Using Eq. (19) and the terms of trade definition, we can rewrite (26) as follows:

$$
\widehat{y}_t^i = \gamma\,\widehat{g}_t^i + (1-\gamma)\,\widehat{c}_t^* - (1-\gamma)\left(p_t^i - p_t^*\right)
\tag{27}
$$

The previous equation establishes that domestic output is positively related to government spending, union-wide consumption (which is an index for the strength of foreign demand), and inversely related to domestic prices (relative to average prices in the union).

Notice that we can integrate Eq. (27) over $i \in [0, 1]$ in order to obtain the union-wide goods market clearing condition:

$$
\widehat{y}_t^* = \gamma\,\widehat{g}_t^* + (1-\gamma)\widehat{c}_t^*
\tag{28}
$$

where $\widehat{y}_t^* \equiv \int_0^1 \widehat{y}_t^i\,di$, and $\widehat{g}_t^* \equiv \int_0^1 \widehat{g}_t^i\,di$.

Similarly, integrating Eq. (14) over $i \in [0, 1]$ and combining the resulting difference equation with Eq. (28), yields the following union-wide dynamic IS equation:

$$
\widehat{y}_t^* = E_t\{\widehat{y}_{t+1}^*\} - (1-\gamma)\left(r_t^* - E_t\{\pi_{t+1}^*\} - \rho\right) - \gamma E_t\{\Delta\widehat{g}_{t+1}^*\}
\tag{29}
$$

where $\pi_t^* \equiv \int_0^1 \pi_t^i\,di$. We can solve the previous equation forward and, under the assumption that $\lim_{T\to\infty}E_t\{\widehat{g}_{t+T}^*\} = \lim_{T\to\infty}E_t\{\widehat{y}_{t+T}^*\} = 0$, write it in level form as:

$$
\widehat{y}_t^* = \gamma\,\widehat{g}_t^* - (1-\gamma)\sum_{k=0}^\infty E_t\{r_{t+k}^* - \pi_{t+k+1}^* - \rho\}
$$

Hence, we see that fluctuations in union-wide output will result from variations in union-wide government spending and expected long-term rates, with the weight attached to both factors being positively and negatively related, respectively, to the steady state share of government spending in output.

3.2. The supply side: marginal cost and inflation dynamics

Given our assumption of price setting à la Calvo, the dynamics of *domestic* inflation in terms of real marginal cost in each individual country are described by the difference equation

$$
\pi_t^i = \beta E_t\left\{\pi_{t+1}^i\right\} + \lambda\widehat{mc}_t^i
\tag{30}
$$

where $\widehat{mc}_t^i = mc_t^i + \mu$ denotes the (log) deviation of real marginal cost from its steady state, and $\lambda \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$.[11]

---

[10] The derivation makes use of a first order Taylor expansion of $\log(Y_t^i - G_t^i)$, as shown in the Appendix A. We also use the fact that in a symmetric steady state $S^i = 1$ (and hence $s^i = 0$) for all $i \in [0, 1]$.

[11] Notice that under our assumptions the fact that each individual economy is open does not affect the form of the equation relating domestic inflation to the real marginal cost. See Galí and Monacelli (2005) for further discussion and a formal derivation.

Using some of our previous results, we can further derive the following expression for marginal cost:

$$mc_t^i = w_t^i - p_t^i - a_t^i + log(1 - \tau^i) = \left(w_t^i - p_{c,t}^i\right) + \left(p_{c,t}^i - p_t^i\right) - a_t^i + log(1 - \tau^i) = c_t^i + \varphi n_t^i + \alpha s_t^i - a_t^i + log(1 - \tau^i) - log(1 - \chi)$$

(31)

We can now combine Eq. (31) with Eqs. (22) and (26) to obtain an expression for the marginal cost as a function of output and government spending, all expressed in deviations from steady state (and up to a first order approximation):

$$\widehat{mc}_t^i = \left(\frac{1}{1 - \gamma} + \varphi\right)\widehat{y}_t^i - \frac{1}{1 - \gamma}\widehat{g}_t^i - (1 + \varphi)a_t^i$$

(32)

The intuition for the negative relationship between marginal cost and government spending is easy to grasp: *given output*, an increase in government spending crowds out domestic consumption and/or generates a real appreciation, both of which tend to reduce real marginal cost through their negative effect on the product wage.[12] In addition, we see that the elasticity of real marginal cost with respect to output is increasing in the government share γ. The reason is simple: in response to a given percent increase in output, and given an unchanged current level of current government spending $\widehat{g}_t^i$ and technology a_t^i, a larger γ is associated with a larger percent increase in consumption and/or the terms of trade. As a result, a larger increase in the product wage and, hence, marginal cost will obtain.

Combining Eqs. (30) and (32) we can derive a version of the new Keynesian Phillips curve (NKPC), applying to each economy in the union:

$$\pi_t^i = \beta E_t\left\{\pi_{t+1}^i\right\} + \lambda\left(\frac{1}{1 - \gamma} + \varphi\right)\widehat{y}_t^i - \frac{\lambda\gamma}{1 - \gamma}\widehat{g}_t^i - \lambda(1 + \varphi)a_t^i$$

(33)

Notice also that by integrating the previous equation over $i \in [0, 1]$ we can obtain the corresponding new Keynesian Phillips curve for the union as a whole:

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \lambda\left(\frac{1}{1 - \gamma} + \varphi\right)\widehat{y}_t^* - \frac{\lambda\gamma}{1 - \gamma}\widehat{g}_t^* - \lambda(1 + \varphi)a_t^*$$

(34)

where $a_t^* \equiv \int_0^1 a_t^i di$.

We have now derived the set of log-linear equilibrium conditions for inflation and output in each individual country (summarized by Eqs. (27) and (33)), s a whole (given by Eqs. (29) and (34)), as a function of government spending (local and union-wide) and the common interest rate. Given the equilibrium path for those variables, one can use Eq. (14) (or, equivalently, (12)) to back out the equilibrium consumption in each country.

Next we turn to the analysis of optimal policy design in the context of the above model. In the next section, and as a starting point of our analysis, we determine the efficient allocation and its implementation under flexible prices.

4. The efficient allocation

In the present section we derive the efficient allocation and show how it can be supported in equilibrium when prices are fully flexible. This will prove a useful benchmark for the analysis of optimal policy in the presence of nominal rigidities, to which we turn later.

4.1. The social planner's problem

The union's optimal allocation in any given period can be described as the solution to the following social planner's problem:

$$\max \int_0^1 U\left(C_t^i, N_t^i, G_t^i\right)di$$

subject to the technological and resource constraints

$$Y_t^i = A_t^i N_t^i$$
$$Y_t^i = C_{i,t}^i + \int_0^1 C_{i,t}^f df + G_t^i$$

(35)

for all $i \in [0, 1]$. Notice that the previous constraints already embed the optimal condition whereby the different good types in any given country should be produced and consumed in identical quantities.[13]

Under our specification of preferences, the optimality conditions for the social planner's problem are:

$$\frac{(N_t^i)^\varphi}{A_t^i} = \frac{(1 - \chi)(1 - \alpha)}{C_{i,t}^i} = \int_0^1 \frac{(1 - \chi)\alpha}{C_{i,t}^f}df = \frac{\chi}{G_t^i}$$

---

[12] Notice that the corresponding elasticity is increasing in γ, since the greater the weight of government spending in aggregate demand the larger will be the percent decline in consumption needed to keep output constant.

[13] That condition in turn implies that $Z_t^i = 1$ in Eq. (21), for all $i \in [0, 1]$.

for all $i \in [0, 1]$. In words, the marginal loss of utility for a household in country i of producing an additional unit of the composite good, given by $(N_t^i)^\varphi / A_t^i$, must be equal, at the margin, to the utility gain resulting from any of the three possible uses of that additional output: consumption by domestic households, consumption by all households in the union, and domestic government spending.

Using the resource constraint (35), and the fact that $Y_t^i = A_t^i N_t^i$, we can guess and verify that the solution to the social planner's problem is given by:

$$N_t^i = 1 \tag{36}$$

$$Y_t^i = A_t^i \tag{37}$$

$$C_{i,t}^i = (1 - \chi)(1 - \alpha)A_t^i \tag{38}$$

$$C_{i,t}^f = (1 - \chi)\alpha A_t^i \tag{39}$$

$$G_t^i = \chi A_t^i \tag{40}$$

for all $i, f \in [0, 1]$, and all t.

Combining Eqs. (38) and (39), together with definition of country i's total consumption index (2), we can derive an expression for the latter under the optimal allocation (in logs):

$$c_t^i = (1 - \alpha)a_t^i + \alpha \int_0^1 a_t^f df + \log(1 - \chi)$$

or, in levels,

$$C_t^i = (1 - \chi)\left(A_t^i\right)^{1-\alpha}\left(A_t^*\right)^\alpha$$

where $A_t^* \equiv \exp \int_0^1 a_t^f df$ is an index of union-wide productivity.

Aggregating over countries, we obtain the corresponding optimal allocation for the union as a whole:

$$Y_t^* = A_t^*$$
$$C_t^* = (1 - \chi)A_t^*$$
$$G_t^* = \chi A_t^*$$

4.2. Decentralization of the efficient allocation under flexible prices

Next we show how the union-wide optimal allocation derived above can be supported as an equilibrium in the presence of *flexible prices*. Letting variables with an upper bar denote their values in a flexible price equilibrium we have

$$
\begin{aligned}
1 - \frac{1}{\epsilon} &= \overline{MC}_t^i \\
&= \frac{(1 - \tau^i)}{A_t^i (1 - \chi)} \overline{C}_t^i \left(\overline{N}_t^i\right)^\varphi \left(\overline{S}_t^i\right)^\varphi \\
&= \frac{(1 - \tau^i)}{A_t^i (1 - \chi)} \overline{C}_t^i \left(\overline{N}_t^i\right)^\varphi \frac{\overline{Y}_t^i - \overline{G}_t^i}{\overline{C}_t^i} \\
&= \frac{1 - \tau^i}{1 - \chi} \left(1 - \left(\overline{G}_t^i / \overline{Y}_t^i\right)\right) \left(\overline{N}_t^i\right)^{1+\varphi}
\end{aligned} \tag{41}
$$

In order for the equilibrium allocation under flexible prices to correspond to the union's socially optimal allocation the following conditions must be satisfied for all $i \in [0, 1]$ and t. First, the subsidy τ^i must be set at a level

$$\bar{\tau}^i = \frac{1}{\epsilon} \tag{42}$$

Secondly, government spending must be set according to the rule[14]

$$\bar{G}_t^i = \chi A_t^i \tag{43}$$

---

[14] Or, equivalently, $\bar{G}_t^i = \chi \bar{Y}_t^i$.

If both conditions are satisfied for all $i \in [0, 1]$, the flexible price equilibrium will yield the level of employment and output in each country that is optimal from the union's perspective, i.e., $\overline{Y}_t^i = A_t^i$ and $\overline{N}_t^i = 1$, for all $i \in [0, 1]$, and all t.[15] It is easy to check that the remaining optimality conditions will also be satisfied as a result of households' optimization.

Notice that in the economy with flexible prices, the lack of an autonomous monetary policy is of no consequence for the attainment of the optimal allocation, for monetary policy is neutral in that environment (it can only influence the path of prices). As a result, local fiscal authorities can focus exclusively on the efficient provision of public consumption goods, according to rule (43) (shadowing the central planner's decisions on that front). In our example economy that rule implies a constant government spending share $\overline{G}_t^i / \overline{Y}_t^i = \gamma = \chi$ for all t.

While the level of prices in the union and in each individual country is determined by the monetary policy regime, each country's terms of trade as well as the inflation differentials vis a vis the union are fully determined by real factors in the present scenario. More specifically, note that the path for the terms of trade that will support the efficient allocation is given by:

$$\overline{S}_t^i = \left(\overline{C}_t^i / \overline{C}_t^* \right)^{\frac{1}{1-\alpha}} = A_t^i / A_t^* \tag{44}$$

for all $i \in [0, 1]$, and all t. Given the definition of the terms of trade it follows that the inflation differential will be inversely proportional to the productivity growth differential:

$$\pi_t^i - \pi_t^* = -\left(\Delta a_t^i - \Delta a_t^* \right)$$

We have thus shown how under flexible prices the efficient allocation can be supported in equilibrium through an appropriate choice of a subsidy (to eliminate market power distortion) and government spending policies (focused on providing the efficient level of public goods). In that context, the policy pursued by the common central bank is of no relevance, since it can only influence aggregate inflation, which under our assumptions generates no distortions. Not surprisingly, things are considerably different when nominal rigidities are present, the case to which we turn next.

5. Sticky prices and policy tradeoffs in the currency union

In the presence of nominal rigidities (and, in particular, of sticky prices) it is generally unfeasible for a monetary union to attain the efficient allocation. The reasons are well understood. First, nominal rigidities imply that the level of employment and output within each country may differ from the efficient one both in aggregate terms and, in the case of staggered price setting, in terms of its distribution across sectors (i.e., types of goods). This is true even if, assumed above, the distortion associated with market power is offset by means of a subsidy. Secondly, the sluggish adjustment of prices, combined with the impossibility of nominal exchange rate adjustments (inherent to a currency union), implies that the changes in terms of trade that are required to support the optimal allocation cannot occur instantaneously.

As shown in Galí and Monacelli (2005) in the context of a related model, when each individual country has its own currency and an autonomous monetary policy (as opposed to the monetary union case considered here), stabilization of the domestic price level in each country guarantees that the flexible price equilibrium (and, hence, the optimal allocation) is attained. As we show next, this is no longer possible under a currency union, at least to the extent that different countries experience asymmetric shocks. As a result, several tradeoffs arise, forcing policymakers to settle for a second best outcome. Next we discuss the nature of those tradeoffs, looking in turn at those facing each of the union's members (and, hence, their fiscal authorities) and, subsequently, those facing the union as a whole (and, hence, the common central bank).

5.1. Union members' tradeoffs

Let $\overline{y}_t^i = a_t^i$ and $\overline{g}_t^i = \log \chi + a_t^i$ denote the (logs) of output and government spending in country i associated with the union-wide efficient allocation (or equivalently, with the flexible price equilibrium under an optimal policy). We use the notation $\tilde{y}_t^i$ and $\tilde{g}_t^i$ to denote the log deviations of country i's output and government spending from those benchmark levels, i.e., $\tilde{y}_t^i \equiv y_t^i - \overline{y}_t^i$ and $\tilde{g}_t^i \equiv g_t^i - \overline{g}_t^i$, which we henceforth refer to as country i's *output gap* and *government spending gap*, respectively.

It will prove convenient to define the following measure of the fiscal stance:

$$\tilde{f}_t^i \equiv \tilde{g}_t^i - \tilde{y}_t^i = \left(g_t^i - y_t^i \right) - \log \chi$$

which we henceforth refer to as the *fiscal gap*.[16]

---

[15] In contrast with Galí and Monacelli (2005), where the optimal allocation problem is analyzed from the viewpoint of a small open economy, here the choice of the subsidy is not affected by any desire to influence the terms of trade in a country's favor. The reason is simple: that goal cannot be attained by all countries simultaneously, and hence it serves no purpose when trying to decentralize the solution to the union's social planner problem. As a result the only role played by the subsidy is to offset firms' market power.

[16] Strictly speaking, $\overline{g}_t^i$ and, hence, $\tilde{g}_t^i$ are only well defined if $\chi > 0$, which we assume for the remainder of this section.

Using Eq. (32), together with the fact that $\bar{y}_t^i - y^i = \bar{g}_t^i - g^i = a_t^i$ (where variables without time subscripts denote steady state values), we can derive the following relationship between the real marginal cost, and the output and fiscal gaps:

$$\widehat{mc}_t^i = \left(\frac{1}{1-\chi} + \varphi \right) \tilde{y}_t^i - \frac{\chi}{1-\chi} \tilde{g}_t^i = (1+\varphi)\tilde{y}_t^i - \frac{\chi}{1-\chi} \tilde{f}_t^i$$

where we have imposed an optimal steady state government spending share ($\gamma = \chi$).

We can combine the previous expression with (30) to obtain a version of the new Keynesian Phillips curve for each union member, expressing domestic inflation in terms of the corresponding output and fiscal gaps:

$$\pi_t^i = \beta E_t\left\{ \pi_{t+1}^i \right\} + \lambda(1+\varphi)\tilde{y}_t^i - \frac{\lambda\chi}{1-\chi} \tilde{f}_t^i \tag{45}$$

In addition we can combine Eqs. (27), (28) and (44), to obtain an equation determining the change in the output gap differential as a function of the differentials in fiscal gap changes, inflation and productivity growth:

$$\Delta\tilde{y}_t^i - \Delta\tilde{y}_t^* = \frac{\chi}{1-\chi}\left(\Delta\tilde{f}_t^i - \Delta\tilde{f}_t^* \right) - \left[\left(\pi_t^i - \pi_t^* \right) + \left(\Delta a_t^i - \Delta a_t^* \right) \right] \tag{46}$$

The previous two equations describe the evolution of country i's output gap and price level as a function of the domestic fiscal gap, given the productivity differential and the union wide fiscal and output gaps. They also make clear the nature of the tradeoff facing fiscal authorities of union member countries. To illustrate those tradeoffs, assume that $\tilde{y}_t^* = \tilde{f}_t^* = p_t^* = 0$. Consider Eq. (45), describing the evolution of the price level in country i. That equation implies that by closing the output and fiscal gaps at all times (and thus trying to replicate the flexible price equilibrium allocation), domestic prices would be fully stabilized. Yet, Eq. (46) makes clear that, in the presence of asymmetric productivity shocks, closing the output gap (without creating a fiscal gap) requires that the terms of trade and, hence, domestic prices, adjust.

5.2. Union-wide tradeoffs

The evolution of inflation, the output gap, and the fiscal gap for the currency union is described by two aggregate equilibrium relations. Thus, by integrating (45), we can derive a version of the new Keynesian Phillips curve describing union-wide inflation in terms of the output and fiscal gaps:

$$\pi_t^* = \beta E_t\left\{ \pi_{t+1}^* \right\} + \lambda(1+\varphi)\tilde{y}_t^* - \frac{\lambda\chi}{1-\chi}\tilde{f}_t^* \tag{47}$$

The union's output gap is determined by a dynamic IS-type equation, which we can derive using (29):

$$\tilde{y}_t^* = E_t\left\{ \tilde{y}_{t+1}^* \right\} - (1-\chi)\left(r_t^* - E_t\left\{ \pi_{t+1}^* \right\} - \overline{rr}_t^* \right) - \chi E_t\left\{ \Delta\tilde{g}_{t+1}^* \right\} = \frac{\chi}{1-\chi}\tilde{f}_t^* - \left(r_t^* - E_t\left\{ \pi_{t+1}^* \right\} - \overline{rr}_t^* \right) + E_t\left\{ \tilde{y}_{t+1}^* \right\} - \frac{\chi}{1-\chi}E_t\left\{ \tilde{f}_{t+1}^* \right\} \tag{48}$$

where $\overline{rr}_t^*$ is the union's natural rate of interest, given by

$$\overline{rr}_t^* = \rho + (1-\chi)^{-1}\left(E_t\left\{ \Delta\bar{y}_{t+1}^* \right\} - \chi E_t\left\{ \Delta\bar{g}_{t+1}^* \right\} \right) = \rho + E_t\left\{ \Delta\bar{y}_{t+1}^* \right\} = \rho + E_t\left\{ \Delta\bar{a}_{t+1}^* \right\}$$

Notice that, to the extent that the union's aggregate fiscal gap $\tilde{f}_t^*$ remains stable at zero, there is no tradeoff between stabilization of the output gap and inflation for the union as a whole. In that case the outcome $\tilde{y}_t^* = \tilde{\pi}_t^* = 0$ could be easily attained by having the central bank follow a rule of the sort

$$r_t = \overline{rr}_t^* + \phi_\pi \pi_t^*$$

On the other hand, if the *aggregated* decisions of the local fiscal authorities lead to fluctuations in the union-wide fiscal gap, the job of the single central bank is considerably more difficult. To illustrate this formally, notice that we can integrate (48) and combine it with (47) to yield:

$$\pi_t^* = \beta E_t\{\pi_{t+1}^*\} + \frac{\lambda\varphi\chi}{1-\chi}\tilde{f}_t^* - \sum_{k=0}^{\infty} E_t\left(r_{t+k}^* - \pi_{t+k+1}^* - \overline{rr}_{t+k}^* \right) = \frac{\lambda\varphi\chi}{1-\chi}\sum_{k=0}^{\infty}\beta^k E_t\left\{ \tilde{f}_{t+k}^* \right\} - \frac{\lambda(1+\varphi)}{1-\beta}\sum_{k=0}^{\infty} \left(1 - \beta^{k+1} \right) E_t\left\{ \left(r_{t+k}^* - \pi_{t+k+1}^* - \overline{rr}_{t+k}^* \right) \right\}$$

Notice that a positive union-wide fiscal gap, current and/or anticipated, will generate upward pressure on current inflation. That pressure can only be partly offset by having the central bank run a tighter monetary policy, which would require raising current and/or future interest rates above their natural level, thus dampening the expansionary impact of members' fiscal policies

on the union's output gap and inflation. Below we show that this is indeed the sort of rule that the union's central bank should adopt, as part of the optimal monetary-fiscal policy mix for the union.

6. Optimal monetary and fiscal policy in the currency union

Next we derive and characterize the optimal fiscal-monetary regime in the currency union. This regime involves full coordination of the monetary and fiscal authorities, *as if* all policy decisions were centralized in a single policymaker, whose objective is to maximize the average welfare of union households.

Note first that the policymakers' joint objective function corresponds to that of the social planner considered in Section 3. Thus it should be clear that they will choose the same efficient steady state, which is feasible and can be supported by means of the constant subsidy (42). Fluctuations about that steady state will in general be inefficient, for the reasons discussed in the previous section. In the Appendix A we show that a second order approximation to the average utility losses of union households resulting from fluctuations about the efficient steady state takes the form:

$$\mathbb{W} \simeq \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \int_0^1 \left(\frac{\epsilon}{\lambda} \left(\pi_t^i \right)^2 + (1 + \varphi) \left(\tilde{y}_t^i \right)^2 + \frac{\chi}{1-\chi} \left(\tilde{f}_t^i \right)^2 \right) di + \text{tips} \tag{49}$$

where *tips* denotes terms that are independent of policy.[17]

We define the optimal policy as the set of rules for the fiscal gaps $\{\tilde{f}_t^i\}$ for all $i \in [0, 1]$ and the common interest rate $\{r_t^*\}$, along with the associated second best outcomes $\{\pi_t^i, \tilde{y}_t^i\}$ for all $i \in [0, 1]$, that maximize Eq. (49), subject to Eqs. (45), (46), and the "aggregation" constraints

$$\pi_t^* = \int_0^1 \pi_t^i di; \quad \tilde{y}_t^* = \int_0^1 \tilde{y}_t^i di; \quad \tilde{f}_t^* = \int_0^1 \tilde{f}_t^i di \tag{50}$$

The optimal policy problem can be solved in two stages. First, we determine the processes $\{\pi_t^i, \tilde{y}_t^i, \tilde{f}_t^i\}$, for all $i \in [0,1]$; that maximize Eq. (49) subject to Eqs. (45), (46) and (50). Secondly, given the solution to that first-stage problem, we determine the interest rate rule that will support the implied paths for the union-wide inflation, output gap and fiscal gap, using Eq. (48).

The optimality conditions associated with the first-stage problem are given by:

$$\frac{\epsilon}{\lambda} \pi_t^i + \Delta \psi_{\pi,t}^i + \psi_{y,t}^i - \psi_{\pi,t}^* = 0 \tag{51}$$

$$(1 - \varphi) \tilde{y}_t^i - \lambda (1 - \varphi) \psi_{\pi,t}^i + (1 - \beta L^{-1}) \psi_{y,t}^i - \psi_{y,t}^* = 0 \tag{52}$$

$$\tilde{f}_t^i + \lambda \psi_{\pi,t}^i - (1 - \beta L^{-1}) \psi_{y,t}^i - \left(\frac{1-\chi}{\chi} \right) \psi_{f,t}^* = 0 \tag{53}$$

$$-\int_0^1 \psi_{y,t}^i di + \psi_{\pi,t}^* = 0 \tag{54}$$

$$-(1 - \beta L^{-1}) \int_0^1 \psi_{y,t}^i di + \psi_{y,t}^* = 0 \tag{55}$$

$$\frac{\chi}{1-\chi} (1 - \beta L^{-1}) \int_0^1 \psi_{y,t}^i di + \psi_{f,t}^* = 0 \tag{56}$$

for all $i \in [0, 1]$ and $t = 0, 1, 2, \ldots$, where $\{\psi_{\pi,t}^i, \psi_{y,t}^i\}$, $\psi_{\pi,t}^*$, $\psi_{y,t}^*$, and $\psi_{f,t}^*$ are the (discounted) Lagrange multipliers associated with constraints in Eqs. (45), (46) and (50), and where $\psi_{\pi,-1}^i = 0$.

Integrating Eq. (51) over $i \in [0, 1]$, combining the resulting equation with Eq. (54), we obtain:

$$\frac{\epsilon}{\lambda} \pi_t^* + \int_0^1 \Delta \psi_{\pi,t}^i di = 0 \tag{57}$$

Similarly, integrating Eq. (52) over $i \in [0, 1]$, combining the resulting equation with Eq. (55), we obtain:

$$\tilde{y}_t^* - \lambda \int_0^1 \psi_{\pi,t}^i \, di = 0$$

Both can be combined to yield

$$\epsilon \pi_t^* + \Delta \tilde{y}_t^* = 0 \tag{58}$$

for t = 1, 2, 3, … whereas for t = 0 we have $\epsilon \pi_0^* + \tilde{y}_0^* = 0$.

Integrating Eq. (53) over $i \in [0, 1]$, combining the resulting equation with Eq. (56) and the result above, we obtain:

$$\tilde{f}_t^* = -\tilde{y}_t^* \tag{59}$$

Notice that Eqs. (58) and (59), together with the union-wide equilibrium conditions (47) and (48), imply that the equilibrium under the optimal policy will satisfy

$$\pi_t^* = \tilde{y}_t^* = \tilde{f}_t^* = 0 \tag{60}$$

for all t. This is one of the central results emerging from our analysis. In words, we may state it as follows: the combined monetary-fiscal policy mix must be such that, at the union level, inflation, the output gap and the fiscal gap remain at a constant (zero) value, at all times. That condition requires, in turn, that the equilibrium interest rate r_t^* equals the union-wide natural rate $\overline{rr}_t^*$ at all times. As argued above, and *conditional* on $\tilde{f}_t^* = 0$ for all t, the union's central bank can implement the desired outcome by adopting a policy rule of the form:

$$r_t = \overline{rr}_t^* + \phi_\pi \pi_t^*$$

where $\phi_\pi > 1$.

What are the paths of inflation and the output gap for each union member associated with the optimal policy? What fiscal policy will support those paths?

Combining Eqs. (52) and (53), and noticing that Eqs. (55) and (56) imply $\frac{\chi}{1-\chi} \psi_{y,t}^* + \psi_{f,t}^* = 0$ we obtain:

$$(1 + \varphi)\tilde{y}_t^i + \tilde{f}_t^i = \lambda \varphi \psi_{\pi,t}^i \tag{61}$$

In this second best environment, as long as prices are less than fully flexible, we have $\psi_{\pi,t}^i > 0$. Hence Eq. (61) immediately implies that, unlike the union-wide policy prescription (60), setting $\tilde{f}_t^i = \tilde{y}_t^i = 0$ for each member country i cannot be an equilibrium under the optimal policy.

To fully characterize the equilibrium dynamics, we notice that the aggregate multiplier $\psi_{\pi,t}^* = \int_0^1 \psi_{\pi,t}^i \, di$ (from Eq. (54)) must evolve exogenously from the viewpoint of the single member country. By substituting Eqs. (54), (55) and (56) into Eqs. (51), (52) and (53), we define a rational expectations equilibrium for country i as an allocation for $\{\pi_t^i, \tilde{y}_t^i, \tilde{f}_t^i, \psi_{\pi,t}^i, \psi_{y,t}^i\}$ that satisfies Eqs. (45), (46), (51), (52), (53), for any given $\{\psi_{\pi,t}^*\}$ and stochastic processes $\{a_t^i, a_t^*\}$, along with the initial condition $\psi_{\pi,-1}^i = 0$. Next we illustrate the implied equilibrium dynamics and the optimal policy responses by means of some simulations.

6.1. Dynamic simulations

In this section we illustrate the equilibrium behavior for a prototype member economy under the optimal policy arrangement described above. We resort to a series of dynamic simulations, and adopt the following benchmark calibration. We assume $\varphi = 3$, which implies a labor supply elasticity of $\frac{1}{3}$, and a steady-state markup $\mu = 1.2$, which implies that ϵ, the elasticity of substitution between differentiated goods (of the same origin), is 6. Parameter θ is set to a benchmark value of 0.75 (a value consistent with an average period of one year between price adjustments), and report results for alternative values. We assume $\beta = 0.99$, which implies a riskless annual return of about 4% in the steady state. As for the fiscal sector, we parameterize the steady state share of government spending in output as $\gamma = \chi = 0.25$, roughly the average share of government consumption in GDP for the euro zone.

We follow the real business cycle literature (King and Rebelo, 1999) and assume the following autoregressive process for labor productivity in country i:

$$a_t^i = 0.95 a_{t-1}^i + \varepsilon_{i,t} \tag{62}$$

Fig. 1 displays impulse responses for a number of domestic variables to a one percent country-specific rise in productivity for alternative values of the price stickiness parameter θ. In particular, θ = 0 represents the limiting case of full (domestic) price flexibility.

Consider first the case of full price flexibility (θ = 0). In that case there is no loss of efficiency associated with inflation, since the latter no longer creates any relative price distortions. Hence, as shown in the figure, it is optimal to fully close the fiscal gap and the
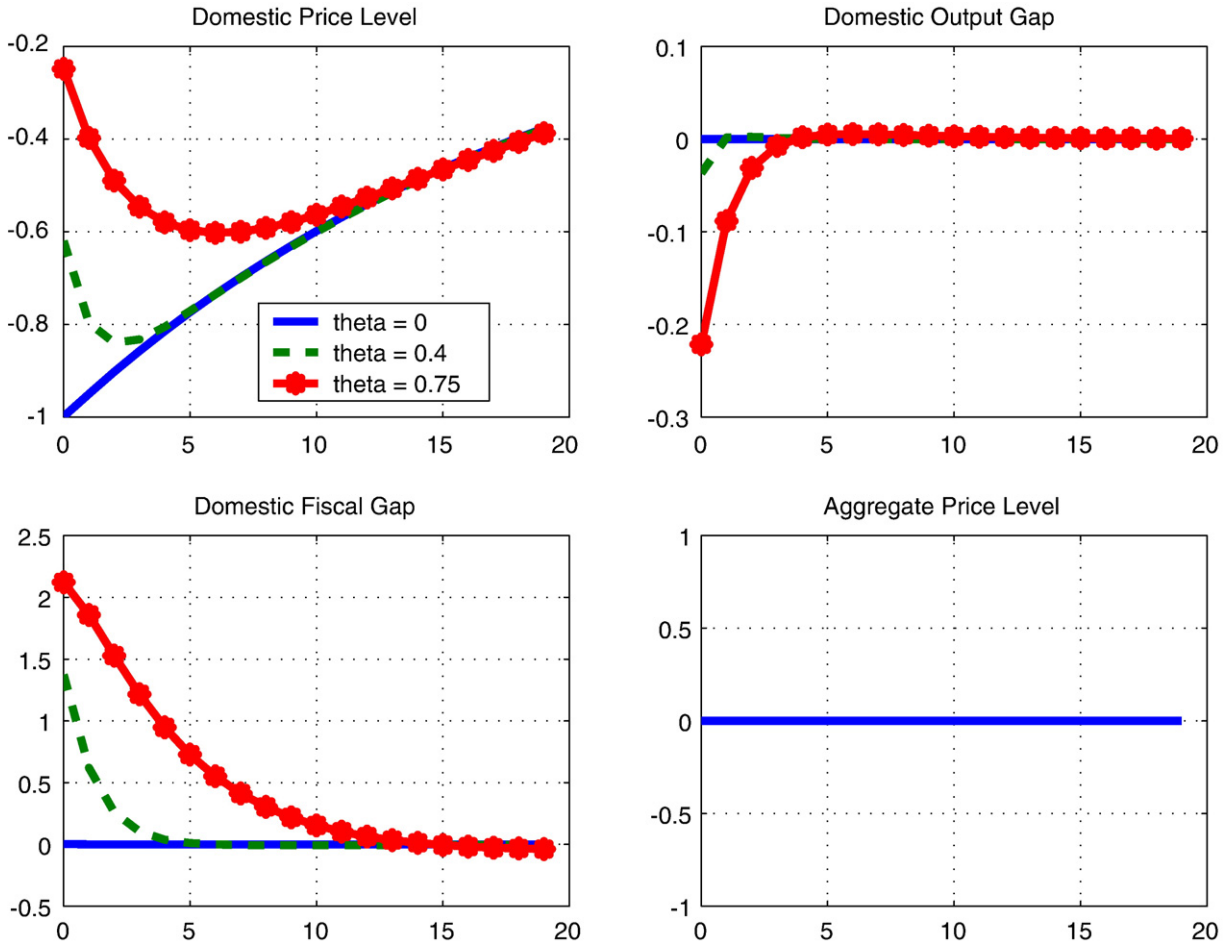
Fig. 1. Domestic productivity shock under the optimal policy.

output gap, in response to asymmetric movements in productivity.[18] As a result, it is optimal for the union member experiencing a productivity increase to fully absorb the latter through an adjustment in the terms of trade brought about by a change in the domestic price level, while maintaining output and government spending at their first-best levels.

To the extent that price stickiness is present ($\theta > 0$), there are welfare losses associated with departures from price stability, in addition to those stemming from nonzero output and fiscal gaps. However – as discussed above – the flexible price/efficient allocation is not feasible under the currency union regime. In particular, the rise in productivity must be absorbed only via a gradual and persistent fall in the price level, with the consequent relative price distortions. As a result, the optimal policy mix requires expanding the fiscal gap to bring about the rise in demand necessary to accommodate the desired expansion in output, thus smoothing the adjustment of prices over time. To see that formally, notice that in the equilibrium under the optimal policy Eq. (46) simplifies to:

$$\tilde{y}_t^i - \frac{\chi}{1-\chi}\tilde{f}_t^i + p_t^i = -\tilde{a}_t^i$$

where $\tilde{a}_t^i \equiv a_t^i - a_t^*$ (and where, without loss of generality, we have normalized $p_t^* = 0$).

Hence, to the extent that the price level reacts gradually, the rise in productivity will be absorbed via a combination of a fall in the output gap and a rise in the fiscal gap. In general, the local fiscal authority is required to trade-off movements in inflation on the one hand with movements in the output and fiscal gap on the other. The higher the degree of price rigidity, the larger the implied fluctuations of both gaps under the optimal policy.[19]

---

[18] In fact, under price flexibility, Eq. (45) does not act as a constraint on the evolution of domestic prices. Hence, optimal policy in this case must satisfy Eq. (61) with $\psi_{\pi,t}^i = 0$.

[19] The impulse response results, at least qualitatively, are similar to the ones obtained in BJ. The welfare interpretation, though, is different, given that the welfare objectives are not readily comparable.

Notice that, under our benchmark calibration, welfare losses from any given output gap variation are of an order of magnitude larger than the ones implied by the same variation in the fiscal gap. This explains why in Fig. 1 the implied volatility of the fiscal gap is larger than the one in the output gap. The optimal balance between the two variables will in general depend on the relative weights attached to the quadratic terms in $\tilde{y}_t^i$ and $\tilde{f}_t^i$ in the welfare loss function (49). These weights depend in turn on parameters φ and χ. The lower the elasticity of labor supply (i.e., the larger φ) the smaller the adjustment in the output gap (relative to the fiscal gap), whereas the larger χ (the share of government spending in the optimal steady state) the lower the adjustment brought about via the fiscal gap (relative to the output gap).

7. Conclusions

We have developed a tractable multicountry framework suitable for monetary and fiscal policy analysis in a currency union. As an application, we have determined the optimal monetary-fiscal policy mix in the presence of idiosyncratic shocks to productivity. Given our assumed nominal rigidities, the presence of those shocks, combined with the impossibility of resorting to nominal exchange rate adjustments, induces an inefficient response of the terms of trade that justifies the use of fiscal policy as a stabilization tool. In particular, the union-wide optimal policy calls for variations in local government spending that go beyond the mere efficient provision of public goods. On the other hand, the union's central bank should seek to stabilize the price level in the union as a whole.

Our framework is amenable to extensions along several dimensions. In order to meet our self-imposed tractability requirement, we have restricted ourselves to less-than-general parametric specifications for utility and technology. The work of Forlati (in press) addresses some of the challenges arising from the relaxation of those assumptions. Our model ignores other aspects that are likely to be relevant for the design of optimal policies. Missing elements include, among others, the presence of sticky wages (along with sticky prices), the need to rely on distortionary taxes, the effects of government debt policies, and the likely existence of non-fully Ricardian behavior on the part of households. Finally, our framework assumes the presence of complete international financial markets. By relaxing the assumption of perfect risk-sharing, one could presumably generate a complementary role for fiscal policy as a cross-country insurance tool. The emergence of a potential conflict between the latter and the stabilization role described in the present paper is likely to constitute an interesting avenue worth exploring in future research. We plan to pursue some of those extensions in future work.

Appendix A. Union's Welfare Loss

For notational simplicity we omit country subscripts, unless needed.

Taylor expansion of $\log(Y_t - G_t)$

Let $\gamma \equiv \frac{G}{Y}$ the steady state government spending share. Define $\hat{y}_t \equiv \log \frac{Y_t}{Y}$ and $\hat{g}_t \equiv \log \frac{G_t}{G}$. A second-order Taylor expansion of $\log(Y_t - G_t)$ about the steady state yields:

$$\log(Y_t - G_t) = \log((1-\gamma)Y) + \frac{1}{1-\gamma}\left(\frac{Y_t - Y}{Y}\right) - \frac{\gamma}{1-\gamma}\left(\frac{G_t - G}{G}\right) - \frac{1}{2}\frac{1}{(1-\gamma)^2}\left(\left(\frac{Y_t - Y}{Y}\right)^2 + \gamma\left(\frac{G_t - G}{G}\right)^2 - 2\gamma\left(\frac{Y_t - Y}{Y}\right)\left(\frac{G_t - G}{G}\right)\right)$$

$$= \log((1-\gamma)Y) + \frac{1}{1-\gamma}(\hat{y}_t - \gamma\hat{g}_t) + \frac{1}{2}\frac{1}{1-\gamma}\left(\hat{y}_t^2 - \gamma\hat{g}_t^2\right) - \frac{1}{2}\frac{1}{(1-\gamma)^2}(\hat{y}_t - \gamma\hat{g}_t)^2$$

$$= \log((1-\gamma)Y) + \frac{1}{1-\gamma}(\hat{y}_t - \gamma\hat{g}_t) - \frac{1}{2}\frac{\gamma}{(1-\gamma)^2}(\hat{g}_t - \hat{y}_t)^2$$

Let $\tilde{y}_t = y_t - \bar{y}_t$ and $\tilde{g}_t = g_t - \bar{g}_t$ denote the output and fiscal gaps, respectively, as defined in the text. Note that $\hat{y}_t = \tilde{y}_t + (\bar{y}_t - y)$ and $\hat{g}_t = \tilde{g}_t + (\bar{g}_t - g)$: Hence, $\hat{g}_t - \hat{y}_t = \tilde{g}_t - \tilde{y}_t + (\bar{g}_t - \bar{y}_t) - \log \gamma$.

Quite generally, g_t and y_t will depend on exogenous shocks only. In the present model, $\bar{g}_t - \bar{y}_t = \log \chi$. Thus, when considering fluctuations about the efficient steady state (with $\gamma = \chi$) we have $\hat{g}_t - \hat{y}_t = \tilde{g}_t - \tilde{y}_t$, allowing to write:

$$\log(Y_t - G_t) \simeq \frac{1}{1-\chi}(\tilde{y}_t - \chi\tilde{g}_t) - \frac{1}{2}\frac{\chi}{(1-\chi)^2}(\tilde{g}_t - \tilde{y}_t)^2 + \text{tips}$$

Taylor expansion of $\int_0^1 \log C_t^i di$

From Eq. (25) in the text we have:

$$\log C_t^i = c_t^i = \log\left(Y_t^i - G_t^i\right) - \alpha s_t^i$$

Using the fact that $\int_0^1 s_t^i di = 0$ and assuming a common (optimal) steady state in all countries we have:

$$\int_0^1 \log C_t^i di = \int_0^1 \log\left(Y_t^i - G_t^i\right) di \simeq \frac{1}{1-\chi}\int_0^1 \left(\tilde{y}_t^i - \chi\tilde{g}_t^i\right) di - \frac{1}{2}\frac{\chi}{(1-\chi)^2}\int_0^1 \left(\tilde{y}_t^i - \tilde{g}_t^i\right)^2 di$$

Taylor expansion of $\frac{N_t^{1+\varphi}}{1+\varphi}$

A second order Taylor expansion of the disutility of labor about a steady state is given by

$$
\frac{N_t^{1+\varphi}}{1+\varphi} \simeq \frac{N^{1+\varphi}}{1+\varphi} + N^{1+\varphi}\left(\frac{N_t - N}{N}\right) + \frac{\varphi}{2}N^{1+\varphi}\left(\frac{N_t - N}{N}\right)^2
$$

$$
= \frac{N_t^{1+\varphi}}{1+\varphi} + N^{1+\varphi}\left(\widehat{n}_t + \frac{1}{2}\widehat{n}_t^2\right) + \frac{\varphi}{2}N^{1+\varphi}\widehat{n}_t^2
$$

$$
= \frac{N_t^{1+\varphi}}{1+\varphi} + N^{1+\varphi}\widehat{n}_t + \frac{1}{2}N^{1+\varphi}(1+\varphi)\widehat{n}_t^2
$$

where $\widehat{n}_t \equiv \log\frac{N_t}{N}$. In the model in the text, the steady state about which the economy fluctuates under the optimal policy is given by $\bar{N} = 1$. Hence, we have

$$
\frac{N_t^{1+\varphi}}{1+\varphi} \simeq \widehat{n}_t + \frac{1}{2}(1+\varphi)\widehat{n}_t^2 + \text{tips}
$$

The next step consists in rewriting the previous expression in terms of the output gap. Using the fact that $N_t = \left(\frac{Y_t}{A_t}\right)\int_0^1\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}dj$, we have

$$
\widehat{n}_t = \widehat{y}_t - a_t + z_t = \widetilde{y}_t + z_t
$$

where $z_t \equiv \log\int_0^1\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}dj$, and where we use the fact that $\overline{y}_t = a_t$.

The following lemma shows that z_t is proportional to the cross-sectional distribution of relative prices (and, hence, of second order).

Lemma 1.

$$
z_t \simeq \frac{\epsilon}{2}var_j\{p_t(j)\}
$$

Proof. see Appendix B.

Using the previous results we can thus rewrite the second order approximation to the disutility of labor about that steady state in terms of the output gap and the price dispersion terms as:

$$
\frac{N_t^{1+\varphi}}{1+\varphi} \simeq \widetilde{y}_t + z_t + \frac{1}{2}(1+\varphi)\widetilde{y}_t^2 + \text{tips}
$$

Collecting results and reintroducing country subscripts, we can write the second order approximation to aggregate welfare in the monetary union as follows:

$$
U_t \equiv \int_0^1 U\left(C_t^i, G_t^i, N_t^i\right)di
$$

$$
= (1-\chi)\int_0^1 \log C_t^i di + \chi\int_0^1 \log G_t^i di - \int_0^1 \frac{(N_t^i)^{1+\varphi}}{1+\varphi}di
$$

$$
\simeq \int_0^1 \left(\widetilde{y}_t^i - \chi\widetilde{g}_t^i\right)di - \frac{1}{2}\frac{\chi}{(1-\chi)}\int_0^1 \left(\widetilde{g}_t^i - \widetilde{y}_t^i\right)^2 di
$$

$$
+\chi\int_0^1 \widetilde{g}_t^i di - \int_0^1 \left(\widetilde{y}_t^i + z_t^i + \frac{1}{2}(1+\varphi)\left(\widetilde{y}_t^i\right)^2\right)di + \text{tips}
$$

$$
= -\int_0^1 \left(z_t^i + \frac{1}{2}(1+\varphi)\left(\widetilde{y}_t^i\right)^2 + \frac{1}{2}\frac{\chi}{1-\chi}\left(\widetilde{g}_t^i - \widetilde{y}_t^i\right)^2\right)di + \text{tips}
$$

In order to express utility in terms of inflation we make use of the following Lemma:

Lemma 2.

$$
\frac{1}{2}\sum_{t=0}^{\infty}\beta^t z_t^i = \frac{\epsilon}{\lambda}\sum_{t=0}^{\infty}\beta^t\left(\pi_t^i\right)^2
$$

Proof. see Appendix B.

Now we can write the discounted sum of utilities across households as:

$$
\mathbb{W}_t = \int_0^1 \sum_{t=0}^{\infty}\beta^t U\left(C_t^i, G_t^i, N_t^i\right)di \simeq -\frac{1}{2}\sum_{t=0}^{\infty}\beta^t \int_0^1 \left(\frac{\varepsilon}{\lambda}\left(\pi_t^i\right)^2 + (1+\varphi)\left(\widetilde{y}_t^i\right)^2 + \frac{\chi}{1-\chi}\left(\widetilde{g}_t^i - \widetilde{y}_t^i\right)^2\right)di
$$

Appendix B. Proofs of Lemmas 1 and 2

Lemma 1.

$$z_t \simeq \frac{\epsilon}{2} var_j\{p_t(j)\}$$

Proof. let $\hat{p}_t(j) \equiv p_t(j) - p_t$. Notice that,

$$\left(\frac{P_t(j)}{P_t}\right)^{1-\epsilon} = exp\big[(1-\epsilon)\hat{p}_t(j)\big] \simeq 1 + (1-\epsilon)\hat{p}_t(j) + \frac{(1-\epsilon)^2}{2}\hat{p}_t(j)^2$$

Furthermore, from the definition of P_t, we have $1 = \int_0^1 \left(\frac{P_t(i)}{P_t}\right)^{1-\epsilon} di$. Hence, it follows that

$$E_j\{\hat{p}_t(j)\} = \frac{(\epsilon-1)}{2} E_j\left\{\hat{p}_t(j)^2\right\}$$

In addition, a second order approximation to $\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon}$, yields:

$$\left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} \simeq 1 - \epsilon\hat{p}_t(j) + \frac{\epsilon^2}{2}\hat{p}_t(j)^2$$

Combining the two previous results, it follows that

$$\int_0^1 \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon} di = 1 + \frac{\epsilon}{2}E_j\left\{\hat{p}_t(j)^2\right\} = 1 + \frac{\epsilon}{2} var_j\{p_t(j)\}$$

from which it follows that $z_t \simeq \frac{\epsilon}{2} var_j\{p_t(j)\}$

Lemma 2.

$$\sum_{t=0}^{\infty} \beta^t z_t = \frac{1}{2}\frac{\epsilon}{\lambda} \sum_{t=0}^{\infty} \beta^t \pi_t^2$$

Proof. we make use of the following property of the Calvo model, as shown in Woodford (2001, NBER wp 8071):

$$\sum_{t=0}^{\infty} \beta^t var_j\{p_t(j)\} = \frac{1}{\lambda}\sum_{t=0}^{\infty} \beta^t \pi_t^2$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta}$, as in the text. The desired result follows trivially from Lemma 1.

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